16 The Origin and Development of Laboratory Models and Analogues of the Ocean Circulation

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16.1 A Brief Philosophy of Laboratory Experimentation

"No one believes a theory, except the theorist. Everyone believes an experiment—except the experimenter." This often used adage, although not aways accurate in detail, carries a certain element of truth. Here we consider certain experiments—analogs, models, or fundamental studies of basic fluid dynamics—that are intended to be relevant to one or another aspect of the ocean circulation. These are physical, fluid models—as opposed to numerical, analytical, abstract, or conceptual models—that make use of a *real* fluid. When the fluid in a container is subjected to some driving force, the fluid moves. It is observed by the experimenter. It is there. It is a real fluid circulation. Apart from its relevance, why should it not be believed?

The experienced experimenter is well aware of the pitfalls of his trade. Aside from the accuracy of his reported observations there are many questions. Were the boundary conditions well controlled? Were the physical properties of the materials and their variations during the experiment known? Did the methods of observation (probes, dyes, tracers, lighting) influence the results? Were the reported observations complete, or at least representative? or were they filtered, massaged, interpreted, selectively reported, etc.? In fact there is always an element of judgment, selectivity, and interpretation concerning what to observe, and having observed, what to record, and what to report. Thus, despite the apparent confidence that may be exuded in publication, the cautious experimenter maintains a restraint born of continual reflection on the extensive preparations necessary for an apparently simple experiment and the many opportunities for error and misinterpretation.

We can broadly classify experiments into four categories, according to their intent, under these headings: (1) simulation, (2) abstraction, (3) verification, and (4) extension. In the first category the experimenter attempts to represent nature in miniature in so far as possible. An effort is made to include all of the relevant driving mechanisms, and the geometry is scaled as in nature, although some distortion may be necessary. Using theoretical guides such as the matching of the appropriate nondimensional ratios, the intent is to learn by trial and error to what extent the ocean circulation can be reproduced as a scale model. If it were possible to reproduce known features of the North Atlantic circulation, for example, such a model could be used to predict similarly scaled features of the ocean circulation in less accessible regions of the world. The predictions would serve as a guide to further exploration and would be compared with observations as they became available. The simulation mode of experimentation is appealing to the eye of the layman who can rather easily be convinced of its possible application and relevance.

The second mode of experimentation is rather like abstract art. The artist draws out (abstracts) from some natural subject those features that he imagines to be of significance, and he displays his interpretation of those features on canvas or in stone for the reaction of his peers and his public. The experimental scientist conceptually isolates one or more processes that he believes to be significant in nature, and he displays and tests them in the form of an operational laboratory experiment. Just as it may be difficult for the artist to persuade his lay audience of the sincerity of his efforts (although his fellow artists understand), so also the scientist may have difficulty convincing his nonprofessional audience that his experiment is relevant to the grand scheme of things past, present, and future.

Abstract experiments may be particularly successful in systems that can be decomposed linearly without doing violence to the essential dynamics, i.e., systems in which the abstracted phenomenon can be isolated by virtue of a lack of coupling with other processes. This may be possible because of a smallness of amplitude of potentially interactive processes, or because of mismatch in the temporal and spatial scales of the various processes. But even in those situations where decomposition is not warranted, one can say, "If .... Perhaps a planet will be found where these conditions prevail; or perhaps some machinery in a chemical plant, somewhere, generates the conditions that I am studying; or perhaps I can build upon this experiment to incorporate the interactive processes necessary for a more realistic representation of the oceanic circulation (simulation?). In any case I will publish my abstract results for the benefit of posterity."

Abstraction experiments, in contrast to direct simulation, are more readily subject to a posteriori mathematical analysis because of their relative simplicity. They may more directly lead to the advancement of theoretical aspects of the problem. Either mode may be regarded as exploratory, for it is likely that certain new aspects of the fluid circulations will emerge that were not anticipated and that will require rationalization. Here again the abstract experiment has the advantage of its simplicity, for deviations from the anticipated behavior will be more clearly recognized. The simulation experiment, however, will generally pose a larger variety of unanticipated phenomena because of its inherently greater complexity.

The verification mode of laboratory experiment implies an apparatus designed to test (and verify) a specific analytical or numerical model. A certain theoretical model predicts a steady-state circulation, the temporal development of a flow, or perhaps an instability. Apparatus is designed to match the conditions of the theory in so far as possible, and, as often as not, the theory is modified to conform to the limitations of the experiment. But in all cases under this category there is a detailed theory capable of a priori predictions and the physical conditions of the theory and of the experiment are closely matched. If this matching is sufficiently precise, the experiment will exactly verify the predictions. From this viewpoint the situation may at first appear to be rather sterile, but this is not the case at all. For following the adage, "No one believes a theory . . . ; everyone believes an experiment . . . ," we find that the theorist has arrived. For now everyone believes the theory, and rightly so, because it is confirmed by the experiment! The verification mode of laboratory experiment is widely endorsed by theorists.

The fourth type of experiment is the extension mode. The experimenter, having set up the theorist by collaboration and verification of his theory, now attempts to do him in by varying the parameters of the experiments, ϵ and δ, until they lie outside the range of validity of the theory. It is then incumbent upon the theorist to expand everything in powers of ϵ and δ to take into account the nonlinear aspects of the flow, but here the experimenter always has the upper hand and can easily keep one step ahead of the theorist. It is only necessary to do this in a manner in keeping with accepted scientific and engineering practice, expressing the experimental results in terms of the minimum number of nondimensional ratios appropriate to the case at hand. Oftentimes, of course, the experimenter and the theorist are one and the same person.

This short excursion into the philosophy of laboratory experimentation may be found useful in the interpretation of the remainder of this chapter. Perhaps not all experimental studies are cleary motivated and can be identified as belonging exclusively to one or the other of the categories discussed above, but such a classification seems to be by and large appropriate.

16.2 Introduction

Perhaps the first rotating laboratory experiment specifically directed toward understanding some aspect of the ocean circulation was that of C. A. Bjerknes in 1902, as reported by Ekman (1905). Because this report is not well known and because a discussion of its results will have later application, I have reproduced it here:

> The late Prof. C. A. Bjerknes at Kristiania, whose vivid interest seems to have been bestowed on every extension of knowledge in his branch, also made in the autumn of 1902, some experiments with the object of verifying some of the results to be found in Section 1 of this paper.

His apparatus consisted of a low cylinder (12 or 17 cm. high and 36 or 44 cm. wide) made of metal or glass, and resting on a table which could be put into uniform rotation (about 7 turns a minute against the sun) by means of a water-turbine. To the upper edge of the rotating cylinder was attached a jet having a horizontal split, 10 cm. long and 1 mm. broad; and through this a stream of air was forced from a pump to produce a wind diametrically across the cylinder over a 10 cm. broad belt.

The motion of the water was observed by means of small balls which just floated in it. As a result of the rotation of the cylinder the current always swept towards the right and thus formed a large whirl-pool occupying, when seen in the wind's direction, the middle as well as the right half of the cylinder.

The direction of motion at different depths was observed at the centre of the cylinder, on a sensitive vane (4 cm. long and 1 mm. high) which could be raised and lowered during the experiment without disturbing the motion. The direction of the vane was read against a glass square divided into radians and laid on top of the cylinder. The following table taken from Prof. Bjerknes' note-book kept in his laboratory gives as an example a series of measurements made during such an experiment. The first column gives the depth in cm. below the surface, the second column the deviation of the current to the right or left of the wind's direction.

Surface (cm)	20–25° right
0.5	45–50° "
1	45–50° "
1.5	25–30° "
2	0–10° "
2.5	0° "
3	5–10° left
3.5	5–10° "
4	5–10° "
4.5	10° "
5	10–15° "
5.5	15–20° "
6	20° "
6.5	20° "
7	20–25° "

The circumstances under which these experiments were made were in any case such as to satisfy the conditions for stationary motion but very roughly; and an exact interpretation may furthermore be difficult owing to the shape of the vessel, etc. It is certain however that their real object was to obtain merely qualitative verification of the reality of the phenomena considered, and as such they are very striking and instructive. Both the deflection of the surface-current and the increase of the angle of deflection increases only in the very uppermost layer; and this is explained as a result of the rapid rotation of the vessel. Indeed a value of $\mu = 0.3$ (which would not appear to be too small for motion on such a small scale) would give $D_{Ek} = 2$ cm only. [*Author's note*: Here I have used D_{Ek} in place of D to avoid later confusion. As used here and in other recent literature $D = D_{Ek}/\pi$ is the mathematically more convenient measure of boundary-layer thickness.] The directions of motion below this level have very much the appearance of a "midwater-current" produced by a pressure-gradient.

Apparently Ekman determined the effective viscosity $\mu = 0.3$ indirectly from his estimate $D_{Ek} = 2$ cm, for with a rotation rate $\Omega = 0.73\ s^{-1}$ and with the viscosity of water $\mu = 0.01$ we find $D_{Ek} = 0.37$ cm and $D = 0.117$ cm. Thus the laminar Ekman depth would be considerably shallower than that inferred from the experimental data, as Ekman must have recognized. Departing temporarily from the historical development to consider this question, these early results possibly may be explained in the light of recent laboratory experiments and numerical calculations on the stability of the Ekman boundary layer.

Laboratory studies have mostly been confined to flow over a rigid boundary (Faller, 1963; Faller and Kaylor, 1967; Tatro and Mollo-Christensen, 1967; Caldwell and van Atta, 1970; Cerasoli, 1975), and most comparative theoretical studies also have been concerned with the rigid-boundary case.

The Reynolds number for the Ekman layer is defined as $Re = UD/\nu$, where U is the difference between the speed at the boundary and the speed of the interior flow. Over a rigid boundary the critical value for instability is approximately $Re_c = 55$. For the Ekman layer with a free boundary, the profile of flow is exactly the same, but potential instabilities are not constrained by a no-slip boundary condition. A preliminary estimate given by Faller and Kaylor (1967) was $Re_c = 12 \pm 3$ for the first onset of instability. This result has recently been confirmed with a more accurate calculation by Iooss, Nielsen, and True (1978), who found $Re_c = 11.816$. In C. A. Bjerknes's experiment the corresponding critical surface water speed would be approximately $U = 1\ \mathrm{cm\ s^{-1}}$, and the corresponding critical wind stress would be $\tau_c = 0.12\ \mathrm{dyn\,cm^{-2}}$. Still another possibility is that of Langmuir circulations, a subject that will be touched upon briefly later, in section 16.6.10 and in this connection it would be interesting to know whether the air jet had sufficient speed to raise capillary waves. Unfortunately Ekman's account gives no means of estimating the air or water speeds, but it seems likely that the surface Ekman layer was unstable, thus accounting for the approximate $D_{Ek} = 2$ cm and the lack of an idealized spiral.

In determining the scope of this chapter it has been found necessary to restrict the material arbitrarily to laboratory experiments rather directly concerned with large-scale oceanic circulations, and a number of significant developments and important areas of research have been omitted from consideration. With one exception we shall be concerned exclusively with rotating experiments, omitting laboratory studies of the generation and interaction of surface and internal waves, thermohaline (double-diffusive) phenomena, thermal convection, turbulence, and other fundamental fluid dynamic studies that may be directly related to oceanic processes. Even in the realm of rotating fluids it is not possible to consider all studies of interest in as much detail as may be warranted, and there will be those overlooked in the literature. Moreover, the author confesses to a certain prejudice, which will be evident, in emphasizing his own contributions and those with which he is most intimately familiar.

After the experiments of C. A. Bjerknes, the first serious attempt at the isolation of an oceanic circulation in a rotating laboratory experiment appears to have been that of Spilhaus (1937). C. G. Rossby had proposed that the Gulf Stream might be similar to an inertial jet emerging from the Straits of Florida. Differences between the Gulf Stream and the usual laboratory jet might be expected from the effects of the earth's rotation (with the consequent adjustment to geostrophic balance) and the stratification of the ocean.

Spilhaus's attempts to verify certain aspects of Rossby's concepts consisted of (1) some preliminary trial experiments in a small cylindrical tank with a jet of water injected from a slot in the wall, (2) more elaborate uniform-fluid experiments in a 6-ft-diameter tank with the jet emerging from an axial tube in the center and with excess water overflowing at the rim, and (3) experiments with a two-layer system (immiscible fluids) in a 4-ft-diameter tank with a central jet confined to the upper layer. The resulting circulations were more complex than anticipated and could not be interpreted satisfactorily in terms of Rossby's theoretical work. Nevertheless it is interesting to read Spilhaus's account of the experiments in the light of our present understanding of source–sink flow in rotating experiments.

16.3 The Experiments of W. S. von Arx

Serious and sustained efforts at modeling the ocean circulation began with von Arx circa 1950. At about the same time, and quite independently, major laboratory efforts were being initiated by D. Fultz and R. Long at the University of Chicago, who were primarily interested in atmospheric circulations, and by R. Hide at King's College, the University of Newcastle, who was attempting to understand the fundamentals of circulation in the molten interior of the earth. The eventual collaboration and interchange of information between these investigators and their students became part of the rapid development of the field of study that today we refer to as geophysical fluid dynamics. The rather bold initiatives of these scientists eventually led to the realization by a broader base of theoretically oriented meteorologists and oceanographers that it was indeed possible to simulate certain features of complex geophysical circulations in the laboratory.

The first major experiments reported by von Arx (1952) were conducted in a paraboloidal basin in which the northern hemisphere continental boundaries were modeled in sponge rubber. The trade winds were derived from the relative motion of the air in the laboratory at the rotation rate $\Omega = 3.18\ s^{-1}$, and the midlatitude westerlies were generated by three stationary vacuum cleaner blowers with appropriately directed nozzles. With this basic system it was possible to generate many of the essential features of the northern ocean circulations that were then known to exist. These and subsequent experiments had certain shortcomings, but their role in the stimulation and development of further laboratory studies should be recognized.

The paraboloidal models were not entirely satisfactory in several ways: geometrical distortion was inevitable; the wind fields could not be precisely controlled or measured; at wind speeds of about $2\ m\ s^{-1}$ the water surface was wavy and it is likely that the surface Ekman layer was unstable; and with the complex geometry a slight tilt of the rotation axis produced large undesirable oscillations. Moreover, a severe critic might argue that in attempting to approach the complexity of nature, the ability to achieve understanding of fundamental processes was sacrificed. Nevertheless, in the developing field of oceanography there was still at that time what seemed to be a residual dichotomy of opinion between advocates of the wind-generation theory of ocean currents and advocates of the thermal-generation theories, which tended to divide theoretical and observational oceanographers; von Arx's experiments were a convincing verification of the theories of Stommel, Munk, and others with respect to wind generation of the primary ocean circulation.

The von Arx experiments helped stimulate further laboratory studies in various ways. First, there was the development of experimental techniques for use with rotating laboratory models. Second, the rotating apparatus that he built was used in later experiments by this author and a succession of others, and the basic rotating system is today still available for use at the Woods Hole Oceanographic Institution. Third, through a paradox that was observed in the early studies with the paraboloid, the possibility of simulating the β-effect by a radial variation of depth was first realized.

Von Arx originally employed his carefully constructed paraboloidal basin at the equilibrium rotation rate, i.e., the rotation rate at which the water surface had the same paraboloidal shape as the container. Strangely, at this speed no western boundary current could be found. But at higher speeds, the western intensification was obviously present and at lower speeds there was eastern itensification! This behavior was explained by C. G. Rossby (von Arx, 1952) and eventually led to the proposition, by the present writer, that the paraboloidal shape was unnecessary—that a flat tank with a paraboloidal free surface, produced by a suitable rotation rate, would work equally well.

Further experiments using a flat-bottomed tank (von Arx, 1957) allowed lower rotation rates and a generally better overall experimental control. Studies of the southern hemisphere were undertaken and the representation of natural circulations was somewhat better than in the paraboloidal basin.

Success with the flat geometry encouraged von Arx to build a much larger apparatus, a floating cylinder with a working area having a diameter of 4 m and driven by impeller blades beneath the tank. A photograph and description of this apparatus appear in von Arx (1957). In an effort to improve the analogy with nature, infrared heaters were installed above the tank with the intention of heating the surface layers of water in the lower latitudes, thus simulating solar heating of the upper layers of the ocean. Unfortunately, the production of a warm surface layer essentially destroyed the induced β-effect associated with the paraboloidal depth variation and eliminated the western intensification!

Although this mammoth rotating tank did not improve significantly the ability to reproduce nature in miniature, the availability of this large apparatus encouraged this author to undertake experiments on the instability of the Ekman boundary layer (Faller, 1963). With the large-diameter tank the Reynolds numbers necessary for instability of the Ekman layer could be achieved at much lower Rossby numbers, i.e., with a more nearly geostrophic, as opposed to gradient, circulation.

As an aside, to this author's knowledge the Ekman number, which figures prominently in all rotating laboratory experiments, was first introduced by H. Lettau in a turbulence course at MIT in 1953 that was attended by both von Arx and this writer. Lettau's definition was $E = D/H$, the geometric ratio of the depth of an Ekman layer to the total depth of fluid. This definition was introduced into the literature by Faller and von Arx (1958) and by Bryan (1960) although the ratio L/D, where L is an arbitrary characteristic length, was defined as the Ekman number by von Arx (1957). The ratio D/H is, of course, also the inverse of the fourth root of the Taylor number; and Fultz (1953) referred to L^2/D^2 as a rotation Reynolds number. Stern (1960b) used Ekman number $E = \Omega H^2/\nu = (H/D)^2$, but it appears that now the generally accepted definition is $E = \nu/\Omega H^2 = (D/H)^2$ or a definition in which H is replaced by some characteristic horizontal scale. This writer prefers the definition $E = D/H$ as originally given by Lettau because of its simple geometric significance and because the scaled Ekman depth is then proportional simply to E rather than $E^{1/2}$, although other definitions may sometimes be mathematically more convenient.

16.4 The SAF Model

In reviewing theories of the ocean circulation, Stommel (1957b) recognized the essential analogy between the precipitation theories of Hough and Goldsborough and more recent theories of the wind-driven ocean circulation. In brief, each mechanism may be considered in terms of distributed sources and sinks of fluid at or near the ocean surface. Moreover, isolating the deep ocean, one could imagine this layer as being driven by the sinking of cold water in selected Arctic and Antarctic regions with more or less uniformly rising motion out of the top of this layer to satisfy the continuity of mass.

At that time this writer was engaged in the study of laboratory models of the atmospheric circulation, using the rotating turntable of von Arx. At the suggestion of H. Stommel and A. B. Arons it was a relatively straightforward matter to arrange a simple source-sink experiment in analogy with the precipitation mechanism. It was predicted that with distributed precipitation over a portion of the interior of a bounded basin and with the β-effect simulation found by von Arx, the geostrophic constraint in the interior of the basin would require some rather bizarre flow patterns. In particular, with the geostrophic condition as the overriding constraint on the interior flow, continuity would have to be satisfied by a combination of east-west geostrophic currents along contours of constant depth and frictionally balanced western boundary currents, unspecified in detail.

Figure 16.1 shows the very first experiment of the series eventually presented in SAF, that is, Stommel, Arons, and Faller (1958), and in F, that is, Faller (1960). In this initial trial the depth variation was provided in part by a spherical-cap polar dome (built of wood) that was 5 cm high in the center, tapering to zero height at the radius $r = 91.4$ cm in a tank of total radius $r = 106.7$ cm. An additional depth variation was provided by the counterclockwise rotation rate of $\Omega = 0.84\ s^{-1}$. The average water depth was $h = 10$ cm at $\Omega = 0$, measured above the flat portion of the tank bottom near the rim.

The side walls of the 30° sector were carved to fit the polar dome and were puttied in place with a small gap between the two sides at the apex to allow water to escape into the remainder of the circular tank as necessary to balance the source. An area of precipitation was arranged by dribbling dyed water (from a supply vessel) through a perforated coffee can at a flow rate of

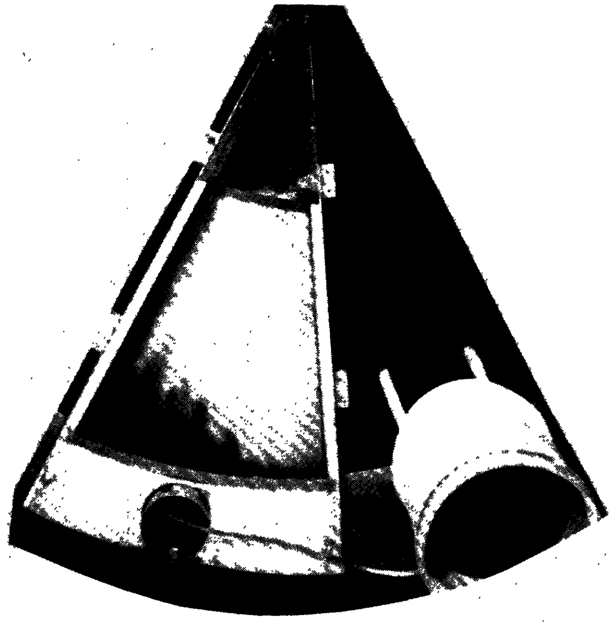

Figure 16.1 The original experiment of Stommel, Arons, and Faller (1958). The black area on the right shows a portion of a spherical-cap polar dome that was 5 cm high at the center and tapered to zero depth at 91.4-cm radius. This dome was painted white within the 30° sector (on the left) that was formed by radial side walls shaped to fit the bottom boundary and puttied in place. The entire circular tank contained water approximately 3 cm deep at the center and 11 cm deep at the rim at the rotation rate $\Omega = 0.84\ s^{-1}$, counterclockwise as viewed from above. Dyed water from the bucket on the right drained through a tube into a perforated coffee can and "precipitated" at the rate of 200 $cm^3 min^{-1}$ into the sector, near the rim. A small gap between the side walls at the apex allowed flow out of the sector. The western boundary current is clearly evident as a dark band of dyed water. (See figure 5.3).

approximately 200 $cm^3 min^{-1}$. The experimental conditions were not precisely controlled, and figure 16.1 shows an area of disturbance near the precipitation source. Similar disturbances occurred near the apex, but these are obscured in figure 16.1 by a partial masonite cover. Nevertheless, the predicted western boundary current was obvious, thus confirming the theoretical prediction and leading to an extended series of controlled experiments.

The SAF experiments established the validity of the following basic concepts for slow, steady motion of a uniform fluid in a bounded basin without closed depth contours:

(1) If there is no local source (sink) of fluid and the local depth remains constant, the interior geostrophic flow is constrained to follow contours of constant depth.
(2) If there is a local source (sink) of fluid or if there is a gradual decrease (increase) in the local fluid depth, a column of fluid moves toward greater (lesser) depths satisfying the geostrophic condition of zero horizontal divergence.
(3) Failure of the interior motion to satisfy an overall mass balance produces a frictional western boundary current, the interior flow moving along lines of constant depth to the western boundary as required for overall continuity.

Perhaps the most striking result of these experiments was the verification of the recirculation phenomenon illustrated in figure 16.2. With a source of strength S_0 at the apex of the sector and a distributed sink (represented by the gradual rise of the free surface), the predicted western boundary current flow was twice the source strength! In the laboratory experiment this current was composed half from the colored source water and half from the recirculated clear water.

Elaboration of the SAF experiments (Faller, 1960) demonstrated several additional features of the flow:

(1) In east-west currents (along contours of constant depth) the effect of bottom friction causes westward-moving currents to diverge in a laminar and geostrophic Gaussian plume. This divergence is forced by bottom Ekman layer suction (and pumping) together with the effect of the radial depth variation. More sur-

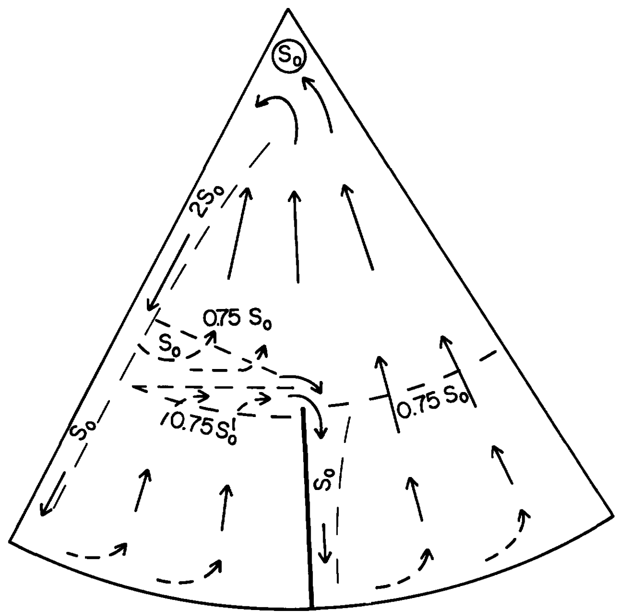

Figure 16.2 An example of the partial radial-barrier experiments of Faller (1960) for a source at the apex and a uniformly rising free surface. The source S_0 is joined by recirculated interior water to give a total western boundary current of $2S_0$. At the radius of the inner end of the partial radial barrier the WBC divides, one S_0 continuing along the boundary and one S_0 separating. Of the latter, $0.25S_0$ reaches the radial barrier and participates in the WBC there, while $0.75S_0$ moves northward as the required interior flow. A transport of $0.75S_0$ from the southwest basin joins in the eastward-moving jet to complete the source water for the transport of one S_0 along the radial barrier.

prisingly, the theory, being linear and reversible, predicts that an eastward-moving current of constant mass flux should converge into a narrower and more intense jet. As shown in the experiments, catastrophy is avoided in the converging jet because the flow spreads out in the western boundary current to whatever extent is necessary for its eventual convergence as it flows eastward.

(2) With an interior source (or sink) there was an intense recirculation analogous to the effects of the wind-spun vortex described by Munk (1950). The intensity of this recirculation phenomenon was found to be in excellent agreement with a theoretical analysis.

(3) The concepts predicted and verified in SAF were extended to somewhat more complex geometries involving partial radial barriers. An example of such a flow, qualitatively verified by experiment, is shown in figure 16.2, where the indicated transports are the theoretical predictions. Further application to an Antarctic-type geometry illustrated the important role of the radial barriers in allowing the buildup of pressure gradients essential to the maintenance of the geostrophic flow.

(4) In certain source–sink experiments with closed contours of constant fluid depth, it is not possible to construct purely geostrophic regimes of flow. In such cases, lacking resolution of the problem by western boundary currents, the effects of bottom friction dominate the flow. Examples of the complex set of spiral flow patterns necessary in order to transport fluid from an interior source to an interior sink illustrated the dominance of frictional effects.

(5) One feature that would not stand the test of time was the observation of an eastern boundary current when the flow was injected through the eastern boundary of the sector. In later experiments (Faller and Porter, 1976) it was found that this current was probably a density-driven flow due to incomplete thermal control of the injected water.

The analogy between these laboratory experiments and theoretical models of steady wind-driven ocean circulation may be illustrated most clearly by a comparison of the governing equations from three studies. These equations are

Stommel (1948) $$\alpha\frac{\partial\psi}{\partial x} + \nabla^2\psi = \gamma\sin\pi y/b, \qquad (16.1)$$

Munk (1950) $$\beta\frac{\partial\psi'}{\partial x} - A\,\nabla^4\psi' = \mathrm{curl}_z\boldsymbol{\tau}, \qquad (16.2)$$

Faller (1960) $$\frac{\partial h}{\partial r}\left(\frac{1}{r}\frac{\partial\psi}{\partial\theta}\right) + \frac{D}{2}\nabla^2\psi = Q - \frac{\partial h}{\partial t}. \qquad (16.3)$$

In (16.1) $\alpha = (H^*/R)(\partial f/\partial y)$, where H^* is a constant effective ocean depth and R a linear drag coefficient; and $\gamma = F\pi/(Rb)$, where F is the magnitude of the maximum wind stress per unit mass and b the north–south dimension of the model. In (16.2) A is a horizontal Austausch coefficient, $\boldsymbol{\tau}$ the wind stress per unit mass, and ψ' the mass transport stream function. In (16.3) h is the variable fluid depth and Q an internal or surface source of fluid per unit horizontal area.

To illustrate the close correspondence of these equations we may multiply (16.1) by R/H^* and convert the right-hand side to the derivative of a cosine; introduce $\psi = \psi'/H$ as the velocity streamfunction in (16.2), H being the ocean depth; and multiply (16.3) by f/h, reorient the coordinate axes with $r\,d\theta = dx$ and $dy = -dr$, and add the horizontal viscous term. The three equations then become

$$\frac{\partial f}{\partial y}\frac{\partial\psi}{\partial x} + \frac{R}{H^*}\nabla^2\psi = -\frac{1}{H^*}\frac{\partial F\cos\pi y/b}{\partial y}, \qquad (16.4)$$

$$\beta\frac{\partial\psi}{\partial x} - A\,\nabla^4\psi = \frac{1}{H}\mathrm{curl}_z\boldsymbol{\tau}, \qquad (16.5)$$

$$\left(-\frac{f}{h}\frac{\partial h}{\partial y}\right)\frac{\partial\psi}{\partial x} + \frac{fD}{2h}\nabla^2\psi - \nu\nabla^4\psi = -\frac{f}{h}Q + \frac{f}{h}\frac{\partial h}{\partial t}. \qquad (16.6)$$

Equations (16.4)–(16.6) clearly demonstrate the analogy between the laboratory experiment and the basic theories of the steady wind-driven ocean circulation. In particular, $\beta = \partial f/\partial y$ is modeled by $-fh^{-1}\partial h/\partial y$, Stommel's linear drag law corresponds to the Ekman layer effect with R/H^* the equivalent of $fD/2h$, and a positive curl of the wind stress in (16.4) or (16.5) is represented in (16.6) by either an internal (or surface) sink of fluid $-fQ/h$ or by a local rate of increase of depth $(f/h)\,\partial h/\partial t$. Note also that with the addition of the lateral viscous term to (16.6) this equation is capable of describing steady, linear western boundary currents analogous to those of Munk.

16.5 Experiments with Rotating Covers

The source–sink experiments were more controlled than the wind-driven experiments, and the regular boundary conditions of the 60° sector allowed a clear distinction between the interior flow and the western boundary current (WBC); turbulence was essentially eliminated and the interior flow could be guaranteed to be essentially geostrophic by controlling the flow rates. The introduction of a rotating cover (rotating lid) as the driving mechanism produced a certain degree of additional control by completely enclosing the fluid, by eliminating the possibility of density differences in