D.U.S.P.
11.203 Microeconomics

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## A Note on the Tax Incidence Problem

This note gives the two equations that can be solved to determine what part of a tax is born by consumers and what part is born by producers.

To restate the problem, we start with a commodity - say a big cheeseburger - that costs $\$ 5.00$ and we impose a tax of $\$ .50$ cents on each cheeseburger. The questions are:

By how much does the price to consumers rise?
By how much does the price to producers fall?
As we saw in class, the diagram of the problem looks like the figure below. When the tax is imposed, there are now two demand curves. The original demand curve is the one consumers still see. The second demand curve is the one producers see (dotted line). At every quantity, the second demand curve lies $\$ .50$ below the original demand curve since the producer gets $\$ .50$ less than whatever the consumer pays. The equilibrium quantity occurs where this second demand curve intersects the supply curve. The rest of the graph is detailed below.


We can solve for $\Delta \mathrm{P}^{\mathrm{D}}$ (the increased price for consumers) and $\Delta \mathrm{P}^{\mathrm{S}}$ (the decreased price for producers) by setting up the following equations (the superscripts $D$ and $S$ refer to demand and supply, respectively).

First, we know that the sum of the two terms must equal the $\$ .50$ tax. Because $\Delta \mathrm{P}^{\mathrm{S}}$ represents a price fall, it is a negative quantity and so we write this equation:

$$
\text { 1) } \Delta \mathrm{P}^{\mathrm{D}-} \Delta \mathrm{P}^{\mathrm{S}}=\$ .50 \quad \text { (i.e. we multiply } \Delta \mathrm{P}^{\mathrm{S}} \text { by }-1 \text { to turn it into a positive number) }
$$

Next, we turn to the elasticity of the demand curve, $\mathrm{e}_{\mathrm{D}}$ and the elasticity of the supply curve, $\mathrm{e}_{\mathrm{S}}$.

Let the original (pre-tax) equilibrium price and quantity be denoted as $\mathrm{P}^{0}$ and $\mathrm{Q}^{0}$ and the change in quantity after the tax denoted as $\Delta \mathrm{Q}$.

With respect to the demand curve, we can say:

$$
\left(\Delta \mathrm{Q} / \mathrm{Q}^{0}\right) /\left(\Delta \mathrm{P}^{\mathrm{D}} / \mathrm{P}^{0}\right)=\mathrm{e}_{\mathrm{D}}
$$

In other words, just considering the demand curve, the percentage change in quantity purchased divided by the change in price paid by consumers must equal the elasticity of demand (this is our standard equation).

For reasons we will see in a moment, I want to multiply both sides of this equation by
Which will leave me with: $\left(\Delta \mathrm{Q} / \mathrm{Q}^{0}\right)=\left(\mathrm{e}_{\mathrm{D}}\right)\left(\Delta \mathrm{P}^{\mathrm{D}} / \mathrm{P}^{0}\right)$

We can then write a similar equation for the supply curve involving the change in price seen by the producers:

$$
\left(\Delta \mathrm{Q} / \mathrm{Q}^{0}\right) /\left(\Delta \mathrm{P}^{\mathrm{S}} / \mathrm{P}^{0}\right)=\mathrm{e}_{\mathrm{S}} \quad \text { or }\left(\Delta \mathrm{Q} / \mathrm{Q}^{0}\right)=\left(\mathrm{e}_{\mathrm{S}}\right)\left(\Delta \mathrm{P}^{\mathrm{S}} / \mathrm{P}^{0}\right)
$$

We can combine these two equations by noting that $\left(\Delta Q / Q^{0}\right)$ is the same in both equations - i.e. producers and consumers started at the same original quantity and saw the same change in quantity. This means we can set the two equations equal to each other.

$$
\left(\mathrm{e}_{\mathrm{D}}\right)\left(\Delta \mathrm{P}^{\mathrm{D}} / \mathrm{P}^{0}\right)=\left(\mathrm{e}_{\mathrm{S}}\right)\left(\Delta \mathrm{P}^{\mathrm{S}} / \mathrm{P}^{0}\right)
$$

We can simplify a little further by noting that $\mathrm{P}^{0}$, the price in the original equilibrium, was the same for both consumers and producers - i.e. they started at the same point so we can cancel that term out. This leaves us with:

$$
\left(\mathrm{e}_{\mathrm{D}}\right)\left(\Delta \mathrm{P}^{\mathrm{D}}\right)=\left(\mathrm{e}_{\mathrm{s}}\right)\left(\Delta \mathrm{P}^{\mathrm{S}}\right)
$$

or: $\quad(2)\left(\Delta \mathrm{P}^{\mathrm{D}}\right) /\left(\Delta \mathrm{P}^{\mathrm{S}}\right)=\left(\mathrm{e}_{\mathrm{S}} / \mathrm{e}_{\mathrm{D}}\right)$

This is our second equation. We are almost at the solution but before we go further, we can see if this makes any sense. First of all, we know the two sides of the equation will have the same sign: $\Delta \mathrm{P}^{\mathrm{D}}$ is positive, $\Delta \mathrm{P}^{\mathrm{S}}$ is negative, $\mathrm{e}_{\mathrm{S}}$, the elasticity of the supply curve is positive (think about that) and we know $\mathrm{e}_{\mathrm{D}}$. the elasticity of the demand curve is negative.

Now think about the magnitudes. Suppose that the supply curve is inelastic and the demand curve is elastic. That means term on the right hand side of the equation lies between 0 and -1 .

It follows that the left hand side of the equation must also lie between 0 and -1.This means that the rise in price to consumers must be smaller than the fall in price to producers.

That is what we argued in class: If supply is more inelastic than demand, then producers will absorb the greater part of the tax. If demand is more inelastic than supply, then consumers will absorb the greater part of the tax. Satisfy yourself that this what equation (2) says.

We now can solve the problem. We are given the two elasticities so we can use equation (2) to compute the ratio of the two price changes. We also know from equation (1) that the sum of the two price changes (correcting for the fact that $\Delta \mathrm{P}^{\mathrm{S}}$ is a price fall) must equal . $\$ 50$. Together, these two equations are enough to solve the problem.
(To combine these equations, see that $\Delta \mathrm{Q}, \mathrm{Q}^{0}$ and $\mathrm{P}^{0}$ are the same in the two equations. In other words, producers and consumers started off from the same original equilibrium and they have seen the same reduction in quantity. This means we can write

$$
\left(\Delta \mathrm{Q} / \mathrm{Q}^{0}\right) /\left(\Delta \mathrm{P}^{\mathrm{S}} / \mathrm{P}^{0}\right)=\mathrm{e}_{\mathrm{S}}
$$

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