Experiment 07: Momentum and Collisions

TEAL
MIT Studio Physics
8.01T

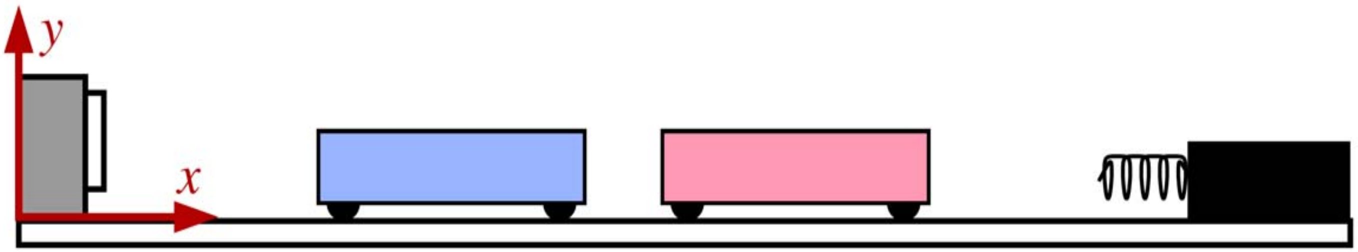
Goals

- Investigate the conservation of momentum in elastic and inelastic collisions

- Investigate the conservation of energy in an elastic collision

- Investigate the amount of kinetic energy available for non-conservative work in an inelastic collision

Equipment setup

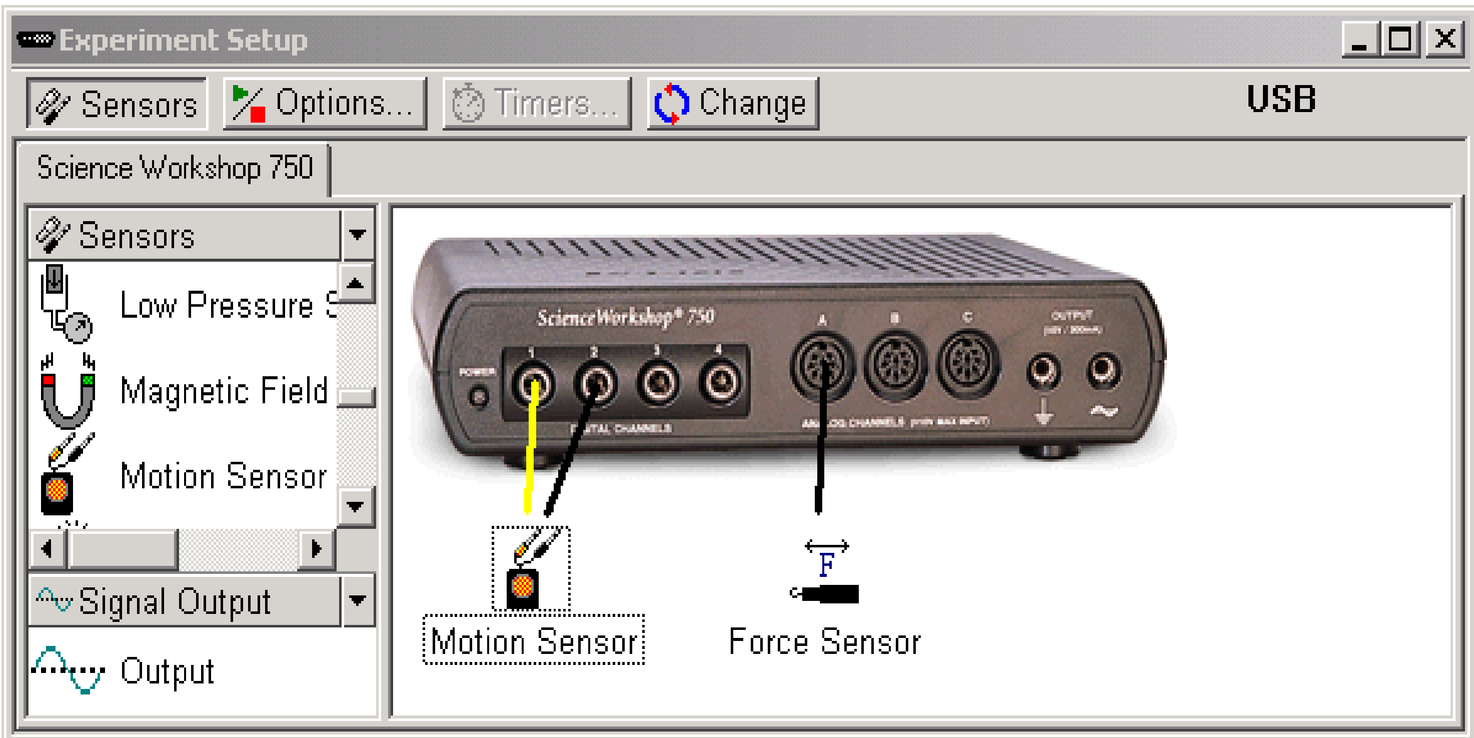


- Use the lighter spring on the force sensor.
- Clip the motion sensor to the end of the track.
- Level the track.

- Place Target cart at rest about 10cm from the spring.
- Place Incident cart about 16-20cm from motion sensor.
- Note: Velcro facing = inelastic and magnets facing = elastic.
- Roll incident cart just hard enough to come back to its starting point. Practice this first before you take your data!
- Make measurements with different weights of incident and target cart.

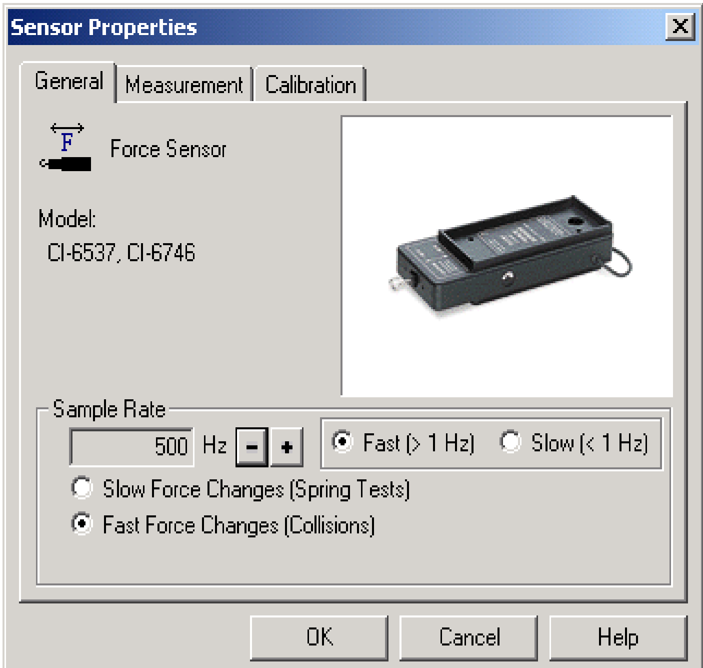
Starting DataStudio

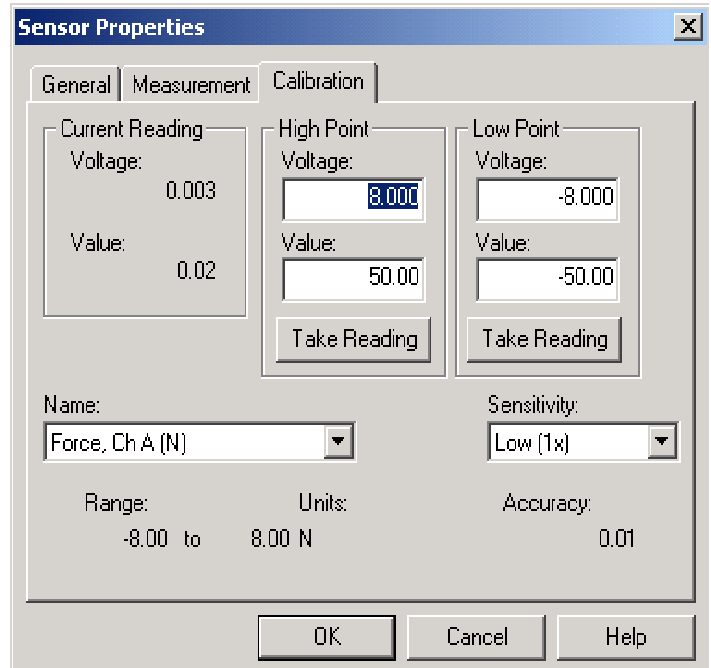
- Create a new experiment.
- Plug force and motion sensors into the 750 and
- drag their icons to inputs in the Setup window.



- Double-click the Force Sensor icon.

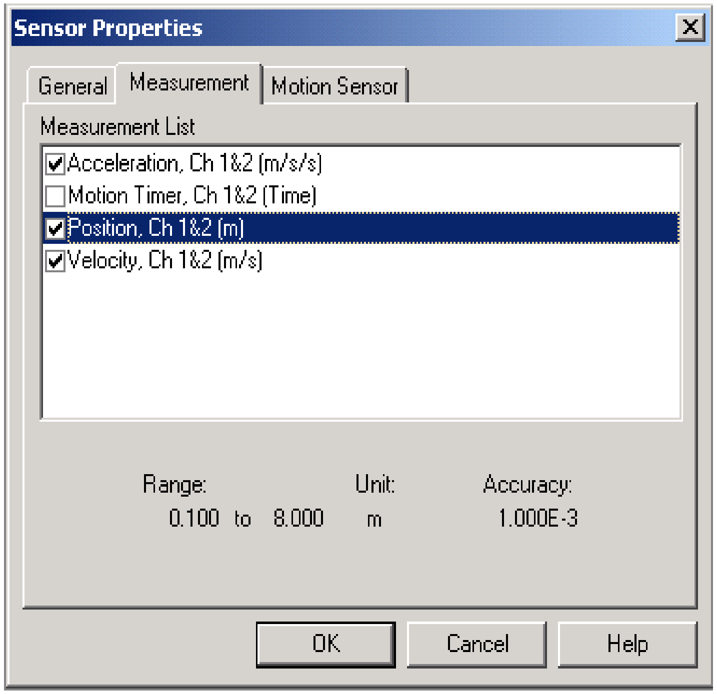
Force Sensor

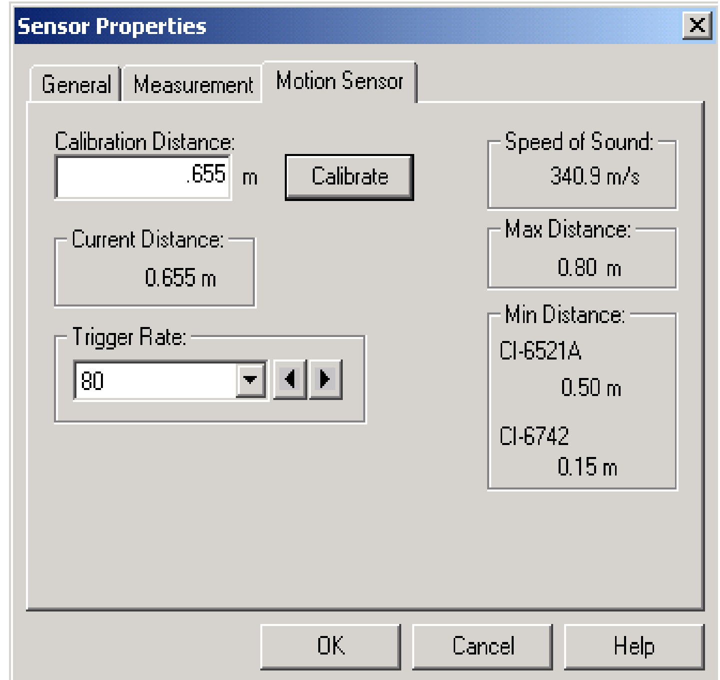




- ❑ Set Sample Rate to 500Hz and Sensitivity to Low.
- ❑ Double-click the Motion Sensor Icon.

Motion Sensor

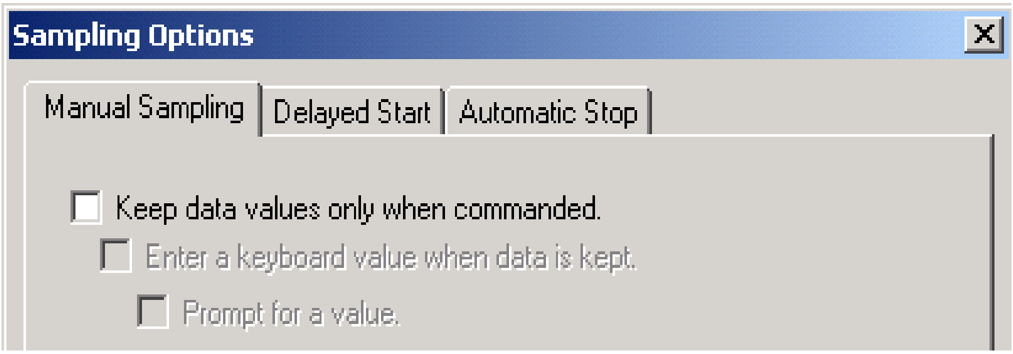




- ❑ Ensure to have Acceleration, Position and Velocity checked
- ❑ Set Trigger Rate to 80Hz

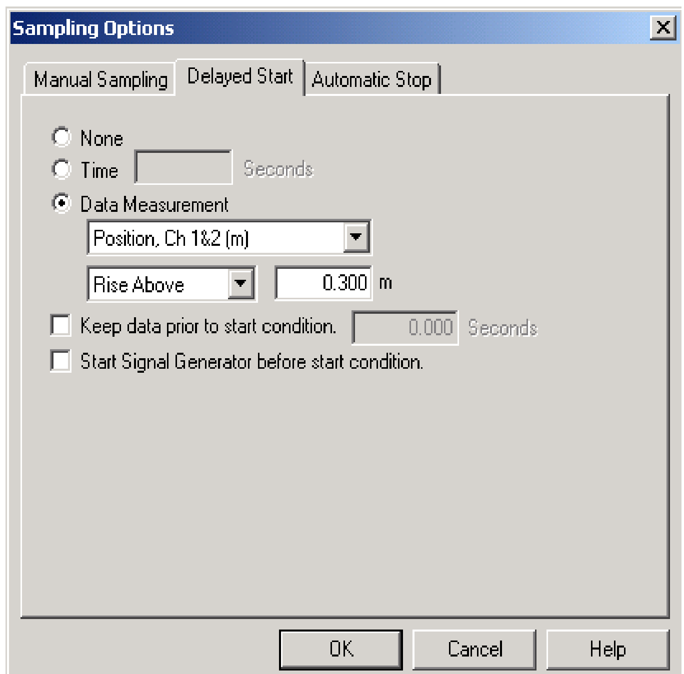
Click 

Sampling Options

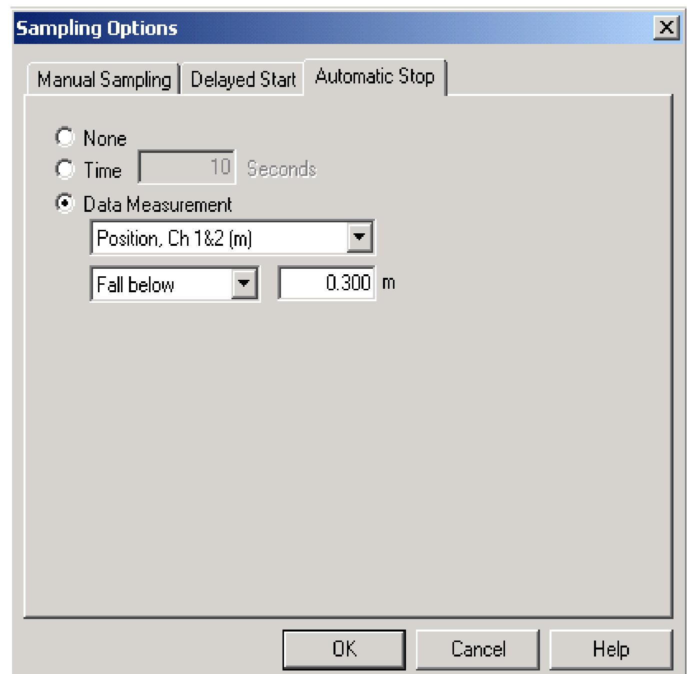


No boxes checked!

Delayed Start and Automatic Stop!



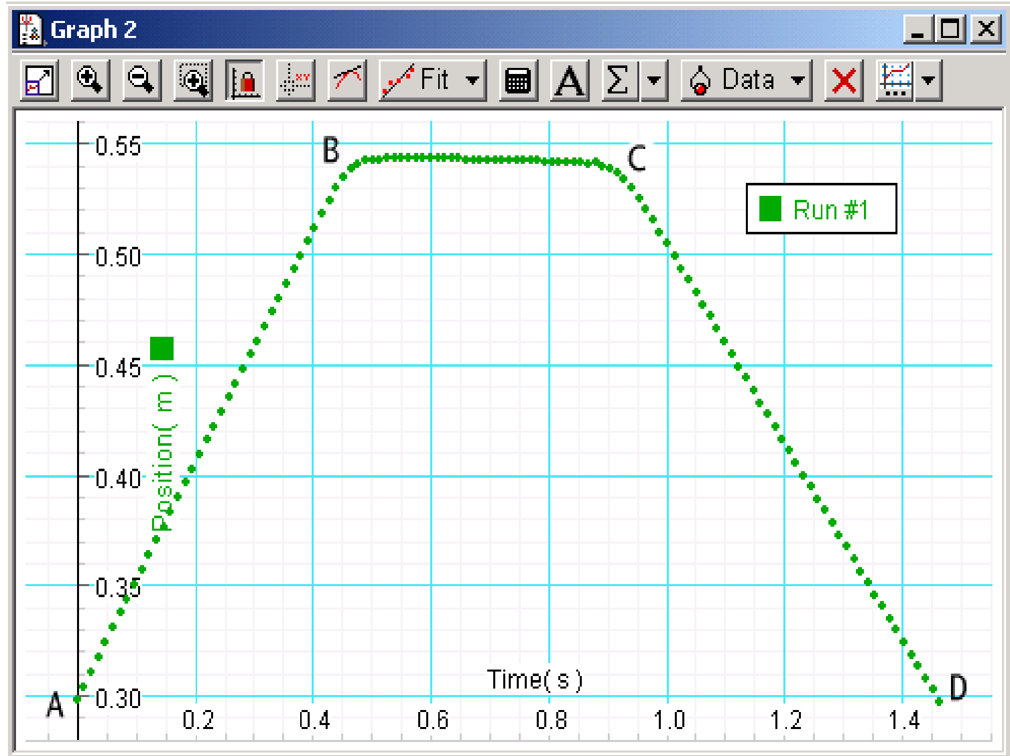
Position rises above 0.3m.



Position falls below 0.3m.

Graph 1

Two equal mass carts A and B collide.
This is X_A vs. time.



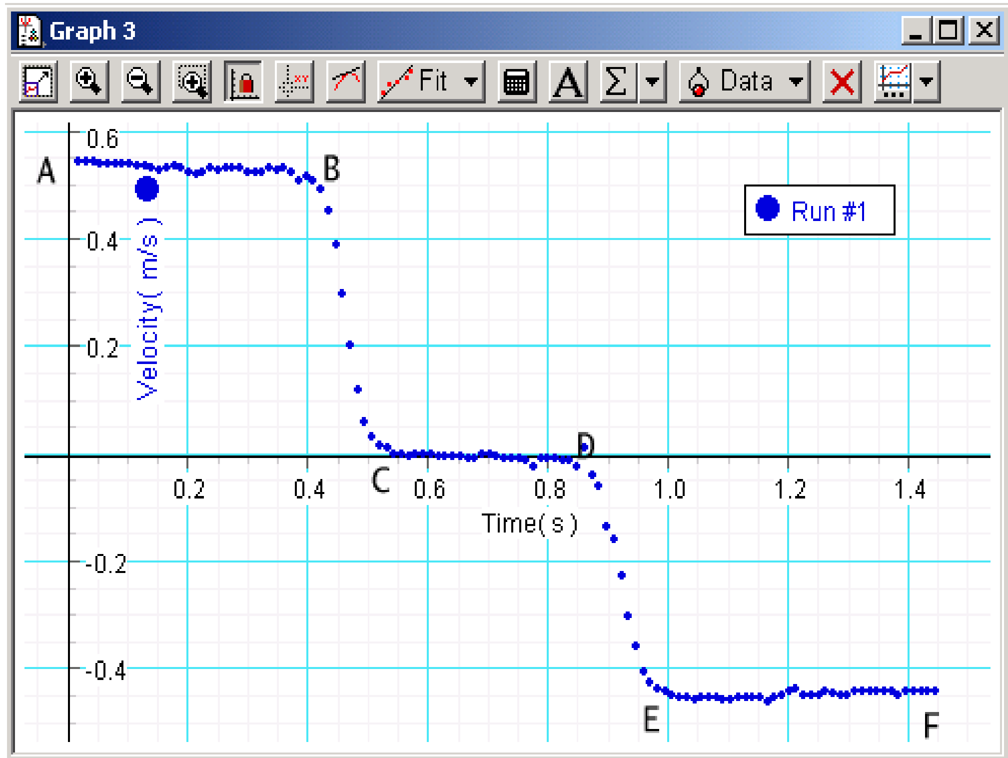
What happens…:

1. Along line AB?
2. At point B?
3. Along line BC?
4. At point C?
5. Along line CD?

At approximately what time does cart B hit the spring?

Graph 2

Two equal mass carts A and B collide.
This is V_A vs. time.

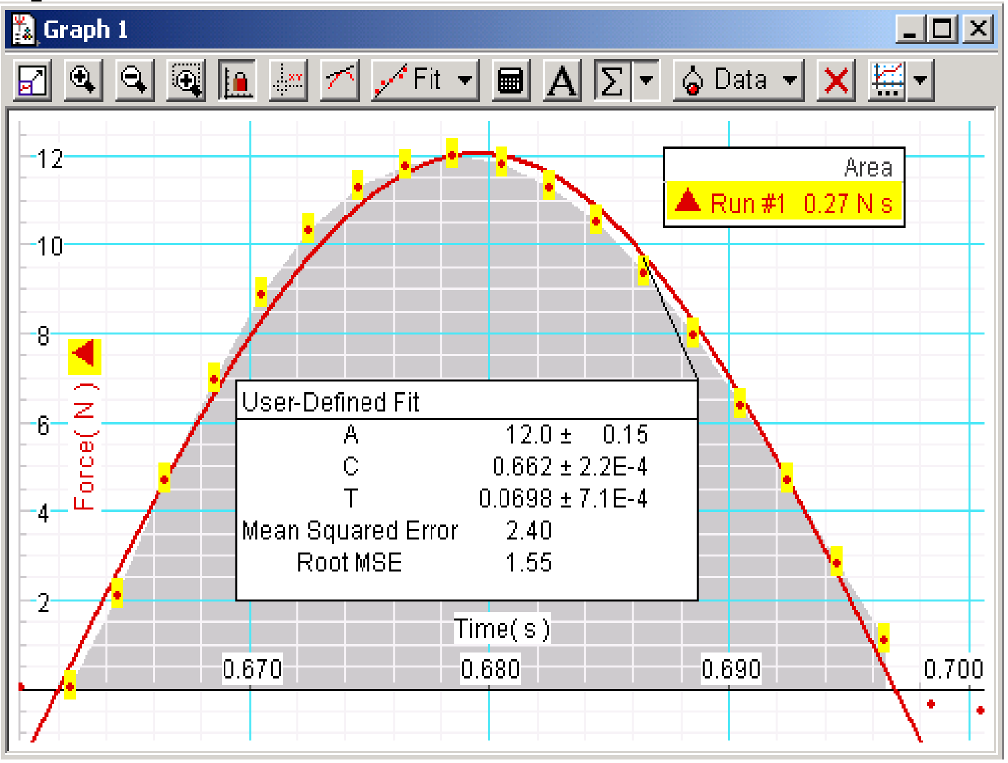


What happens…:

1. Along line AB?
2. Along line BC?
3. Along line CD?
4. Along line DE?
5. Along line EF?

Graph 3

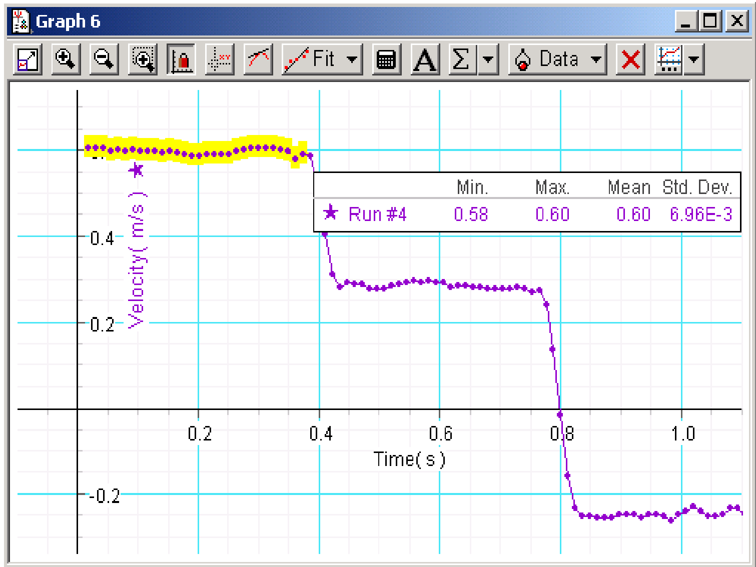
A cart of mass 0.25kg collides with a spring on the force sensor. Here is the force during the collision. The fit is to: $A*\sin(2\pi(x-C)/T)$

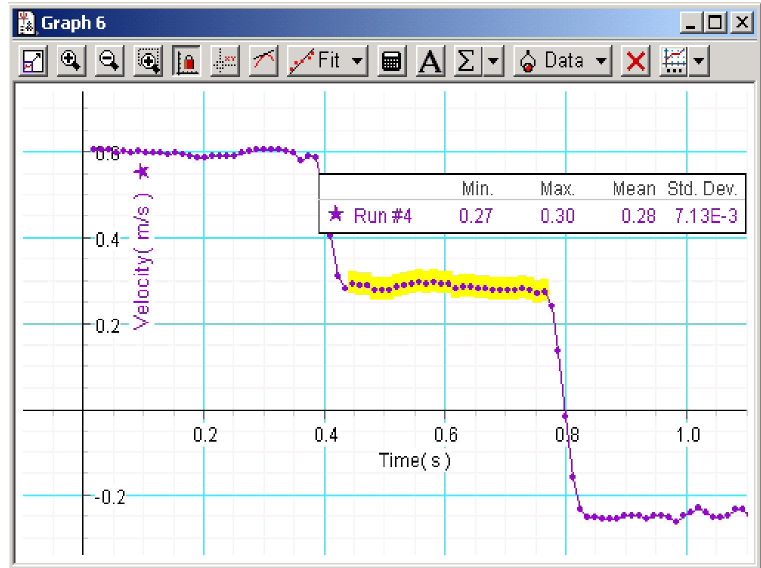


What does the area under the curve tell you?

What can you learn from the parameter T?

Inelastic collisions





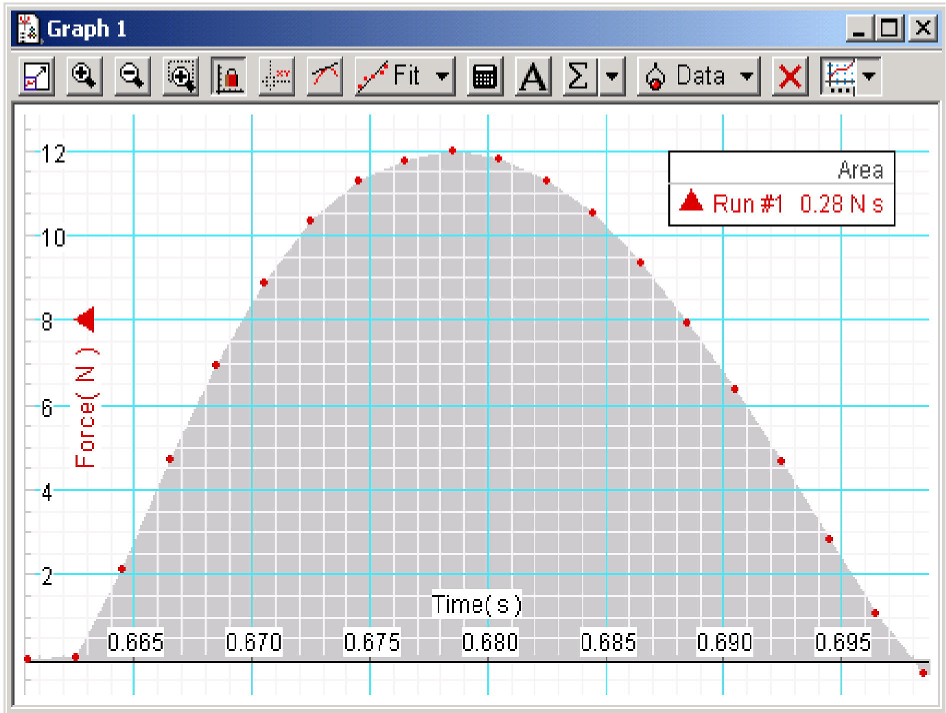
- Use Statistics Tool (Σ) to measure velocities $v_{A,1}$ and v_2, before and after

the collision. Complete the table.

m_A	m_B	$v_{A,1}$	v_2
0.25 kg	0.25kg		
0.25 kg	0.50 kg		
0.50 kg	0.25 kg		

Elastic collisions

- Measure $v_{A,1}$ and $v_{A,2}$ as before.
- Determine the spring impulse J. (Σ tool)
- Find $v_{B,2}$ from J.



m_A	m_B	$v_{A,1}$	$v_{A,2}$	J
0.25 kg	0.25kg			
0.25 kg	0.75 kg			
0.75 kg	0.25 kg			

Inelastic analysis (Homework)

m_A	m_B	$v_{A,1}$	v_{CM}	K_1	K_{CMCS}	v_2	K_2	W_{NC}
0.25 kg	0.25 kg							
0.25 kg	0.50 kg							
0.50 kg	0.25 kg							

Fill in the table!

$$v_{CM} = \frac{m_A v_{A,1}}{m_A + m_B} \qquad K_{CMCS} = \frac{1}{2} m_A (v_{A,1}^2 - v_{CM})^2 + \frac{1}{2} m_B v_{CM}^2$$

$$K_1 = \frac{1}{2} m_A v_{A,1}^2 \qquad K_2 = \frac{1}{2}(m_A + m_B) v_2^2 = K_1 - W_{NC}$$

Elastic analysis (Homework)

m_A	m_B	$v_{A,1}$	v_{CM}	K_1	$v_{A,2}$	J	$v_{B,2}$	K_2
0.25 kg	0.25 kg							
0.25 kg	0.75 kg							
0.75 kg	0.25 kg							

Fill in the table. Part of Problem Set 9.

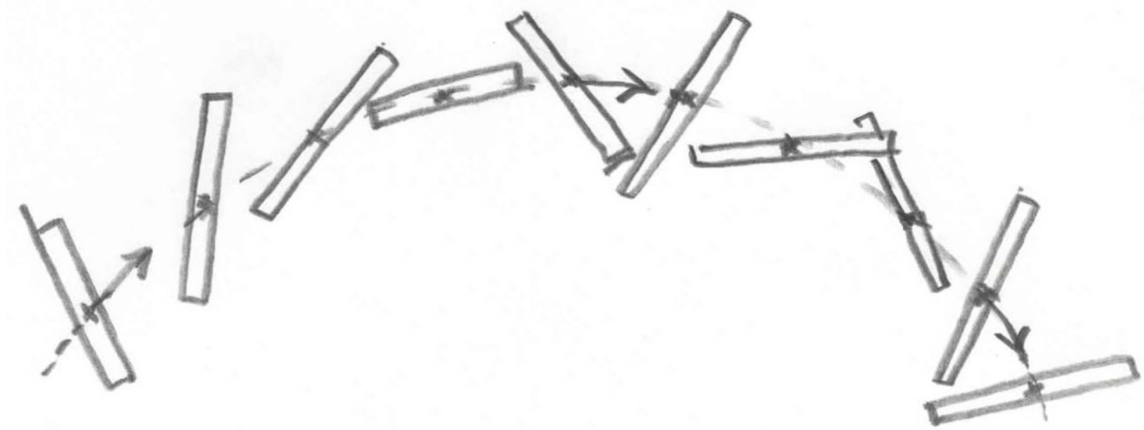
Rotational Motion, Torque, Angular Acceleration, and Moment of Inertia

8.01t

Nov 3, 2004

Rotation and Translation of Rigid Body

Motion of a thrown object



Translational Motion of the Center of Mass

- Total momentum $$\vec{\mathbf{p}}^{total} = m^{total}\vec{\mathbf{V}}_{cm}$$

- External force and acceleration of center of mass

$$\vec{\mathbf{F}}_{ext}^{total} = \frac{d\vec{\mathbf{p}}^{total}}{dt} = m^{total}\frac{d\vec{\mathbf{V}}_{cm}}{dt} = m^{total}\vec{\mathbf{A}}_{cm}$$

Rotation and Translation of Rigid Body

- Torque produces angular acceleration about center of mass

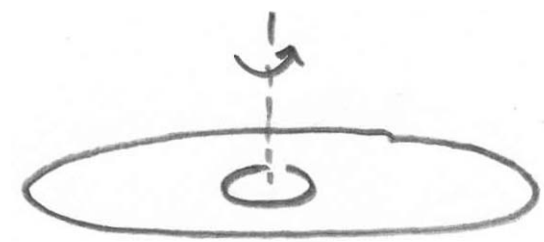
$$\tau_{cm}^{total} = I_{cm}\alpha_{cm}$$

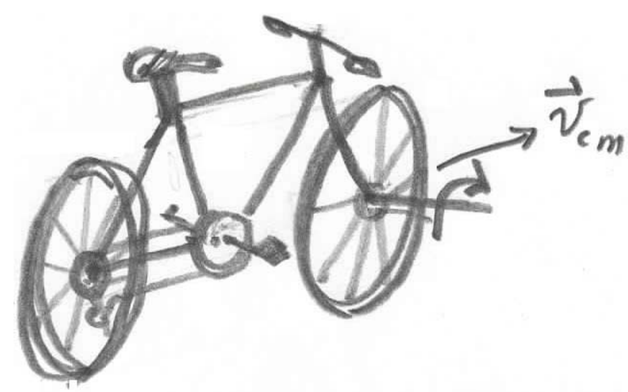
- I_{cm} is the moment of inertial about the center of mass

- α_{cm} is the angular acceleration about center of mass

Fixed Axis Rotation

- CD is rotating about axis passing through the center of the disc and is perpendicular to the plane of the disc.
- For straight line motion, bicycle wheel rotates about fixed direction and center of mass is translating

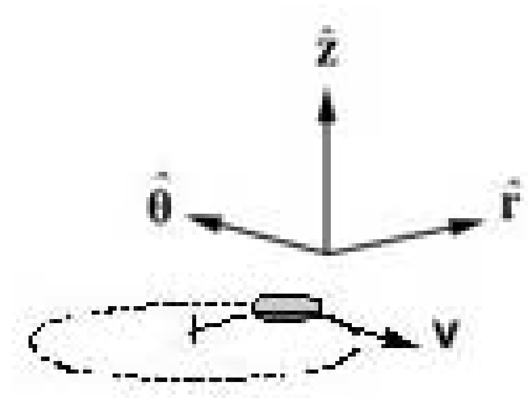




PRS Question

Consider the uniformly rotating object shown below. If the object's angular velocity is a vector (in other words, it points in a certain direction in space) is there a particular direction we should associate with the angular velocity?

1. yes, $\pm\ \hat{\boldsymbol{\theta}}$ direction (tangential
2. yes, $\pm\ \hat{\mathbf{r}}$ direction (radial)
3. yes, $\pm\ \hat{\mathbf{z}}$ direction (perpendicular to plane)
4. yes, some other direction
5. no, the choice is really arbitrary



Fixed Axis Rotation: Angular Velocity and Angular Acceleration

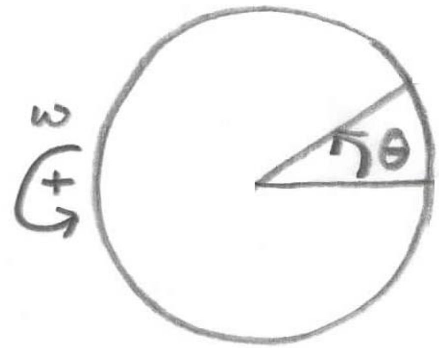
Angle variable θ

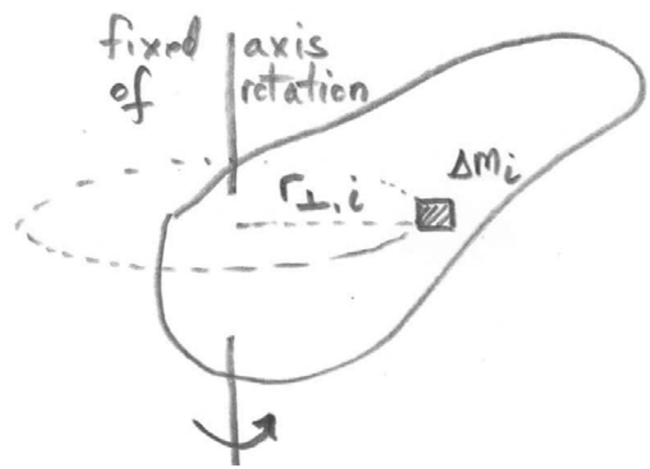
Angular velocity $\omega \equiv \dfrac{d\theta}{dt}$

Angular acceleration $\alpha \equiv \dfrac{d^2\theta}{dt^2}$

Mass element Δm_i

Radius of orbit $r_{\perp,i}$





PRS Question

A ladybug sits at the outer edge of a merry-go-round, and a gentleman bug sits halfway between her and the axis of rotation. The merry-go-round makes a complete revolution once each second. The gentleman bug's angular speed is

1. half the ladybug's.
2. the same as the ladybug's.
3. twice the ladybug's.
4. impossible to determine

PRS Question

A ladybug sits at the outer edge of a merry-go-round that is turning and slowing down. The tangential component of the ladybug's (Cartesian) acceleration is

1. in the direction of the velocity.
2. is opposite the direction of the velocity.
3. is perpendicular to the plane of the merry-go round.
4. is towards the center of the merry-go round.
5. zero

Fixed Axis Rotation: Tangential Velocity and Tangential Acceleration

- Individual mass elements Δm_i

- Tangential velocity $v_{\tan,i} = r_{\perp,i}\omega$

- Tangential acceleration $a_{\tan,i} = r_{\perp,i}\alpha$

- Radial Acceleration

$$a_{rad,i} = \frac{v_{\tan,i}^2}{r_{\perp,i}} = r_{\perp,i}\omega^2$$

Class Problem

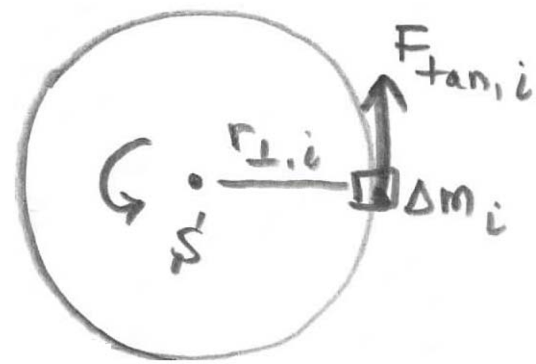
A turntable is a uniform disc of mass 1.2 kg and a radius $1.3 \times 10^{-1} m$. The turntable is spinning initially at a constant rate of (33 rpm). The motor is turned off and the turntable slows to a stop in 8.0 s. Assume that the angular acceleration is constant.

a) What is the initial angular velocity of the turntable?

b) What is the angular acceleration of the turntable?

Newton’s Second Law

- Tangential force on mass element produces torque



- Newton’s Second Law

$$F_{\tan,i} = \Delta m_i a_{\tan,i}$$

$$F_{\tan,i} = \Delta m_i r_{\perp,i} \alpha$$

- Torque

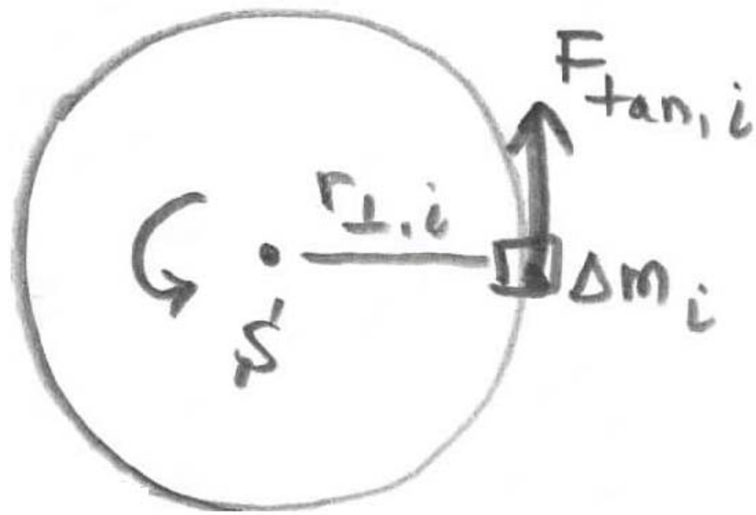
$$\vec{\boldsymbol{\tau}}_{S,i} = \vec{\mathbf{r}}_{S,i} \times \vec{\mathbf{F}}_i$$

Torque

Torque about is S:

$$\vec{\boldsymbol{\tau}}_{S,i} = \vec{\mathbf{r}}_{S,i} \times \vec{\mathbf{F}}_i$$

- Counterclockwise
- perpendicular to the plane



$$\tau_{S,i} = r_{\perp,i} F_{\tan,i} = \Delta m_i (r_{\perp,i})^2 \alpha$$

Moment of Inertia

- Total torque is the sum over all mass elements

$$\tau_S^{total} = \tau_{S,1} + \tau_{S,2} + ... = \sum_{i=1}^{i=N} \tau_{S,i} = \sum_{i=1}^{i=N} r_{\perp,i} F_{\tan,i} = \sum_{i=1}^{i=N} \Delta m_i (r_{\perp,i})^2 \alpha$$

- Moment of Inertia about S:

$$I_S = \sum_{i=1}^{i=N} \Delta m_i (r_{\perp,i})^2$$

- Continuous Body

$$I_S = \int_{body} dm (r_\perp)^2$$

- Mass Element:

  one dim'l: $dm = \lambda dl$

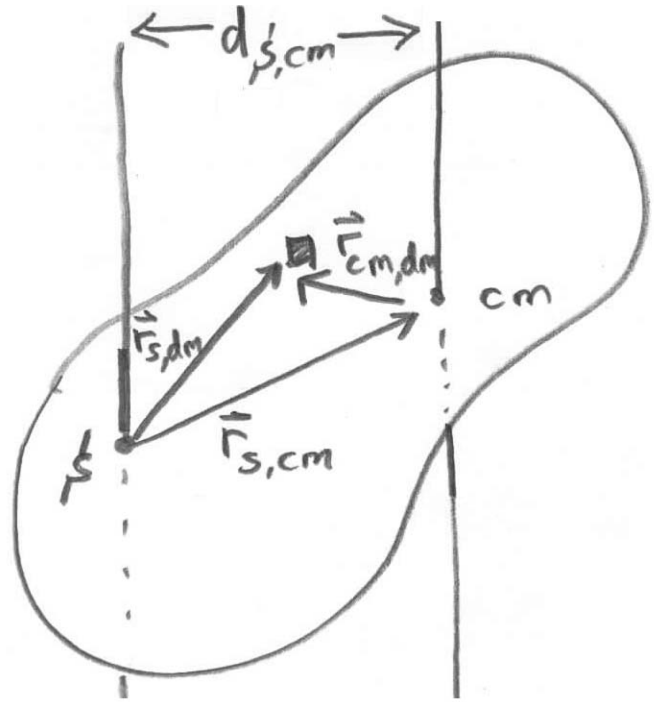
  two dim'l: $dm = \sigma dA$

- Unit: $[kg - m^2]$

- Summary: $\tau_S^{total} = I_S \alpha$

Parallel Axis Theorem

- rigid body of mass m.
- axis one through center of mass of the body.
- parallel axis through point S in body.
- $d_{s,cm}$ perpendicular distance between two parallel axes.



$$I_S = I_{cm} + md_{S,cm}^2$$

Class Problem: Moment of Inertia of a Rod

Consider a thin uniform rod of length L and mass m. In this problem, you will calculate the moment of inertia about two different axes that pass perpendicular to the rod. One passes through the center of mass of the rod and the second passes through an endpoint of the rod.

Strategy: Moment of Inertia

Step 1: Identify the axis of rotation

Step 2: Choose a coordinate system

Step 3*:* Identify the infinitesimal mass element dm.

Step 4: Identify the radius, $r_{\perp}$, of the circular orbit of the infinitesimal mass element dm.

Step 5: Set up the limits for the integral over the body in terms of the physical dimensions of the rigid body.

Step 6: Explicitly calculate the integrals.

Class Problem

A turntable is a uniform disc of mass 1.2 kg and a radius $1.3\times10^{-1}m$. The moment of inertia of the disc is $1.0\times10^{-1}kg-m^2$. The turntable is spinning at an initial constant frequency of $f_0 = 33\ cycles/min$. The motor is turned off and the turntable slows to a stop in 8.0 s due to frictional torque. Assume that the angular acceleration is constant. What is the magnitude of the frictional torque acting on the disc?