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Electromechanical Dynamics

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Appendix F

GLOSSARY OF COMMONLY USED SYMBOLS

Section references indicate where symbols of a given significance are introduced; grouped symbols are accompanied by their respective references. The absence of a section reference indicates that a symbol has been applied for a variety of purposes. Nomenclature used in examples is not included.

Symbol	Meaning	Section
A	cross-sectional area	
A_i	coefficient in differential equation	5.1.1
(A_n^+, A_n^-)	complex amplitudes of components of n th mode	9.2.1
A_w	cross-sectional area of armature conductor	6.4.1
a	spacing of pole faces in magnetic circuit	8.5.1
$a, (a_c, a_s)$	phase velocity of acoustic related waves	13.2.1, 11.4.1
a_b	Alfvén velocity	12.2.3
(a, b, c)	Lagrangian coordinates	11.1
a_i	constant coefficient in differential equation	5.1.1
a_p	instantaneous acceleration of point p fixed in material	2.2.1c
B, B_r, B_s	damping constant for linear, angular and square law dampers	2.2.1b, 4.1.1, 5.2.2
B, B_i, B_0	magnetic flux density	1.1.1a, 8.1, 6.4.2
B_i	induced flux density	7.0
$(B_r, B_{ra}, B_{rb}, B_{rm})$	radial components of air-gap flux densities	4.1.4
$[B_{rf}, (B_{rf})_{av}]$	radial flux density due to field current	6.4.1
b	width of pole faces in magnetic circuit	8.5
b	half thickness of thin beam	11.4.2b
C	contour of integration	1.1.2a
$C, (C_a, C_b), C_0$	capacitance	2.1.2, 7.2.1a, 5.2.1
C	coefficient in boundary condition	9.1.1
C	the curl of the displacement	11.4
(C^+, C^-)	designation of characteristic lines	9.1.1

Symbol	Meaning	Section
c_p	specific heat capacity at constant pressure	13.1.2
c_v	specific heat capacity at constant volume	13.1.2
D	electric displacement	1.1.1a
d	length	
da	elemental area	1.1.2a
$d\mathbf{f}_n$	total elemental force on material in rigid body	2.2.1c
$d\mathbf{l}$	elemental line segment	1.1.2a
$d\mathbf{T}_n$	torque on elemental volume of material	2.2.1c
dV	elemental volume	1.1.2b
E	constant of motion	5.2.1
E	Young's modulus or the modulus of elasticity	9.1
\mathbf{E}, E_0	electric field intensity	1.1.1a, 5.1.2d
E_f	magnitude of armature voltage generated by field current in a synchronous machine	4.1.6a
E_i	induced electric field intensity	7.0
e_{11}, e_{ij}	strain tensor	9.1, 11.2
\dot{e}_{ij}	strain-rate tensor	14.1.1a
F	magnetomotive force (mmf)	13.2.2
F	force density	1.1.1a
\hat{F}	complex amplitude of $f(t)$	5.1.1
F_0	amplitude of sinusoidal driving force	9.1.3
f	equilibrium tension of string	9.2
f	driving function	5.1.1
$f, \mathbf{f}, f^e, f^s, f_j, f_i, f_1$	force	2.2.1, 2.2.1c, 3.1, 5.1.2a, 3.1.2b, 8.1, 9.1
f	arbitrary scalar function	6.1
f'	scalar function in moving coordinate system	6.1
f	three-dimensional surface	6.2
f	integration constant	11.4.2a
G	a constant	5.1.2c
G	shear modulus of elasticity	11.2.2
G	speed coefficient	6.4.1
G	conductance	3.1
g	air-gap length	5.2.1
g, \mathbf{g}	acceleration of gravity	5.1.2c, 12.1.3
$(\mathbf{H}, H_x, H_y, H_z)$	magnetic field intensity	1.1.1a
h	specific enthalpy	13.1.2
$\mathbf{I}, I, (I_r, I_\theta), I_f$	electrical current	10.4.3, 12.2.1a, 4.1.2, 6.4.1
$(i, i_1, i_2, \dots, i_k), (i_{ar}, i_{as}, i_{br}, i_{bs}), i_a, (i_a, i_b, i_c), (i_f, i_d), (i_r, i_\theta)$	electrical current	2.1, 4.1.3, 6.4.1, 4.1.7, 6.4.1, 4.1

Symbol	Meaning	Section
\mathbf{i}_n	unit vector perpendicular to area of integration	6.2.1
\mathbf{i}_s	unit vector normal to surface of integration	6.2.1
$(\mathbf{i}_x, \mathbf{i}_y, \mathbf{i}_z), (\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3)$	unit vectors in coordinate directions	2.2.1c
J, J_f	current density	7.0, 1.1.1a
$J, J_r, (J_x, J_y, J_z)$	moment of inertia	5.1.2b, 4.1.1, 2.2.1c
J_{xx}, J_{yz}	products of inertia	2.2.1c
j	$\sqrt{-1}$	4.1.6a
K	loading factor	13.2.2
K, \mathbf{K}_f	surface current density	7.0, 1.1.1a
K	linear or torsional spring constant	2.2.1a
K_i	induced surface current density	7.0
$k, k_o, (k_r, k_i)$	wavenumber	7.1.3, 10.1.3, 10.0
k	summation index	2.1.1
k	maximum coefficient of coupling	4.1.6b
k_n	n th eigenvalue	9.2
$(L, L_1, L_2), (L_a, L_f), L_m, (L_0, L_2), (L_T, L_s, L_{sr}), L_{ss}$	inductance	2.1.1, 6.4.1, 2.1.1, 4.2.1, 4.1.1, 4.2.4
L	length of incremental line segment	6.2.1
l	value of relative displacement for which spring force is zero	2.2.1a
l, l_w, l_y	length	
M	Hartmann number	14.2.2
M	mass of one mole of gas in kilograms	13.1.2
M	Mach number	13.2.1
M	mass	2.2.1c
M	number of mechanical terminal pairs	2.1.1
M, M_s	mutual inductance	4.1.1, 4.2.4
M	magnetization density	1.1.1a
m	mass/unit length of string	9.2
N	number of electrical terminal pairs	2.1.1
N	number of turns	5.2.2
n	number density of ions	12.3.1
n	integer	7.1.1
\mathbf{n}	unit normal vector	1.1.2
\mathbf{P}	polarization density	1.1.1a
P	power	12.2.1a
p	number of pole pairs in a machine	4.1.8
p	power per unit area	14.2.1
p	pressure	5.1.2d and 12.1.4
P_o, P_o, P_m, P_r	power	4.1.6a, 4.1.6b, 4.1.2, 4.1.6b
Q	electric charge	7.2.1a
q, q_i, q_k	electric charge	1.1.3 and 2.1.2, 8.1, 2.1.2
R, R_i, R_o	radius	

Symbol	Meaning	Section
$R, R_a, R_b, R_f, R_r, R_s$	resistance	
(R, R_g)	gas constant	13.1.2
R_e	electric Reynolds number	7.0
R_m	magnetic Reynolds number	7.0
r	radial coordinate	
\mathbf{r}	position vector of material	2.2.1c
\mathbf{r}'	position vector in moving reference frame	6.1
r_m	center of mass of rigid body	2.2.1c
S	reciprocal modulus of elasticity	11.5.2c
S	surface of integration	1.1.2a
S	normalized frequency	7.2.4
S	membrane tension	9.2
S_z	transverse force/unit length acting on string	9.2
s	complex frequency	5.1.1
(s, s_{mT})	slip	4.1.6b
s_i	i th root of characteristic equation, a natural frequency	5.1.1
T	period of oscillation	5.2.1
T	temperature	13.1.2
$T, T, T^e, T_{em}, T_m, T_0, T_1$	torque	2.2.1c, 5.1.2b, 3.1.1, 4.1.6b, 4.1.1, 6.4.1, 6.4.1
\mathbf{T}	surface force	8.4
T_{ij}^m	mechanical stress tensor	13.1.2
$T_{m\pi}$	the component of the stress-tensor in the m th-direction on a cartesian surface with a normal vector in the π th-direction	8.1
T_{or}	constant of coulomb damping	4.1.1
T_o	initial stress distribution on thin rod	9.1.1
T	longitudinal stress on a thin rod	9.1.1
T_z	transverse force per unit area on membrane	9.2
T_2	transverse force per unit area acting on thin beam	11.4.2b
t	time	1.1.1
t'	time measured in moving reference frame	6.1
U	gravitational potential	12.1.3
U	longitudinal steady velocity of string or membrane	10.2
u	internal energy per unit mass	13.1.1
u	surface coordinate	11.3
$u_0(x - x_0)$	unit impulse at $x = x_0$	9.2.1
u	transverse deflection of wire in x -direction	10.4.3
$u_{-1}(t)$	unit step occurring at $t = 0$	5.1.2b
V, V_m	velocity	7.0, 13.2.3
V	volume	1.1.2
V, V_a, V_f, V_o, V_s	voltage	
V	potential energy	5.2.1

Symbol	Meaning	Section
v, \mathbf{v}	velocity	
(v, v_1, \dots, v_k)	voltage	2.1.1
$v', (v_a, v_b, v_c),$ v_f, v_{oc}, v_t	voltage	
v_n	velocity of surface in normal direction	6.2.1
v_o	initial velocity distribution on thin rod	9.1.1
v_p	phase velocity	9.1.1 and 10.2
\mathbf{v}^r	relative velocity of inertial reference frames	6.1
v_s	$\sqrt{f/m}$ for a string under tension f and having mass/unit length m	10.1.1
v	longitudinal material velocity on thin rod	9.1.1
v	transverse deflection of wire in y -direction	10.4.3
(W_e, W_m)	energy stored in electromechanical coupling	3.1.1
(W'_e, W'_m, W')	coenergy stored in electromechanical coupling	3.1.2b
W''	hybrid energy function	5.2.1
w	width	5.2.2
w	energy density	11.5.2c
w'	coenergy density	8.5
X	equilibrium position	5.1.2a
$(x, x_1, x_2, \dots, x_k)$	displacement of mechanical node	2.1.1
x	dependent variable	5.1.1
x_p	particular solution of differential equation	5.1.1
$(x_1, x_2, x_3), (x, y, z)$	cartesian coordinates	8.1, 6.1
(x', y', z')	cartesian coordinates of moving frame	6.1
(α, β)	constants along C^+ and C^- characteristics, respectively	9.1.1
(α, β)	see (10.2.20) or (10.2.27)	
α	transverse wavenumber	11.4.3
(α, β)	angles used to define shear strain	11.2
(α, β)	constant angles	4.1.6b
α	space decay parameter	7.1.4
α	damping constant	5.1.2b
α	equilibrium angle of torsional spring	2.2.1a
γ	ratio of specific heats	13.1.2
γ	piezoelectric constant	11.5.2c
$\gamma, \gamma_0, \gamma'$	angular position	
$\Delta_d(t)$	slope excitation of string	10.2.1b
Δ_0	amplitude of sinusoidal slope excitation	10.2.1b
Δr	distance between unstressed material points	11.2.1a
Δs	distance between stressed positions of material points	11.2.1a
$\delta()$	incremental change in ()	8.5
$\delta, \delta_1, \delta_0$	displacement of elastic material	11.1, 9.1, 11.4.2a
δ	thickness of incremental volume element	6.2.1
δ	torque angle	4.1.6a

Symbol	Meaning	Section
δ_{ij}	Kronecker delta	8.1
(δ_+, δ_-)	wave components traveling in the $\pm x$ -directions	9.1.1
ϵ	linear permittivity	1.1.1b
ϵ_0	permittivity of free space	1.1.1a
η	efficiency of an induction motor	4.1.6b
η	second coefficient of viscosity	14.1.1c
$\theta, \theta_i, \theta_m$	angular displacement	2.1.1, 3.1.1, 5.2.1
θ	power factor angle; phase angle between current and voltage	4.1.6a
θ	equilibrium angle	5.2.1
$\dot{\theta}$	angular velocity of armature	6.4.1
θ_m	maximum angular deflection	5.2.1
$(\lambda, \lambda_1, \lambda_2, \dots, \lambda_k)$	magnetic flux linkage	2.1.1, 6.4.1, 4.1.7,
λ_a		4.1.3, 4.1
$(\lambda_a, \lambda_b, \lambda_c)$		
$(\lambda_{aT}, \lambda_{aS}, \lambda_{bT}, \lambda_{bS})$		
(λ_T, λ_S)		
λ	Lamé constant for elastic material	11.2.3
λ	wavelength	7.1.4
μ	linear permeability	1.1.1a
$\mu, (\mu_+, \mu_-)$	mobility	12.3.1, 1.1.1b
μ	coefficient of viscosity	14.1.1
μ_d	coefficient of dynamic friction	2.2.1b
μ_0	permeability of free space	1.1.1a
μ_s	coefficient of static friction	2.2.1b
ν	Poisson's ratio for elastic material	11.2.2
ν	damping frequency	10.1.4
(ξ, ξ)	continuum displacement	8.5
ξ_0	initial deflection of string	9.2
ξ_d	amplitude of sinusoidal driving deflection	9.2
$(\xi_n(x), \hat{\xi}_n(x))$	n th eigenfunctions	9.2.1b
(ξ_+, ξ_-)	amplitudes of forward and backward traveling waves	9.2
$\dot{\xi}_0(x)$	initial velocity of string	9.2
ρ	mass density	2.2.1c
ρ_f	free charge density	1.1.1a
ρ_s	surface mass density	11.3
Σ	surface of discontinuity	6.2
σ	conductivity	1.1.1a
σ_f	free surface charge density	1.1.1a
σ_m	surface mass density of membrane	9.2
σ_0	surface charge density	7.2.3
σ_s	surface conductivity	1.1.1a
σ_u	surface charge density	7.2.3
τ	surface traction	8.2.1
τ, τ_d	diffusion time constant	7.1.1, 7.1.2a
τ	relaxation time	7.2.1a

Symbol	Meaning	Section
τ_e	electrical time constant	5.2.2
τ_m	time for air gap to close	5.2.2
τ_o	time constant	5.1.3
τ_t	traversal time	7.1.2a
ϕ	electric potential	7.2
ϕ	magnetic flux	2.1.1
ϕ	cylindrical coordinate	2.1.1
ϕ	potential for H when $J_f = 0$	8.5.2
ϕ	flow potential	12.2
χ_e	electric susceptibility	1.1.1b
χ_m	magnetic susceptibility	1.1.1a
ψ	the divergence of the material displacement	11.4
ψ	angle defined in Fig. 6.4.2	6.4.1
ψ	angular position in the air gap measured from stator winding (a) magnetic axis	4.1.4
ψ	electromagnetic force potential	12.2
ψ	angular deflection of wire	10.4.3
Ω	equilibrium rotational speed	5.1.2b
Ω	rotation vector in elastic material	11.2.1a
Ω_n	real part of eigenfrequency (10.1.47)	10.1.4
$\omega, (\omega_r, \omega_s)$	radian frequency of electrical excitation	4.1.6a, 4.1.2
ω	natural angular frequency (Im s)	5.1.2b
ω, ω_m	angular velocity	2.2.1c, 4.1.2
ω_c	cutoff frequency for evanescent waves	10.1.2
ω_d	driving frequency	9.2
ω_n	nth eigenfrequency	9.2
ω_o	natural angular frequency	5.1.3
(ω_r, ω_i)	real and imaginary parts of ω	10.0
∇	nabla	6.1
∇_Σ	surface divergence	6.2.1

1.8. B/c-k