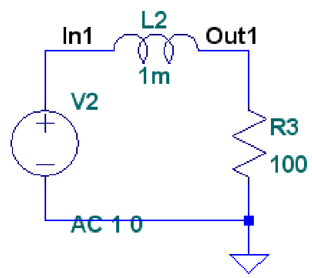
Step by step: Inductor and resistor



The impedance of the resistor is $Z_R = R$ (constant for all frequencies) and the impedance of the inductor is $Z_L = i\omega L$.

1. What's the voltage ratio V_{out}/V_{in}? Compute this like any voltage divider, but keep in mind the answer is a function of frequency. It's called the frequency response.

   Answer: The frequency response of this circuit (and many others like it) is the impedance from the output to ground, divided by the impedance from the input to ground. This is the principle of a voltage divider.
$$\frac{V_{out}}{V_{in}} = \frac{R}{R + i\omega L}$$

2. Find an expression for the frequency where the real and imaginary parts of the frequency response are equal. This the corner frequency, F_c, of the filter.

   Answer: To really figure this out, you'll have to use a common technique in complex numbers: multiplying by a carefully chosen version of 1. Use the conjugate of the denominator:
$$\frac{V_{out}}{V_{in}} = \frac{R}{R + i\omega L}\frac{R - i\omega L}{R - i\omega L} = \frac{R(R - i\omega L)}{R^2 + \omega^2 L^2}$$
   The real and imaginary parts of this are:
$$\mathrm{Re}\left[\frac{V_{out}}{V_{in}}\right] = \frac{R^2}{R^2 + \omega^2 L^2} \qquad \mathrm{Im}\left[\frac{V_{out}}{V_{in}}\right] = \frac{\omega^2 L^2}{R^2 + \omega^2 L^2}$$
   These parts are equal at the frequency where $R^2 = \omega^2 L^2$, so $R = \omega L$ and $\omega = \frac{R}{L}$. (You could also have just looked at the real and imaginary parts of the denominator in the first place, since the numerator is real.) Keeping in mind that the frequency ω is expressed in radians per second, and the frequency F_c needs to be in Hz:
$$F_c = \frac{\omega_c}{2\pi} = \frac{R}{2\pi L}$$
   From that expression you can see that using a larger inductance lowers the corner frequency of the filter. If you imagine putting an inductor in series with a speaker, this means that larger inductors will create an earlier rolloff in the frequency response (and hence, having a stronger effect). The same is not true of capacitors!

3. What kind of filter is this? Lowpass, highpass, bandpass, notch, or something else?

   Answer: This is a lowpass filter. It removes high frequencies and allows low frequencies to pass on to the load (speaker) un-altered.

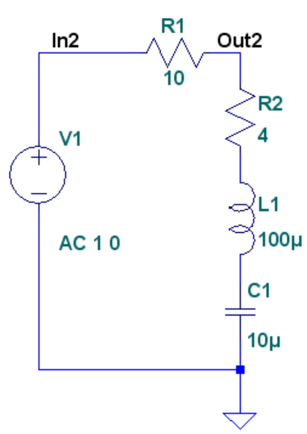
4. Compute the corner frequency of the filter, in Hz, using the component values on the schematic.

   Answer: Using the expression for F_c above:

$$F_c = \frac{R}{2\pi L} = \frac{100}{2\pi \times 10^{-3}} = 1.6 \times 10^4 \text{ Hz} = 16 \text{ kHz}$$

   That's actually a pretty high frequency; it would be difficult to hear the effect of this filter on music. Note, however, that 100 ohms is a lot higher than the impedance of a typical speaker (4 to 8 ohms). If you put a 1 mH inductor in series with a speaker, the corner frequency will be closer to 1 kHz, and you'll definitely be able to hear that.

Still step by step: Notch filter



This circuit is a little more complicated. Let's try to figure out what it does without making a mess of complex numbers.

1. If the L-C combination has a really big impedance, what's the ratio V_{out}/V_{in}?

   Answer: This is a voltage divider too! If the impedance from the output to ground is really big, than it makes up almost all of the impedance from the input to ground.

$$\frac{V_{out}}{V_{in}} = 1$$

2. At what frequency does the L-C combination have zero impedance? (Hint: $\frac{1}{i} = -i$, so $\frac{1}{i\omega C} = -\frac{i}{\omega C}$.)

   Answer: Look at the impedance of an inductor and capacitor in series, set it to 0, and solve for ω:

$$Z_L + Z_C = i\omega L + \frac{1}{i\omega C} = i\omega L - \frac{i}{\omega C} = 0$$
$$\omega L = \frac{1}{\omega C}$$
$$\omega^2 LC = 1$$
$$\omega = \frac{1}{\sqrt{LC}} \to F = \frac{1}{2\pi\sqrt{LC}}$$

3. If the L-C combination has zero impedance, what's the ratio V_{out}/V_{in}?

   Answer: If you know the L-C combination has zero impedance, that means you can act like it's a wire. So we're left with a simple resistive voltage divider. There's a resistor (R_2) from the output to ground, and two series resistors (R_1 and R_2) from input to ground:

$$\frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + R_2}$$

4. What kind of filter is this? Lowpass, highpass, bandpass, notch, or something else?

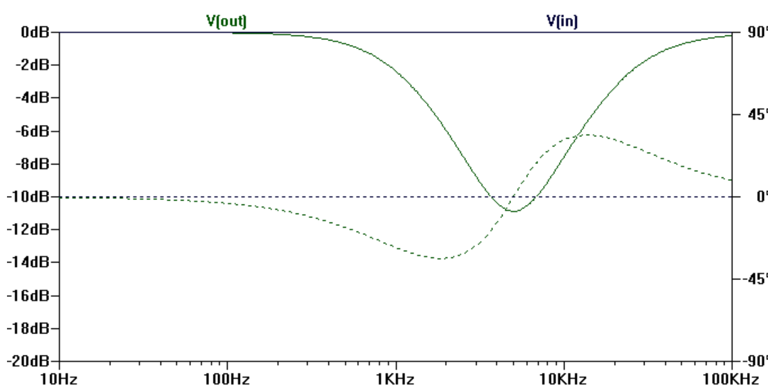
   Answer: This filter doesn't do much (since $V_{out}/V_{in} = 1$) at most frequencies. But it does block out signals to some extent ($V_{out}/V_{in} = \frac{R_2}{R_1+R_2}$) at a particular frequency. That's called a notch filter.

5. Compute the numerical value of your first three answers (output ratios and frequency).

   Answer: All we have to do is plug numbers into the above formulas:
   - Filter gain at most frequencies: $\frac{V_{out}}{V_{in}} = 1$
   - Resonance frequency: $F = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{1\times 10^{-9}}} = 5030$ Hz
   - Filter gain at notch frequency: $\frac{V_{out}}{V_{in}} = \frac{R_2}{R_1+R_2} = \frac{4}{14} = \frac{2}{7}$

6. Try to draw the frequency response graph of this circuit.

   Answer: You can see that the frequency response is flat over most of the audio range, with a dip of about -11 dB (close to $\frac{2}{7}$) at 5 KHz, just as we predicted. (Ask me if you're curious how to simulate circuits this way!)



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