Topic 20

Beam, Plate, and Shell Elements—Part II

Contents:

- **Formulation of isoparametric (degenerate) beam elements for large displacements and rotations**
- **A rectangular cross-section beam element of variable thickness; coordinate and displacement interpolations**
- **Use of the nodal director vectors**
- **The stress-strain law**
- **Introduction of warping displacements**
- **Example analysis: 180 degrees, large displacement twisting of a ring**
- **Example analysis: Torsion of an elastic-plastic cross-section**
- **Recommendations for the use of isoparametric beam and shell elements**
- **The phenomena of shear and membrane locking as observed for certain elements**
- **Study of solutions of straight and curved cantilevers modeled using various elements**
- **An effective 4-node shell element (the MITC4 element) for analysis of general shells**
- **The patch test, theoretical and practical considerations**
- **Example analysis: Solution of a three-dimensional spherical shell**
- **Example analysis: Solution of an open box**
- **Example analysis: Solution of a square plate, including use of distorted elements**
- **Example analysis: Solution of a 30-degree skew plate**
- **Example analysis: Large displacement solution of a cantilever**

Contents: (continued)

- **Example analysis: Collapse analysis of an I-beam in torsion**
- **Example analysis: Collapse analysis of a cylindrical shell**

Textbook: Sections 6.3.4, 6.3.5

Example: 6.18

References:

The displacement functions to account for warping in the rectangular cross-section beam are introduced in

Bathe, K. J., and A. Chaudhary, "On the Displacement Formulation of Torsion of Shafts with Rectangular Cross-Sections," *International Journal for Numerical Methods in Engineering, 18,* 1565–1568, 1982.

The 4-node and 8-node shell elements based on mixed interpolation (i.e., the MITC4 and MITC8 elements) are developed and discussed in

Dvorkin, E., and K. J. Bathe, "A Continuum Mechanics Based Four-Node Shell Element for General Nonlinear Analysis," *Engineering Computations, 1,* 77–88, 1984.

Bathe, K. J., and E. Dvorkin, "A Four-Node Plate Bending Element Based on Mindlin/Reissner Plate Theory and a Mixed Interpolation," *International Journal for Numerical Methods in Engineering, 21,* 367–383, 1985.

Bathe, K. J., and E. Dvorkin, "A Formulation of General Shell Elements—The Use of Mixed Interpolation of Tensorial Components," *International Journal for Numerical Methods in Engineering,* in press.

The I-beam analysis is reported in

Bathe, K. J., and P. M. Wiener, "On Elastic-Plastic Analysis of I-Beams in Bending and Torsion," *Computers & Structures, 17,* 711–718, 1983.

The beam formulation is extended to a pipe element, including ovalization effects, in

Bathe, K. J., C. A. Almeida, and L. W. Ho, "A Simple and Effective Pipe Elbow Element—Some Nonlinear Capabilities," *Computers & Structures, 17,* 659–667, 1983.

FORMULATION OF ISOPARAMETRIC (DEGENERATE) BEAM ELEMENTS

- The usual Hermitian beam elements (cubic transverse displacements, linear longitudinal displacements) are usually most effective in the linear analysis of beam structures.
- When in the following discussion we refer to a "beam element" we always mean the "isoparametric beam element."

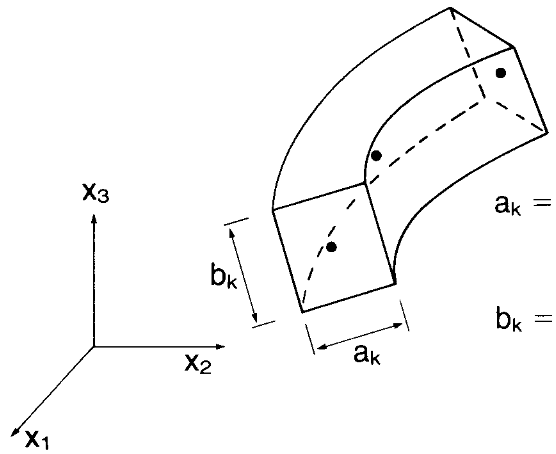
Transparency 20-1

- The isoparametric formulation can be effective for the analysis of
  - Curved beams
  - Geometrically nonlinear problems
  - Stiffened shell structures (isoparametric beam and shell elements are coupled compatibly)
- The formulation is analogous to the formulation of the isoparametric (degenerate) shell element.

Transparency 20-2

Transparency 20-3

Consider a beam element with a rectangular cross-section:

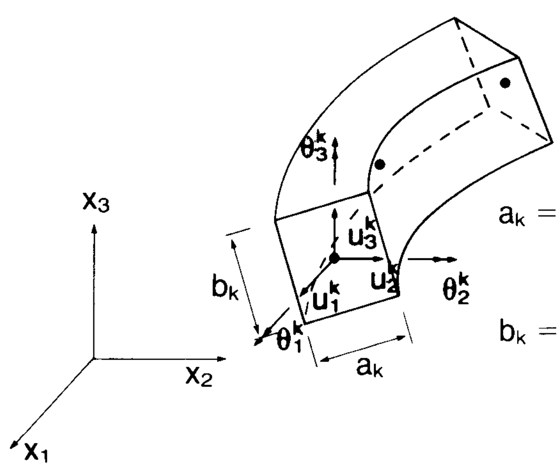


a_k = thickness at node k in t-direction

b_k = thickness at node k in s-direction

Transparency 20-4

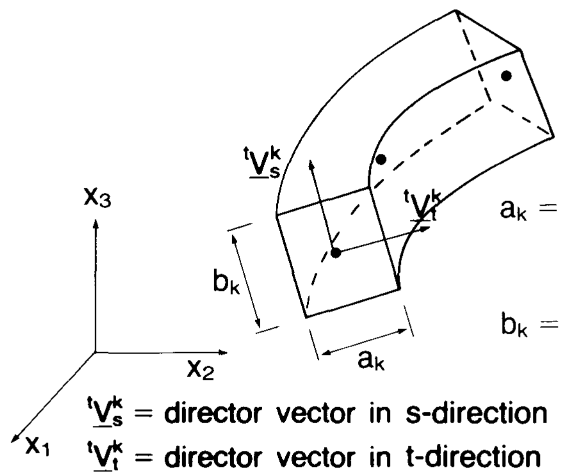
Consider a beam element with a rectangular cross-section:



a_k = thickness at node k in t-direction

b_k = thickness at node k in s-direction

Consider a beam element with a rectangular cross-section:



a_k = thickness at node k in t-direction

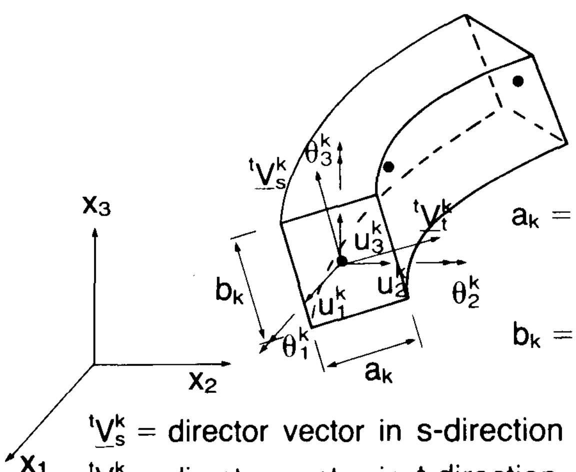
b_k = thickness at node k in s-direction

$^t\underline{V}_s^k$ = director vector in s-direction

$^t\underline{V}_t^k$ = director vector in t-direction

Transparency 20-5

Consider a beam element with a rectangular cross-section:



a_k = thickness at node k in t-direction

b_k = thickness at node k in s-direction

$^t\underline{V}_s^k$ = director vector in s-direction

$^t\underline{V}_t^k$ = director vector in t-direction

Transparency 20-6

Transparency 20-7

The coordinates of the material particles of the beam are interpolated as

$$ {}^{t}x_i = \sum_{k=1}^{N} h_k \, {}^{t}x_i^k + \frac{t}{2} \sum_{k=1}^{N} a_k \, h_k \, {}^{t}V_{ti}^k + \frac{s}{2} \sum_{k=1}^{N} b_k \, h_k \, {}^{t}V_{si}^k $$

where

${}^{t}V_{ti}^k$ = direction cosines of the director vector in the t-direction, of node k at time t

${}^{t}V_{si}^k$ = direction cosines of the director vector in the s-direction, of node k at time t

Transparency 20-8

Since ${}^{t}u_i = {}^{t}x_i - {}^{0}x_i$, we have

$$ {}^{t}u_i = \sum_{k=1}^{N} h_k \, {}^{t}u_i^k + \frac{t}{2} \sum_{k=1}^{N} a_k \, h_k \, ({}^{t}V_{ti}^k - {}^{0}V_{ti}^k) + \frac{s}{2} \sum_{k=1}^{N} b_k \, h_k \, ({}^{t}V_{si}^k - {}^{0}V_{si}^k) $$

The vectors ${}^{0}\underline{V}_t^k$ and ${}^{0}\underline{V}_s^k$ can be calculated automatically from the initial geometry of the beam element if the element is assumed to lie initially in a plane.

Also

$$u_i = {}^{t+\Delta t}x_i - {}^{t}x_i$$

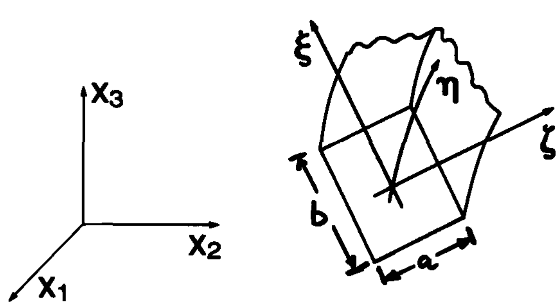
$$= \sum_{k=1}^{N} h_k u_i^k + \frac{t}{2}\sum_{k=1}^{N} a_k h_k V_{ti}^k + \frac{s}{2}\sum_{k=1}^{N} b_k h_k V_{si}^k$$

where V_{ti}^k and V_{si}^k are increments in the direction cosines of the vectors ${}^t\underline{V}_t^k$ and ${}^t\underline{V}_s^k$. These increments are given in terms of the incremental rotations $\underline{\theta}_k$, about the Cartesian axes, as

$$\underline{V}_t^k = \underline{\theta}_k \times {}^t\underline{V}_t^k \;;\; \underline{V}_s^k = \underline{\theta}_k \times {}^t\underline{V}_s^k$$

Transparency 20-9

- Using the above displacement and geometry interpolations, we can develop the strain-displacement matrices for the Cartesian strain components. A standard transformation yields the strain-displacement relations corresponding to the beam coordinates η, ξ, ζ.



Transparency 20-10

Transparency 20-11

- The stress-strain relationship used for linear elastic material conditions is

$$\underline{C}_{beam} = \begin{array}{c} \begin{array}{ccc} \eta\eta & \eta\xi & \eta\zeta \end{array} \longleftarrow \text{components} \\ \begin{bmatrix} E & 0 & 0 \\ 0 & Gk & 0 \\ 0 & 0 & Gk \end{bmatrix} \end{array}$$

k = shear correction factor

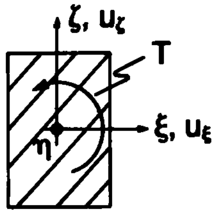
since only the one normal and two transverse shear stresses are assumed to exist.

Transparency 20-12

- The material stress-strain matrix for analysis of elasto-plasticity or creep would be obtained using also the condition that only the stress components $(\eta\eta)$, $(\eta\zeta)$ and $(\eta\xi)$ are non-zero.

Transparency 20-13

- Note that the kinematic assumptions in the beam element do not allow – so far – for cross-sectional out-of-plane displacements (warping). In torsional loading, allowing for warping is important.
- We therefore amend the displacement assumptions by the following displacements:



$$u_\eta = \alpha \underbrace{\xi\zeta}_{\substack{\text{exact warping}\\ \text{displacements}\\ \text{for infinitely}\\ \text{narrow section}}} + \beta \underbrace{\xi\zeta(\xi^2 - \zeta^2)}_{\substack{\text{exact warping}\\ \text{displacements}\\ \text{for square}\\ \text{section}}}$$

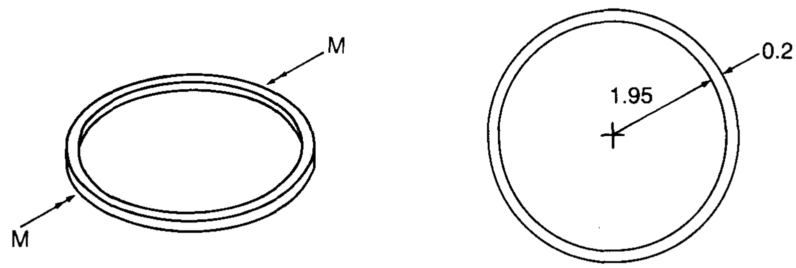
Transparency 20-14

Torsion constant k in formula, $T = k\,G\theta\,a^3 b$

$\frac{b}{a}$	k: Analytical value (Timoshenko)	k: ADINA
1·0	0·141	0·141
2·0	0·229	0·230
4·0	0·281	0·289
10·0	0·312	0·323
100·0	0·333	0·333

Transparency 20-15

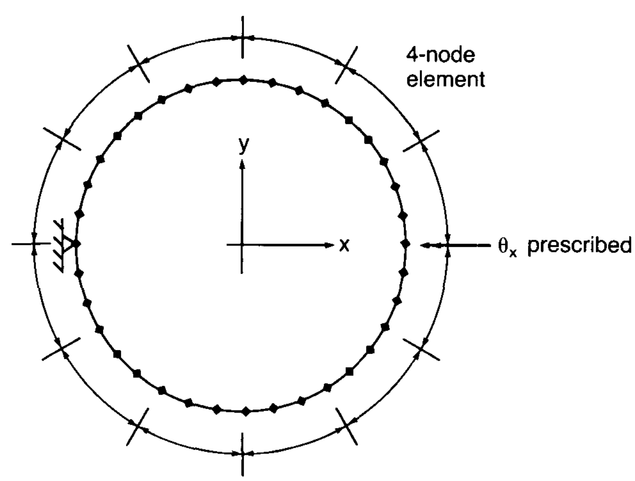
Example: Twisting of a ring

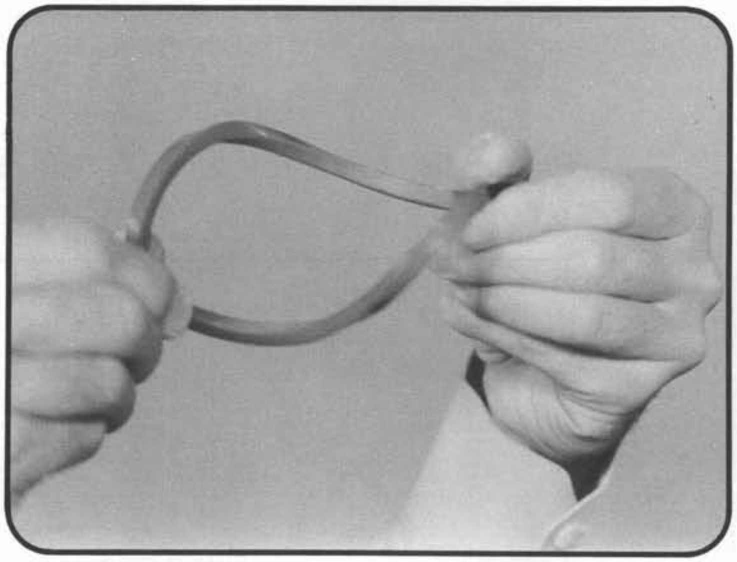


All dimensions in inches
thickness = 0.2
$E = 3 \times 10^5$ psi
$\nu = 0.3$

Transparency 20-16

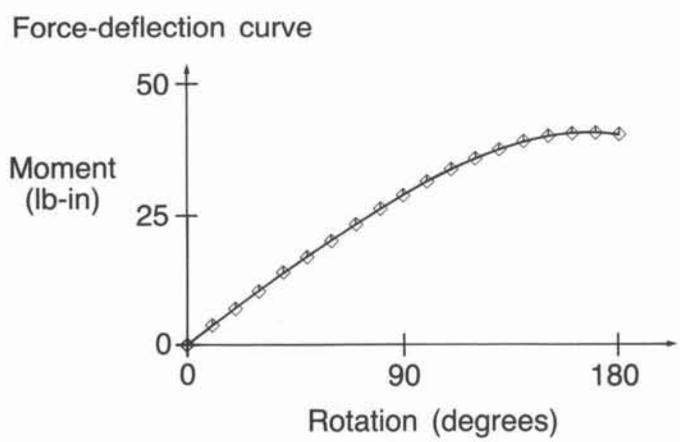
Finite element mesh: Twelve 4-node iso-beam elements





Demonstration Photograph 20-1
Close-up of ring deformations

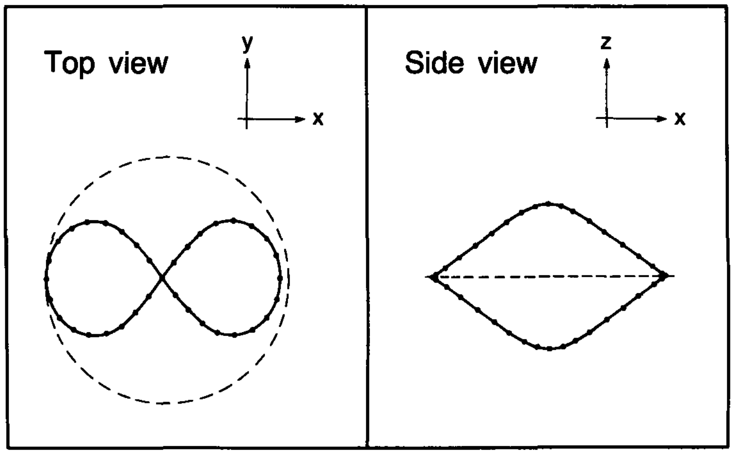
Use the T.L. formulation to rotate the ring 180 degrees:

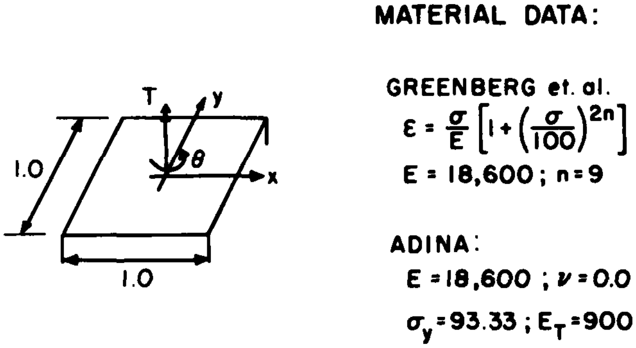


Transparency 20-17

Transparency 20-18

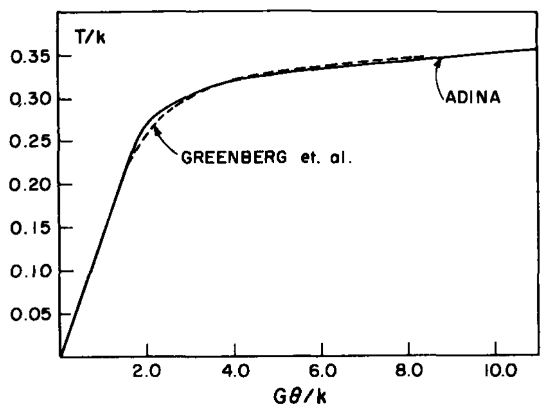
Pictorially, for a rotation of 180 degrees, we have





Elastic–plastic analysis of torsion problem

Slide 20-1



Solution of torsion problem
($k = 100/\sqrt{3}$, θ = rotation per unit length)

Slide 20-2

Transparency 20-19

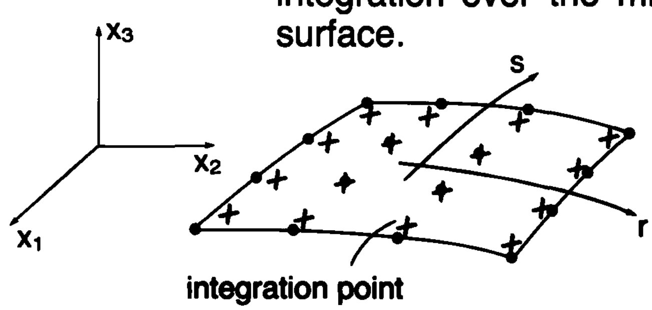
Use of the isoparametric beam and shell elements

- The elements can be programmed for use with different numbers of nodes
  - For the beam, 2, 3 or 4 nodes
  - For the shell, 4, 8, 9, ⋯, 16 nodes
- The elements can be employed for analysis of moderately thick structures (shear deformations are approximately taken into account).

Transparency 20-20

- The elements can be used for analysis of thin structures – but then only certain elements of those mentioned above should be used.

For shells: Use only the 16-node element with 4 × 4 Gauss integration over the mid-surface.



For beams:

Use 2-node beam element with 1-point Gauss integration along r-direction,

or

Use 3-node beam element with 2-point Gauss integration along r-direction,

or

Use 4-node beam element with 3-point Gauss integration along r-direction.

Transparency 20-21

The reason is that the other elements become overly (and artificially) stiff when used to model thin structures and curved structures.

Two phenomena occur:

- Shear locking
- Membrane locking

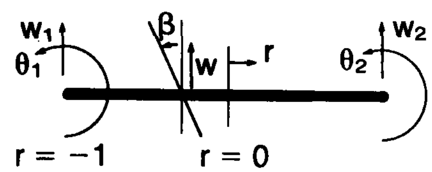
Transparency 20-22

Transparency 20-23

- The 2-, 3- and 4-node beam elements with 1-, 2- and 3-point Gauss integration along the beam axes do not display these phenomena.
- The 16-node shell element with 4×4 Gauss integration on the shell mid-surface is relatively immune to shear and membrane locking (the element should not be distorted for best predictive capability).

Transparency 20-24

- To explain shear locking, consider a 2-node beam element with exact integration (2-point Gauss integration corresponding to the r-direction).



Transverse displacement:

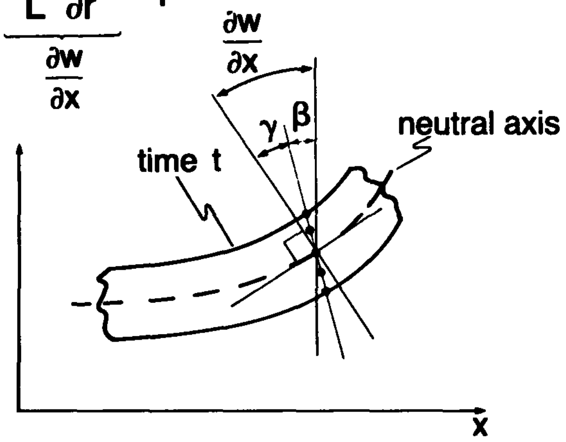
$$w = \frac{1}{2}(1 - r)\, w_1 + \frac{1}{2}(1 + r)\, w_2$$

Section rotation:

$$\beta = \frac{1}{2}(1 - r)\, \theta_1 + \frac{1}{2}(1 + r)\, \theta_2$$

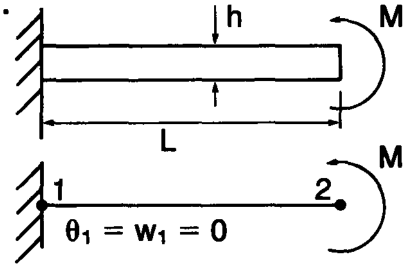
Hence the transverse shear deformations are given by

$$\gamma = \underbrace{\frac{2}{L}\frac{\partial w}{\partial r}}_{\frac{\partial w}{\partial x}} - \beta$$



Transparency 20-25

Consider now the simple case of a cantilever subjected to a tip bending moment, modeled using one 2-node element:



Here $\beta = \frac{1}{2}(1 + r)\,\theta_2$

$$\gamma = \frac{1}{L} w_2 - \frac{1}{2}(1 + r)\,\theta_2$$

Transparency 20-26

Transparency 20-27

We observe:

- Clearly, γ cannot be zero at all points along the beam, unless θ_2 and w_2 are zero. But then also β would be zero and there would be no bending of the beam.
- Since for the beam
  — bending strain energy $\propto h^3$
  — shear strain energy $\propto h$
  any error in the shear strains (due to the finite element interpolation functions) becomes increasingly more detrimental as h becomes small.

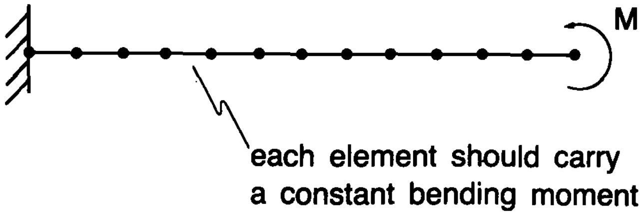
Transparency 20-28

- For the cantilever example, the shear strain energy should be zero. As h decreases, the relative error in the shear strain increases rapidly and in effect, introduces an artificial stiffness that makes the model "lock."

h/L L = 100	$\theta_{analytical}$	finite element solution (exact integration)
0.50	9.6×10^{-7}	3.2×10^{-7}
0.10	1.2×10^{-4}	2.4×10^{-6}
0.01	1.2×10^{-1}	2.4×10^{-5}

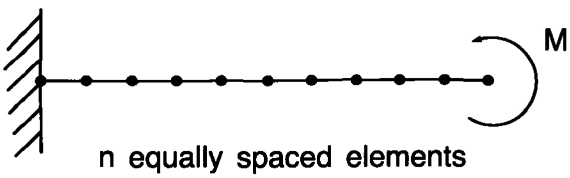
Transparency 20-29

- Although we considered only one element in the solution, the same conclusion of locking holds for an assemblage of elements.



Transparency 20-30

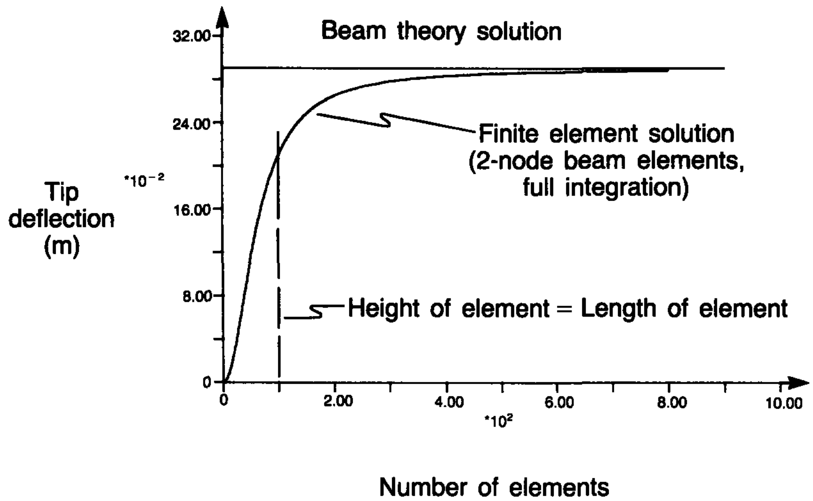
<u>Example:</u> Beam locking study



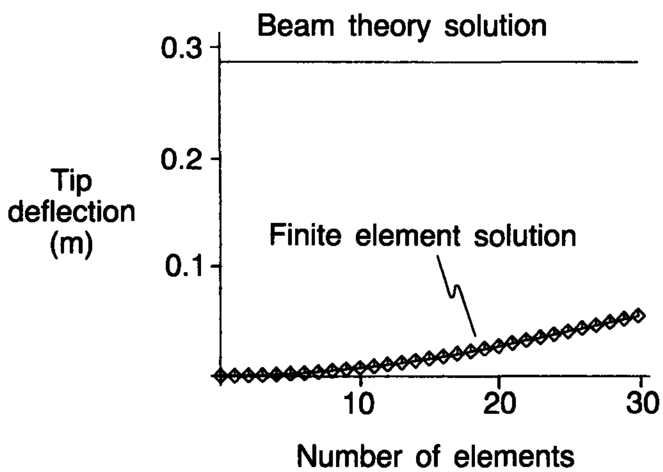
L = 10 m
Square cross-section, height = 0.1 m
Two-node beam elements,
full integration

Transparency 20-31

Plot tip deflection as a function of the number of elements:



Transparency 20-32



A remedy for the 2-node beam element is to use only 1-point Gauss integration (along the beam axis).

This corresponds to assuming a constant transverse shear strain, (since the shear strain is only evaluated at the mid-point of the beam).

The bending energy is still integrated accurately (since $\frac{\partial \beta}{\partial r}$ is correctly evaluated).

h/L L = 100	$\theta_{analytical}$	finite element solution (1-point integration)
0.50	9.6×10^{-7}	9.6×10^{-7}
0.10	1.2×10^{-4}	1.2×10^{-4}
0.01	1.2×10^{-1}	1.2×10^{-1}

Transparency 20-33

- The 3- and 4-node beam elements evaluated using 2- and 3-point integration are similarly effective.
- We should note that these beam elements based on "reduced" integration are reliable because they do not possess any spurious zero energy modes. (They have only 6 zero eigenvalues in 3-D analysis corresponding to the 6 physical rigid body modes).
- The formulation can be interpreted as a mixed interpolation of displacements and transverse shear strains.

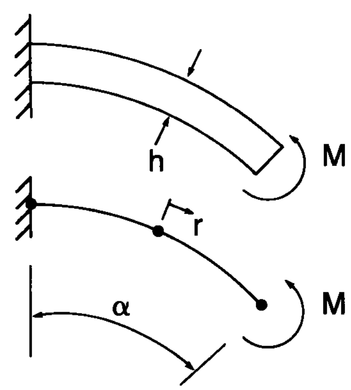
Transparency 20-34

Transparency 20-35

- Regarding membrane-locking we note that in addition to not exhibiting erroneous shear strains, the beam model must also not contain erroneous mid-surface membrane strains in the analysis of curved structures.
- The beam elements with reduced integration also do not "membrane-lock."

Transparency 20-36

Consider the analysis of a curved cantilever:



The exactly integrated 3-node beam element, when curved, does contain erroneous shear strains and erroneous mid-surface membrane strains. As a result, when h becomes small, the element becomes very stiff.

h/R R = 100	$\theta_{analytical}$ ($\alpha = 45°$)	finite element solution: 3-node element, 3-point integration	finite element solution: 3-node element, 2-point integration
0.50	7.5×10^{-7}	6.8×10^{-7}	7.4×10^{-7}
0.10	9.4×10^{-5}	2.9×10^{-5}	9.4×10^{-5}
0.01	9.4×10^{-2}	4.1×10^{-4}	9.4×10^{-2}

Transparency 20-37

- Similarly, we can study the use of the 4-node cubic beam element:

h/R R = 100	$\theta_{analytical}$ ($\alpha = 45°$)	finite element solution: 4-node element, 4-point integration	finite element solution: 4-node element, 3-point integration
0.50	7.5×10^{-7}	7.4×10^{-7}	7.4×10^{-7}
0.10	9.4×10^{-5}	9.4×10^{-5}	9.4×10^{-5}
0.01	9.4×10^{-2}	9.4×10^{-2}	9.4×10^{-2}

We note that the cubic beam element performs well even when using full integration.

Transparency 20-38

Transparency 20-39

Considering the analysis of shells, the phenomena of shear and membrane locking are also present, but the difficulty is that simple "reduced" integration (as used for the beam elements) cannot be recommended, because the resulting elements contain spurious zero energy modes.

For example, the 4-node shell element with 1-point integration contains 6 spurious zero energy modes (twelve zero eigenvalues instead of only six).

Transparency 20-40

Such spurious zero energy modes can lead to large errors in the solution that – unless a comparison with accurate results is possible – are not known and hence the analysis is unreliable.

Transparency 20-41

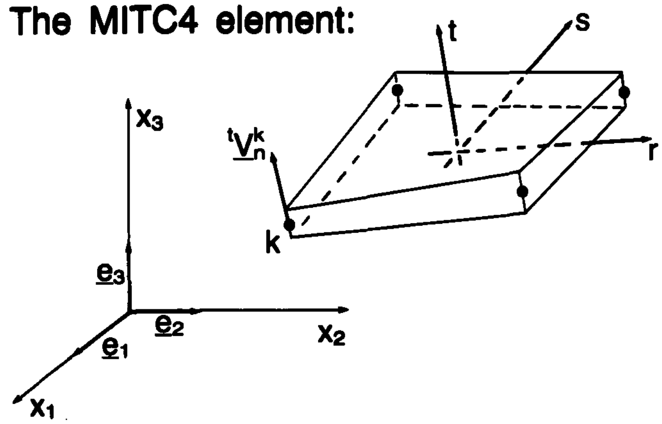
- For this reason, only the 16-node shell element with 4×4 Gauss integration on the shell mid-surface can be recommended.
- The 16-node element should, as much as possible, be used with the internal and boundary nodes placed at their $\frac{1}{3}$rd points (without internal element distortions). This way the element performs best.

Transparency 20-42

- Recently, we have developed elements based on the mixed interpolation of tensorial components.
- The elements do not lock, in shear or membrane action, and also do not contain spurious zero energy modes.
- We will use the 4-node element, referred to as the MITC4 element, in some of our demonstrative sample solutions.

Transparency 20-43

The MITC4 element:

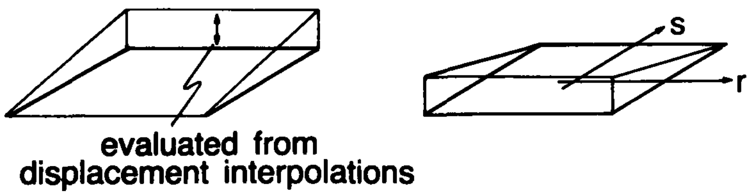


- For analysis of plates
- For analysis of moderately thick shells and thin shells

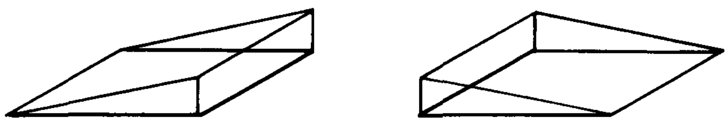
Transparency 20-44

- The key step in the formulation is to interpolate the geometry and displacements as earlier described, but
  - — To interpolate the transverse shear strain tensor components separately, with judiciously selected shape functions
  - — To tie the intensities of these components to the values evaluated using the displacement interpolations

rt transverse shear strain tensor component interpolation



st transverse shear strain tensor component interpolation



Transparency 20-45

The MITC4 element

- has only six zero eigenvalues (no spurious zero energy modes)
- passes the patch test

What do we mean by the patch test?
The key idea is that any arbitrary patch of elements should be able to represent constant stress conditions.

Transparency 20-46

Transparency 20-47

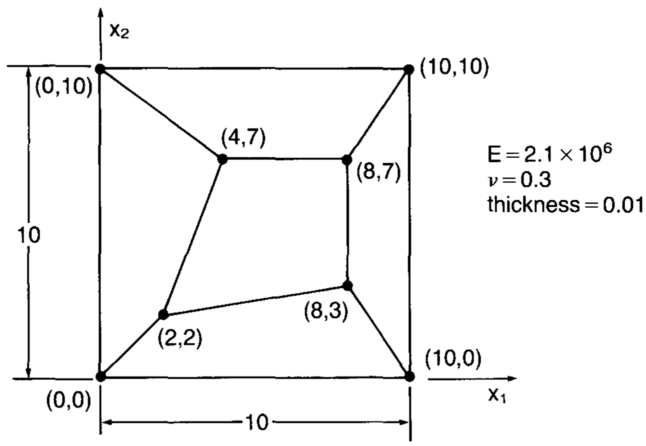
THE PATCH TEST

- We take an arbitrary patch of elements (some of which are geometrically distorted) and subject this patch to
  - — the minimum displacement/rotn. boundary conditions to eliminate the physical rigid body modes, and
  - — constant boundary tractions, corresponding to the constant stress condition that is tested.

Transparency 20-48

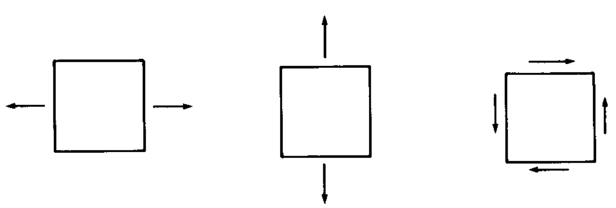
- We calculate all nodal point displacements and element stresses.

The patch test is passed if the calculated element internal stresses and nodal point displacements are correct.

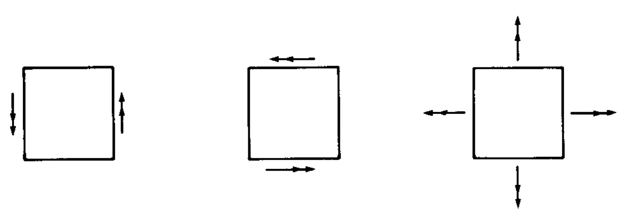


PATCH OF ELEMENTS CONSIDERED

Transparency 20-49



MEMBRANE TESTS

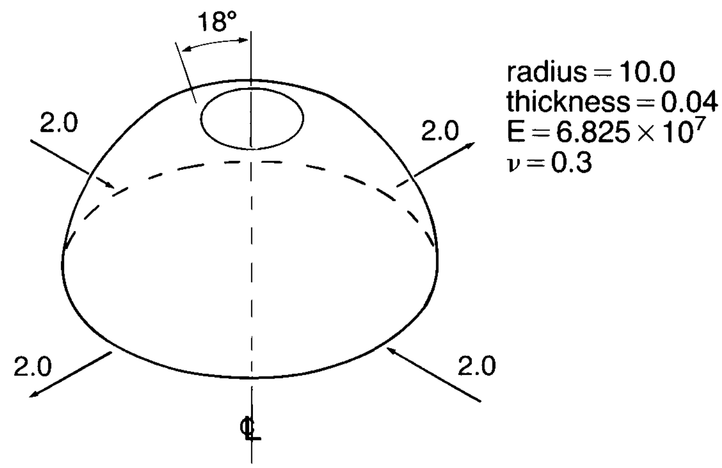


BENDING/TWISTING TESTS

Transparency 20-50

Transparency 20-51

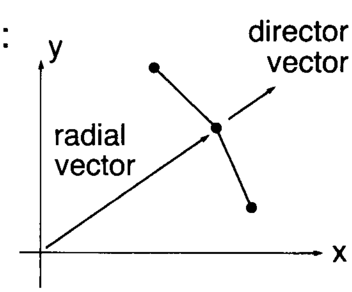
Example: Spherical shell



Transparency 20-52

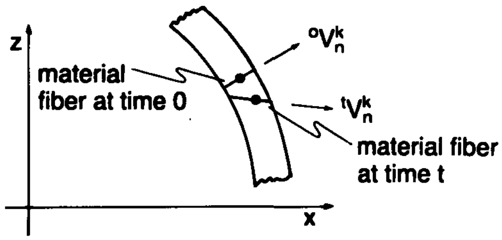
Selection of director vectors:

- One director vector is generated for each node.
- The director vector for each node is chosen to be parallel to the radial vector for the node.
- In two dimensions:



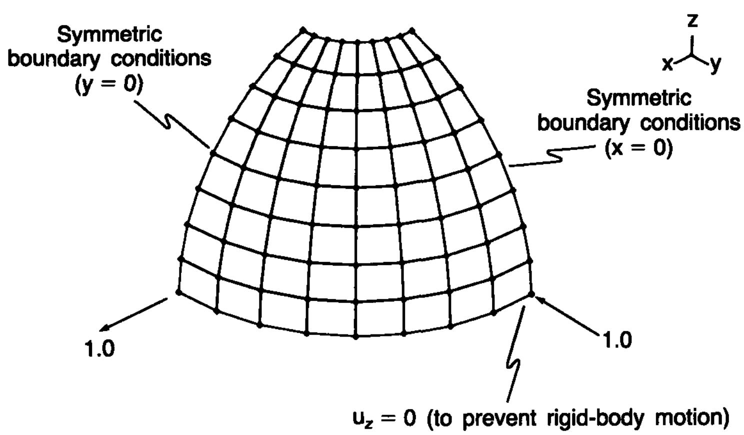
Selection of displacement boundary conditions:

- Consider a material fiber that is parallel to a director vector. Then, if this fiber is initially located in the x-z plane, by symmetry this fiber must remain in the x-z plane after the shell has deformed:



Transparency 20-53

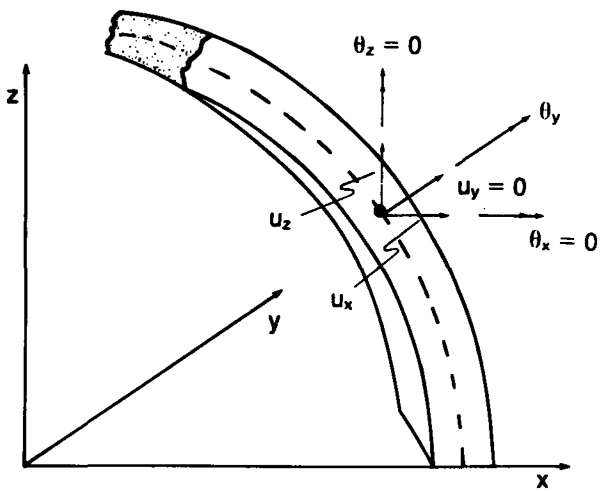
Finite element mesh: Sixty-four MITC4 elements



Transparency 20-54

Transparency 20-55

This condition is applied to each node on the x-z plane as follows:

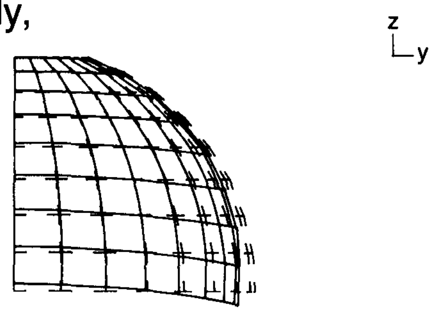


Transparency 20-56

- A similar condition is applied to nodes initially in the y-z plane.
- These boundary conditions are most easily applied by making each node in the x-z or y-z plane a 6 degree of freedom node. All other nodes are 5 degree of freedom nodes.
- To prevent rigid body translations in the z-direction, the z displacement of one node must be set to zero.

Linear elastic analysis results:

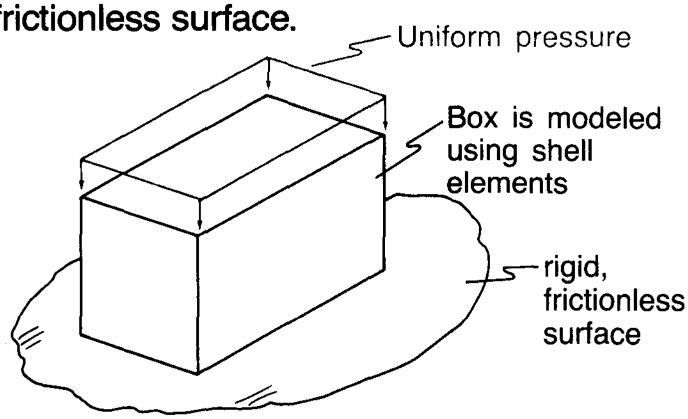
- Displacement at point of load application is 0.0936 (analytical solution is 0.094).
- Pictorially,



Transparency 20-57

<u>Example</u>: Analysis of an open (five-sided) box:

Box is placed open-side-down/Add on a frictionless surface.



Transparency 20-58

Transparency 20-59

Modeling of the box with shell elements:

— Choose initial director vectors.

— Choose 5 or 6 degrees of freedom for each node.

— Choose boundary conditions.

Transparency 20-60

— Instead of input of director vectors, one for each node, it can be more effective to have ADINA generate mid-surface normal vectors.

— If no director vector is input for a node, ADINA generates for each element connected to the node a nodal point mid-surface normal vector at that node (from the element geometry).

— Hence, there will then be as many different nodal point mid-surface normal vectors at that node as there are elements connected to the node (unless the surface is flat).

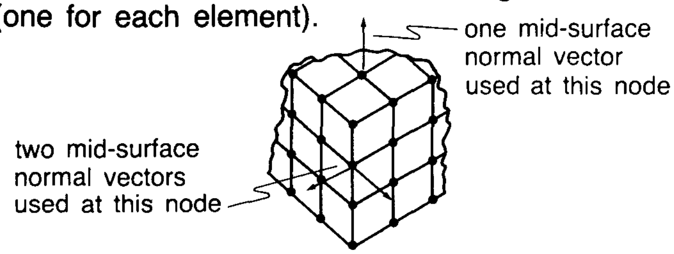
Transparency 20-61

Nodal point mid-surface normal vectors for the box:

— We use the option of automatic generation of element nodal point mid-surface normal vectors.

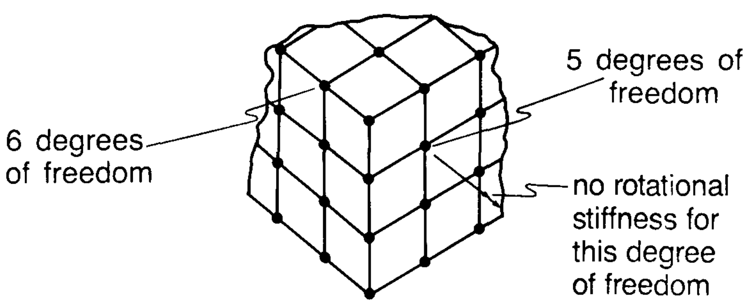
— At a node, not on an edge, the result is one mid-surface normal vector (because the surface is flat).

— At an edge where two shell elements meet, two mid-surface normal vectors are generated (one for each element).



Transparency 20-62

Degrees of freedom:



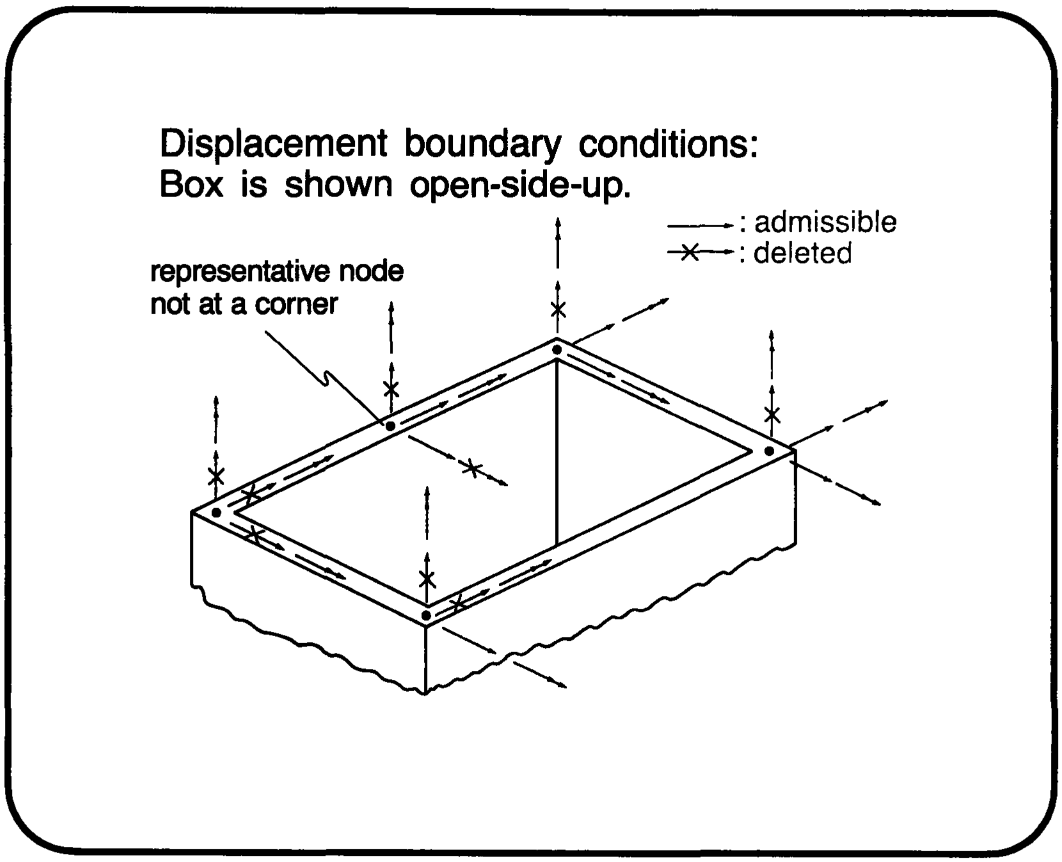
Note added in preparation of study-guide

In the new version of ADINA (ADINA 84 with an update inserted, or ADINA 86) the use of the 5 or 6 shell degree of freedom option has been considerably automatized:

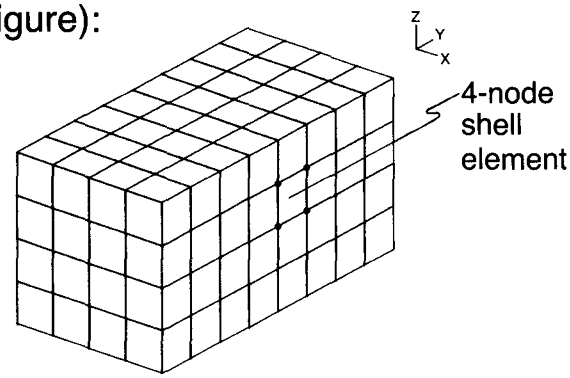
- The user specifies whether the program is to use 5 or 6 degrees of freedom at each shell mid-surface node N
  - IGL(N).EQ.0→6 d.o.f. with the translations and rotations corresponding to the global (or nodal skew) system
  - IGL(N).EQ.1→5 d.o.f. with the translations corresponding to the global (or nodal skew) system but the rotations corresponding to the vectors V_1 and V_2
- The user (usually) does not input any mid-surface normal or director vectors. The program calculates these automatically from the element mid-surface geometries.
- The user recognizes that a shell element has no nodal stiffness corresponding to the rotation about the mid-surface normal or director vector. Hence, a shell mid-surface node is assigned 5 d.o.f. unless
  - a shell intersection is considered
  - a beam with 6 d.o.f. is coupled to the shell node
  - a rotational boundary condition corresponding to a global (or skew) axis is to be imposed
  - a rigid link is coupled to the shell node

For further explanations, see the ADINA 86 users manual.

Transparency 20-63



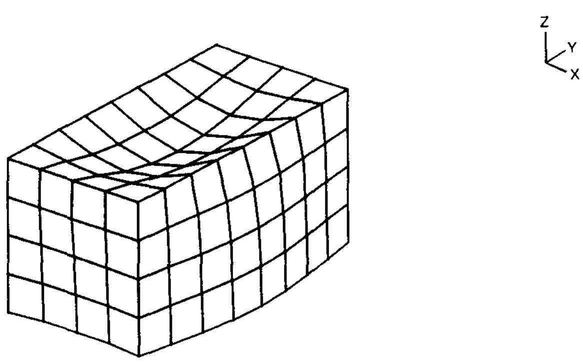
Consider a linear elastic static analysis of the box when a uniform pressure load is applied to the top.
We use the 128 element mesh shown (note that all hidden lines are removed in the figure):



Transparency 20-64

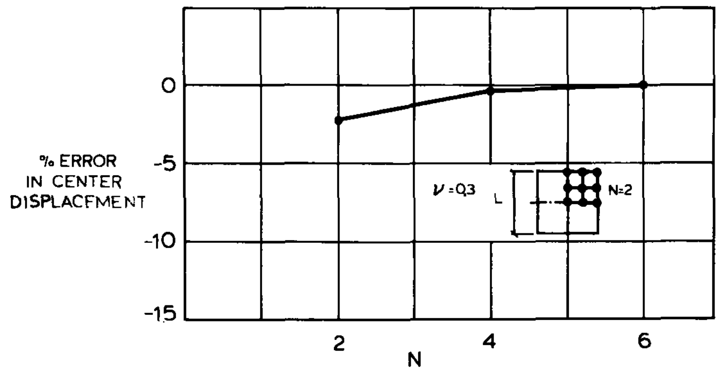
We obtain the result shown below (again the hidden lines are removed):

- The displacements in this plot are highly magnified.



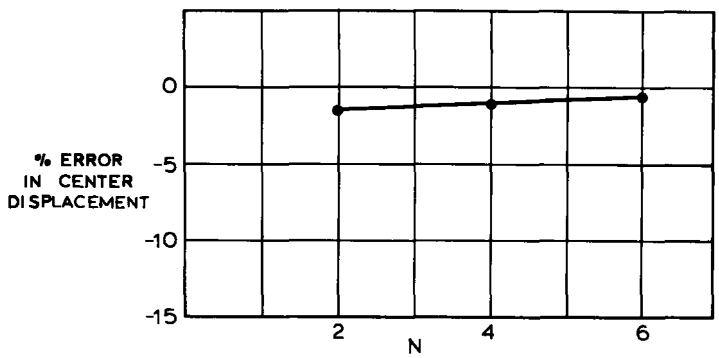
Transparency 20-65

Slide 20-3

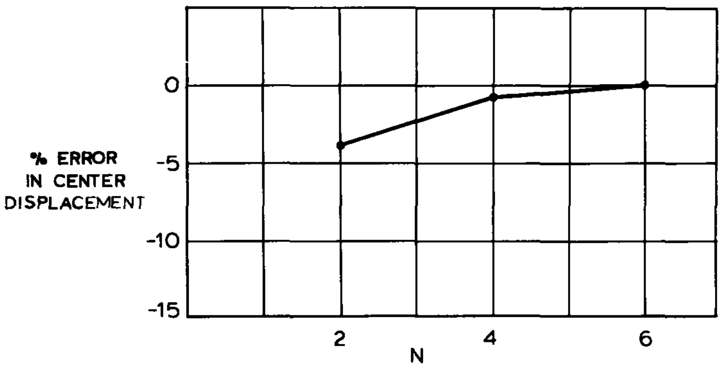


Simply-supported plate under uniform pressure, L/h = 1000

Slide 20-4

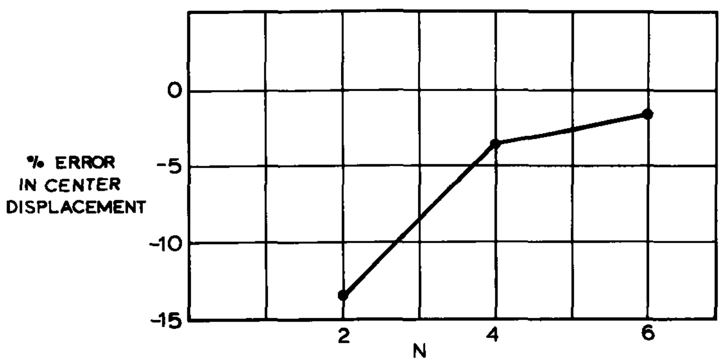


Simply-supported plate under concentrated load at center, L/h = 1000



Clamped plate under uniform pressure, L/h = 1000

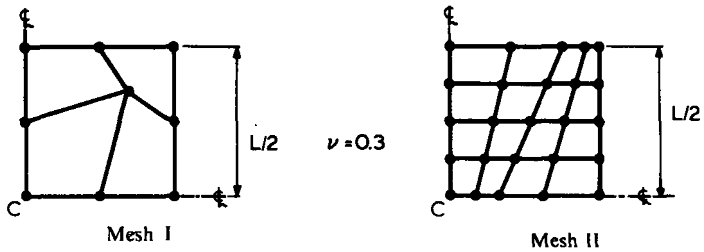
Slide
20-5



Clamped plate under concentrated load at center, L/h = 1000

Slide
20-6

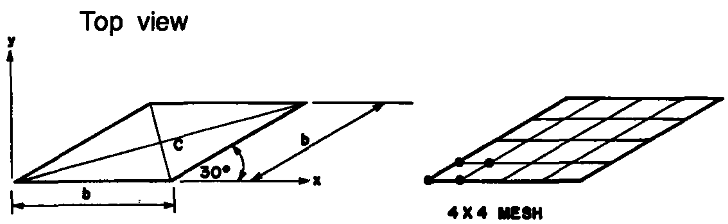
Slide 20-7



$\left.\dfrac{w^{FEM}}{w^{KIRCHHOFF}}\right\|_C$	Mesh I	0·93
	Mesh II	1·01
$\left.\dfrac{M^{FEM}}{M^{KIRCHHOFF}}\right\|_C$	Mesh I	0.85
	Mesh II	1.02

Effect of mesh distortion on results in analysis of a simply-supported plate under uniform pressure ($L/h = 1000$)

Slide 20-8



SIMPLY SUPPORTED EDGES

$E=30 \cdot 10^6$

$\nu=0.3$

$b=1$

thickness=0.01

uniform pressure p=1

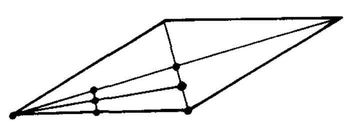
BOUNDARY CONDITION w=0 ON FOUR EDGES

Analysis of skew plate

MESH	w_C^{FEM} / w_C^{MO}	$M_{max}^{FEM} / M_{max}^{MO}$	$M_{min}^{FEM} / M_{min}^{MO}$
4 X 4	0.879	0.873	0.852
8 X 8	0.871	0.928	0.922
16 X 16	0.933	0.961	0.919
32X32	0.985	0.989	0.990

Solution of skew plate at point C using uniform skew mesh

Slide 20-9



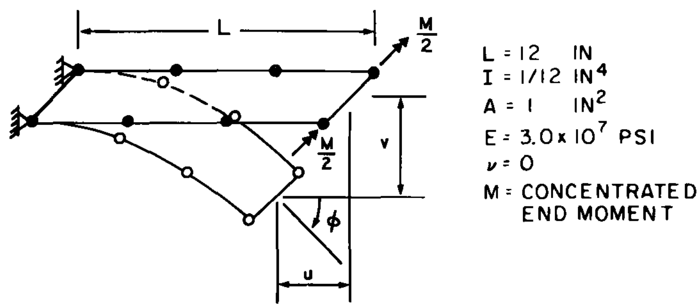
2 X 2 MESH

MESH	w_C^{FEM} / w_C^{MO}	$M_{max}^{FEM} / M_{max}^{MO}$	$M_{min}^{FEM} / M_{min}^{MO}$
2 X 2	0.984	0.717	0.602
4 X 4	0.994	0.935	0.878

Solution of skew plate using a more effective mesh

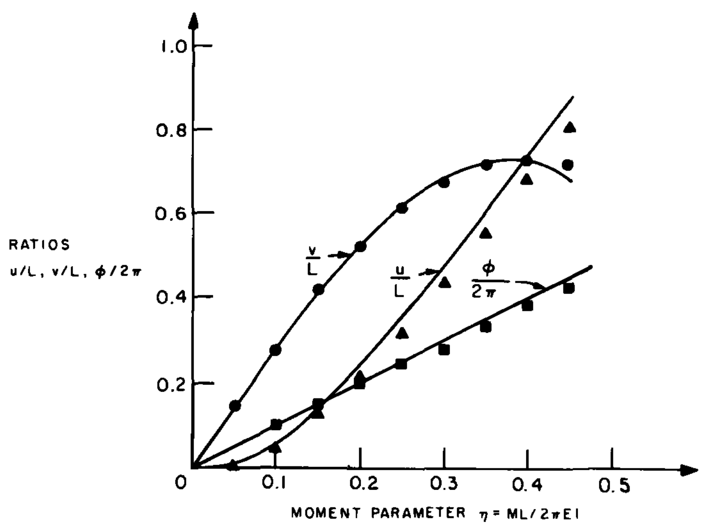
Slide 20-10

Slide 20-11

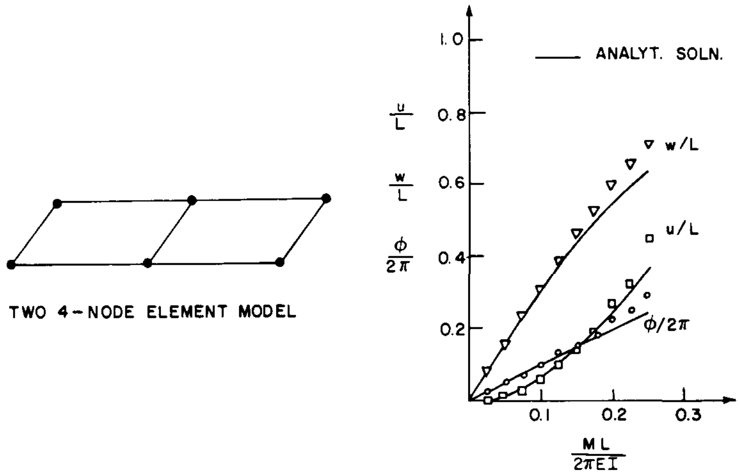


Large displacement analysis of a cantilever

Slide 20-12

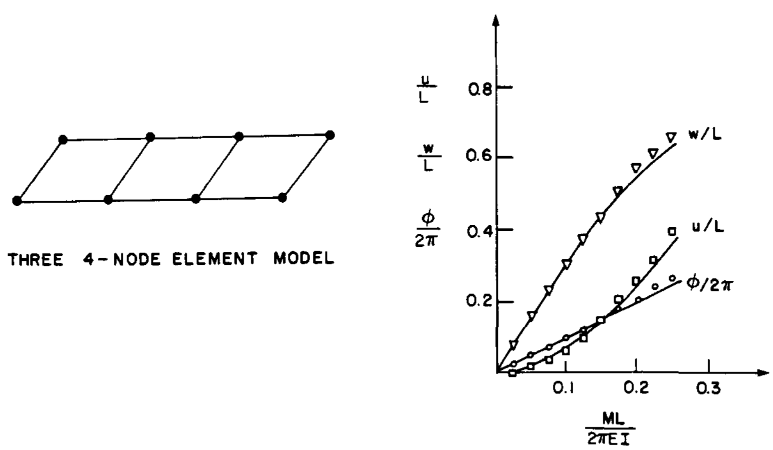


Response of cantilever



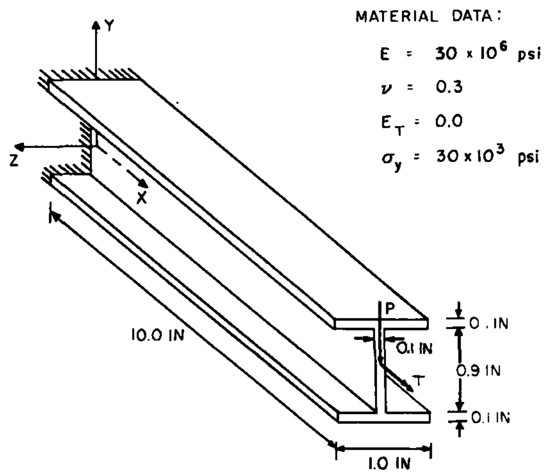
Large displacement/rotation analysis of a cantilever

Slide 20-13



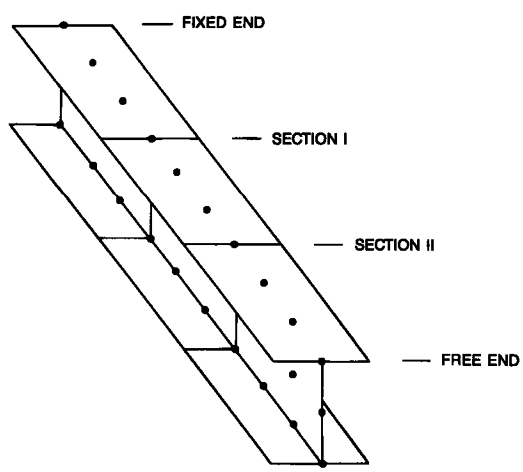
Slide 20-14

Slide 20-15

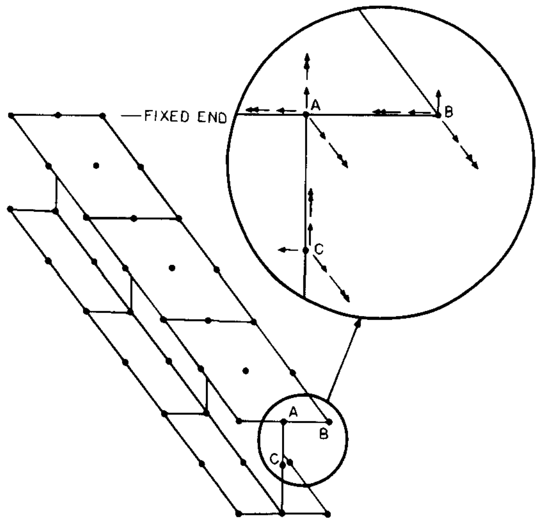


Analysis of I-beam

Slide 20-16

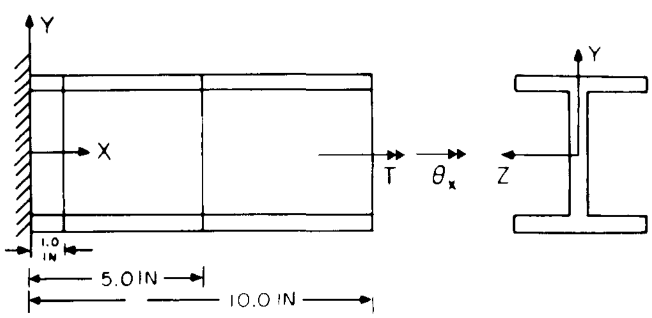


Iso-beam model



Shell model

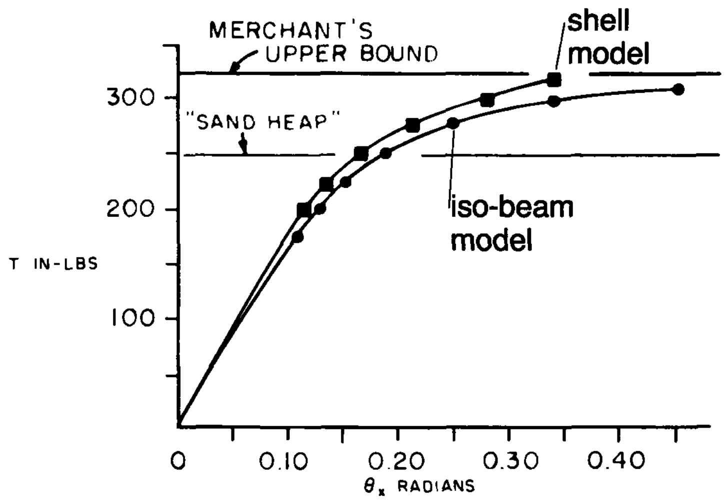
Slide 20-17



I-beam

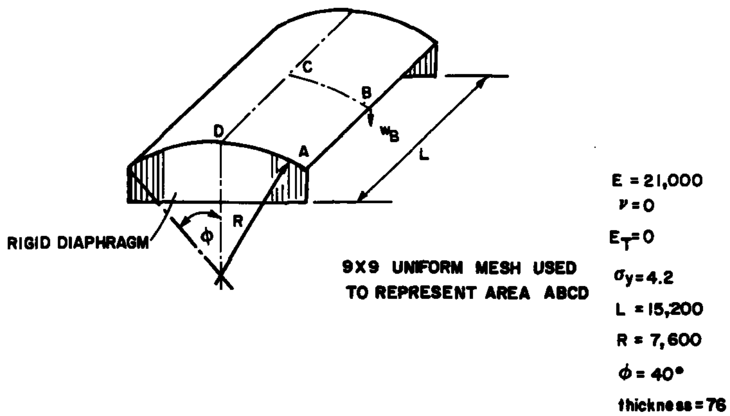
Slide 20-18

Slide 20-19

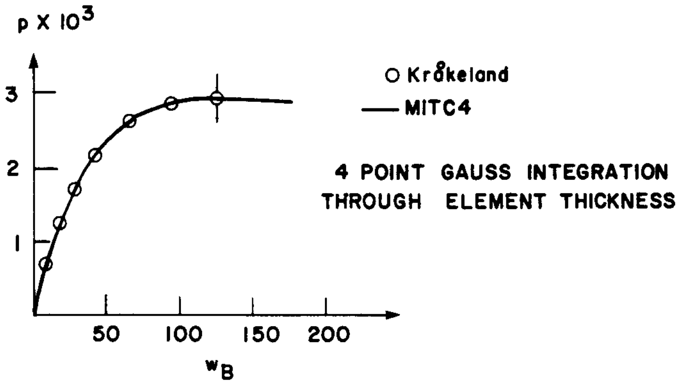


Rotation of *I*-beam about *X*-axis for increasing torsional moment.

Slide 20-20



Large deflection elastic-plastic analysis of a cylindrical shell



Response of shell

Slide 20-21