DESIGN OF IIR DIGITAL FILTERS - PART 2

Solution 15.1

In the absence of aliasing, the transformation from analog to digital frequency response corresponding to impulse invariance is

$$H(e^{j\omega}) = \frac{1}{T} H_a\left(\frac{j\omega}{T}\right) \qquad |\omega| \leq \pi$$

Thus, in addition to a scale factor of 1/T there is a linear mapping between analog and digital frequency given by

$$\omega = \Omega T \qquad |\omega| \leq \pi.$$

The desired analog frequency response can be obtained by reflecting the digital frequency response through this transformation, as indicated in Figure S15.1-1

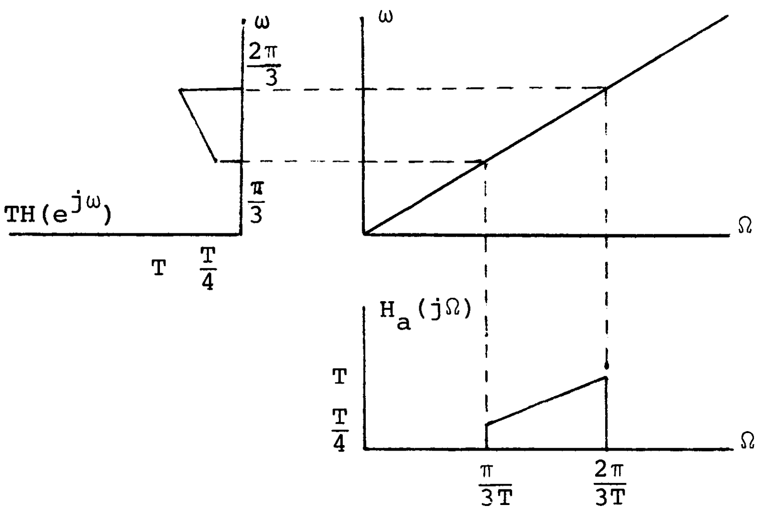


Figure S15.1-1

Note that since the frequency transformation between ω and Ω is linear, the shape of the frequency response is preserved.

(b) For the bilinear transformation the transformation between analog and digital frequency is given by

$$\omega = 2 \arctan\left(\frac{\Omega T}{2}\right).$$

Thus, as in (a) we obtain the corresponding analog frequency response by reflecting the digital frequency response through this transformation as indicated in Figure S15.1-2.

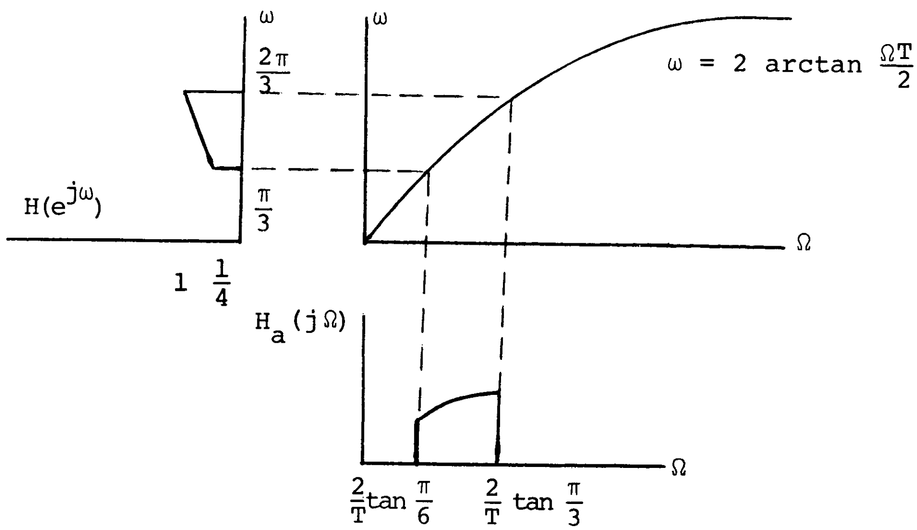


Figure S15.1-2

In this case, since the frequency transformation between ω and Ω is not linear, the linear slope of the digital frequency response does not correspond to a linear slope in the analog frequency response.

Solution 15.2

Figures S15.2-1 and S15.2-2 show the result of reflecting this frequency response characteristic through the frequency transformation for impulse invariance and the bilinear transformation respectively. In this case, since the digital frequency response is piecewise constant, its shape is preserved in the analog frequency response for both cases.

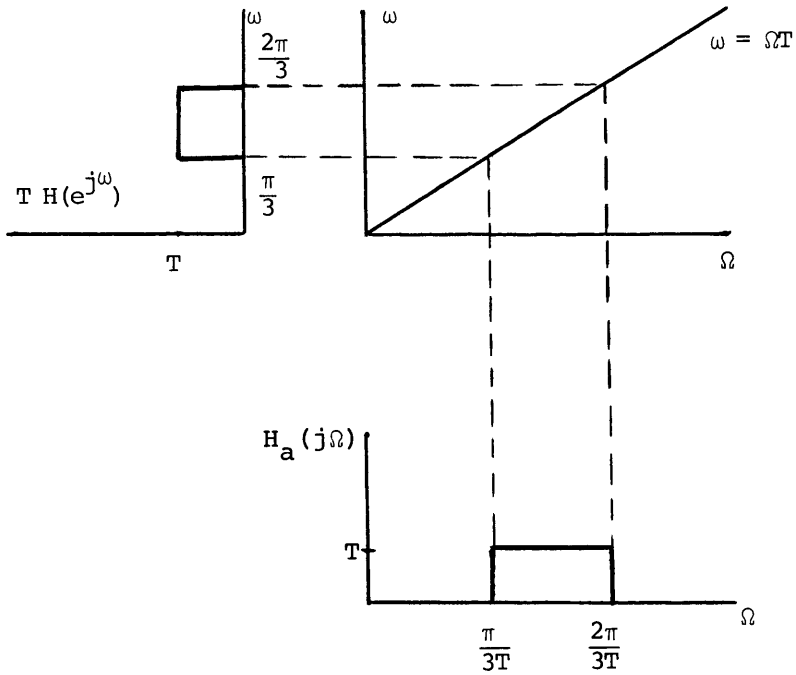


Figure S15.2-1

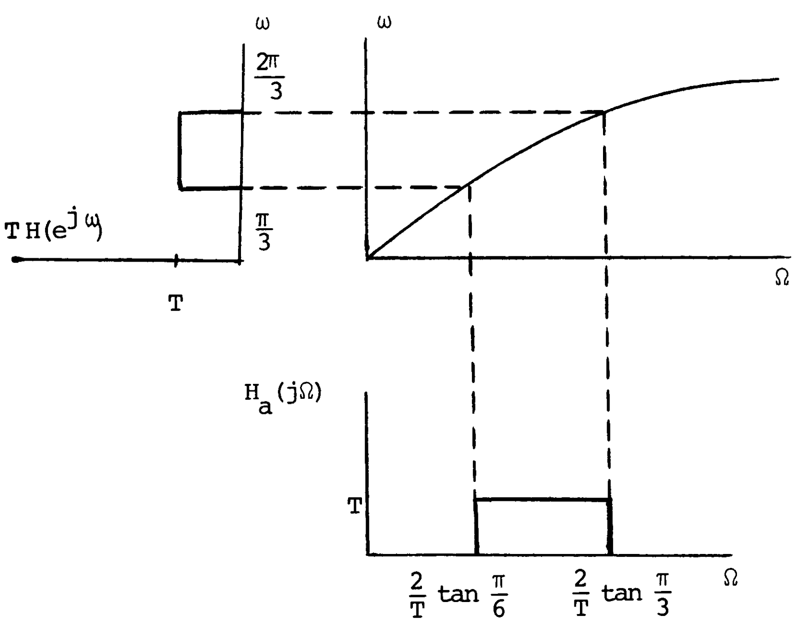


Figure S15.2-2

Solution 15.3

Since the filter is to be designed using the bilinear transformation with $T = 1$, the relation between analog and digital frequency is

$$\Omega = 2\tan\frac{\omega}{2} .$$

Thus we require that:

(i) $(.99) < |H_a(j\Omega)| \le 1$ for $0 \le \Omega < 2\tan\frac{\pi}{16}$

and

(ii) $|H_a(j\Omega)| < .001$ for $2\tan\frac{\pi}{12} < \Omega$

Solution 15.4*

Since $G_a(s)$ is an analog low pass filter, $G(z)$ given by

$$G(z) = G_a\left[\frac{z-1}{z+1}\right]$$

is a digital lowpass filter. Also, since $H_a(s) = G_a(1/s)$ is an analog highpass filter,

$$H(z) = H_a\left[\frac{z-1}{z+1}\right] = G_a\left[\frac{z+1}{z-1}\right] .$$

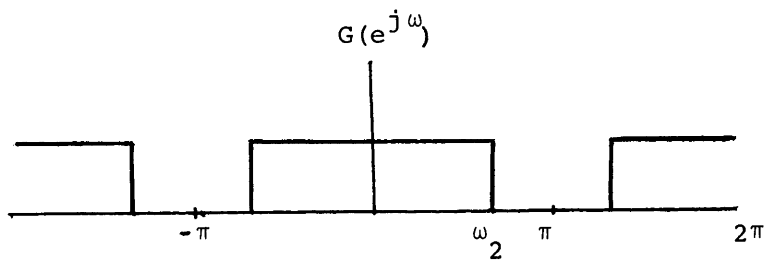
Comparing $H(z)$ with $G(z)$ we observe that

$$H(z) = G(-z).$$

Thus, multiplying the output of each delay by -1 will convert a digital lowpass filter to a digital highpass filter. To determine the relationship between the cutoff frequencies, since $e^{j\pi}=-1$,

$$H[e^{j\omega}] = G[e^{j\pi}e^{j\omega}] = G[e^{j(\pi+\omega)}].$$

Thus the frequency response of the highpass filter is equal to that of the lowpass filter, shifted by π as illustrated in Figure S15.4-1.



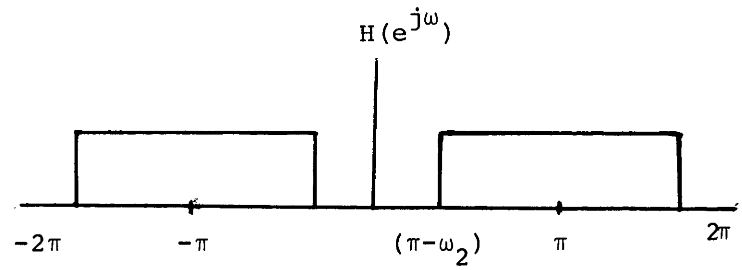


Figure S15.4-1

The cutoff frequency of the high pass filter is $(\pi-\omega_2)$ and thus with $\omega_2 = \frac{\pi}{2}$, ω_H is also $\frac{\pi}{2}$ as desired.

Finally, to determine how to modify each of the coefficients we note that multiplying the output of each delay by -1 is equivalent to changing the sign of all coefficient branches whose input has passed through an odd number of delays. Thus the coefficients A,C and the two coefficients of 2 are multiplied by -1.

Solution 15.5

The coefficient b_0 is chosen to be $h_d(0) = 1$. The coefficients a_1 and a_2 are obtained by solving the equations

$$a_1\phi(1,1) + a_2\phi(1,2) = \phi(1,0)$$

$$a_1\phi(2,1) + a_2\phi(2,2) = \phi(2,0)$$

where

$$\phi(i,r) = \sum_{n=1}^{\infty} h_d(n-r)\, h_d(n-i)$$

For the specified desired unit-sample response

$\phi(1,0) = 9$

$\phi(2,0) = 8$

$\phi(1,1) = 10$

$\phi(1,2) = 9$

$\phi(2,1) = 9$

$\phi(2,2) = 10$

Thus

$10a_1 + 9a_2 = 9$

$9a_1 + 10a_2 = 8$

Solving these equations, we obtain

$a_1 = \frac{18}{19}$

$a_2 = \frac{-1}{19}$.

Resource: Digital Signal Processing
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