## Motion <br> Derivatives and Integrals Series

Instructor's Guide

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Developed by the Teaching and Learning Laboratory at MIT for the Singapore University of Technology and Design

## Introduction

## When to Use this Video

- In Phys 101, during Lecture 8 or in a discussion section connected with that lecture.
- Students should be familiar with linear and parabolic motion, how to handle polar-to-cartestian transformations (though not necessarily rotational motion), and introductory calculus through the chain rule.


## Learning Objectives

After watching this video students will be able to:

- Explain what is meant by the phrase "rigid body."


## Key Information

Duration: 11:10
Narrator: Prof. Dan Frey
Materials Needed:

- Paper
- Pencil
- A computer algebra system, such as Maple or Mathematica, is recommended but not required
- Describe restrictions on the motion of a object by using equations of constraint.
- Use derivatives and integrals to connect different measurements of motion.


## Motivation

- Most videos on motion concentrate on straight-line motion, or two-dimensional linear motion. It is unusual to find a video that deals with angular motion. This video attempts to bridge the gap between linear and angular motion.
- Many SUTD students will be engineers, and may be interested in the design and construction of devices such as robots. Using this context may help to make the video more accessible than one focused on projectiles or other typical physics topics.


## Student Experience

It is highly recommended that the video is paused when prompted so that students are able to attempt the activities on their own and then check their solutions against the video.

During the video, students will:

- Determine the position of a robot gripper from measurements of the robot's components.
- Use derivatives to obtain velocity and position from the above information.
- Find and apply equations of constraint to solve problems.


## Video Highlights

This table outlines a collection of activities and important ideas from the video.

| Time | Feature | Comments |
| :--- | :--- | :--- |
| $0: 50$ | Instructor's introduction | This section describes the prerequisites and the <br> learning objectives of the video. |
| $1: 33$ | Introduction to robotics | A clip from a robotics competition takes up most <br> of this segment. |
| $2: 48$ | Rigid bodies defined | A good example is shown at 4:00. A counter- <br> example is shown at 4:19, with a robot arm that <br> vibrates and flexes as it moves. |
| $4: 04$ | First robot arm problem | This is an extended example problem involving a <br> robot arm that starts at 4:30 and goes to 6:17. The <br> solution is given in the video. |
| $6: 20$ | Constraints and equations of constraint <br> defined | An example of a car on a roller coaster is given <br> starting at 7:36. |
| $8: 03$ | Find the constraint | This is a simple problem in which students find <br> the equation of constraint for the wheel on a cart. |
| $8: 41$ | Second robot arm problem | This is another extended example problem. <br> The robot arm in this case is substantially more <br> complex, and the question focuses on equations <br> of constraint. Solutions are not given. Possible <br> answers should be discussed in class. |
| $10: 27$ | Review |  |

## Video Summary

This video uses robotics as a context for describing rigid body motion and equations of constraint. Illustrative video clips are drawn from the robotics competition in MIT's 2.007 "Design and Manufacturing" course. The video describes both linear and angular motion.

## Phys 101 Materials

## Pre-Video Materials

When appropriate, this guide is accompanied by additional materials to aid in the delivery of some of the following activities and discussions.


## 1. Angular/Linear Worksheet (Appendix A1)

This worksheet has a few simple problems to prime students for the video. Each relates to the transition between Cartesian coordinates and polar coordinates, which plays a big part in the student-active part of the video. This worksheet should ideally be completed immediately prior to watching the video, preferably in-class before the video is shown or as an addition to homework the night before.

Some students may have difficulty with \#3 if they try to use polar coordinates; encourage them to use Cartesian coordinates.


## 2. Acceleration (Appendix A2-1 and A2-2)

This is a matched pair of clicker questions. The intended use is to start with A2-1, collect responses, and then move on to A2-2 before discussing the answer. Students are likely to ask what the difference is between the two questions; go back and forth to show them the differences. Some students may have missed the word "angular"; others may be confused about the difference between the two types of acceleration. This is an excellent pair of problems for probing students' understanding of angular acceleration.

3. Graphs, (Appendix A3-1 and A3-2)

This is another matched pair of clicker questions, intended for use one after another. If the students read carefully, understand the use of derivatives and integrals in basic motion, and are comfortable choosing "none of the above" answers (for the first problem), they should have little difficult with this.

## Post-Video Materials



## 1. Forklift (Appendix A4)

One of the robots in the video does not use a gripper; instead, it "forks" the toy police car and carries it into place like a forklift. The slide has an illustration. Students are asked to determine the maximum acceleration that the robot can obtain before the car falls off the forks. This should be done in symbolic form. Students can work on the problem in small groups and then return to the whole class for a discussion.

If your students complete this task quickly, ask them to take the extra step of relating this acceleration to the rotation of the robot arm (both vertical and horizontal).
2. Thirsty robot (Appendix A5)

This is a discussion question, suitable for use in class or in a separate discussion section. As with the simple robot in the video, this arm can pivot at the joint, and extend its arm. If students have difficulty starting this problem, you may wish to give them the list of quantities below and have them consider each in turn. They should also consider the angular equivalents to each. Discussion should focus on why students believe each value does or does not need to be constrained.
(a) Position
(b) Velocity
(c) Acceleration
(d) Jerk (the time derivative of acceleration)

Students may not have heard of jerk before; introducing the concept will be useful as it occasionally appears in later engineering courses.

## Additional Resources

## Going Further

The Lagrangian approach to mechanics handles equations of constraint particularly well. Learning Lagrangian mechanics can be an excellent goal for students who are ready for a more mathematically sophisticated method. The MIT course 16.07, Dynamics, includes lecture notes, assignments, and exams on MIT's OpenCourseWare site. The section on Lagrangian dynamics begins in lecture 20. Course 2.003J likewise includes material on the Lagrangian, beginning with lecture 13.

- Sheila Widnall, S., Deyst, J., \& Greitzer, E.16.07 Dynamics, Fall 2009. (Massachusetts Institute of Technology: MIT OpenCouseWare), http://ocw.mit.edu (Accessed March 5th 2012). License: Creative Commons BY-NC-SA
- Hadjiconstantinou, N., So, P., Sarma, S., \& Peacock, T. 2.003J Dynamics and Control I, Spring 2007. (Massachusetts Institute of Technology: MIT OpenCouseWare), http://ocw. mit.edu (Accessed March 5th 2012). License: Creative Commons BY-NC-SA


## References

Introductory mechanics has yielded a rich vein of physics education research. The first reference is a resource letter that contains dozens of further resources. Later items were chosen for their specific relevance to either the video or the supplementary materials.

- McDermott, L. C., and Redish, E. F. (1999). Resource letter PER-1. Phys. Educ. Res. Am. J. Phys. 67(9), 755-76
- Clement, J (1982). Students' Preconceptions in Introductory Mechanics. Am. J. Phys 50(1), 66-71
- McDermott, L. C., Rosenquist, M. L., and van Zee, E. H. (1987). Student Difficulty in Connecting Graphs and Physics: Examples from Kinematics. Am. J. Phys. 55(6), 503-513

For those interested in pursuing non-rigid motion, the following textbook uses an approach that considers intermolecular and interatomic forces very early on. Such approaches have been beneficial in building students' mental models of how objects respond to shear, torsion, tension, and the like.

- Chabay, R., and Sherwood, B. (2010). Matter and Interactions. Hoboken, NJ: Wiley.


1. Write the location of these two
points in polar coordinates $(r, \theta)$.
2. Write the equation for the line
connecting these points in polar
coordinates.
3. Write the equations describing the
motion of a point that travels along this
line, starting at $(-1,4)$ at time $t=0$ and
arriving at $(3,2)$ at time $t=10$. You may
use the coordinate system of your
choice.
The mass below is moving in a circle, with $\theta$ increasing
at a constant rate. Describe its angular acceleration.


The mass below is moving in a circle, with $\theta$ increasing
at a constant rate. Describe its linear acceleration.





The robot in this picture uses a "forklift" style arm. Determine the maximum acceleration it can achieve without losing the police car it is carrying.


A robot arm is shown below from the top and
side. The robot carries a cup of water. What
values would we have to constrain in order to
prevent the robot from spilling its water?


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