8.851 Course 2006 (Lecture Notes, I. Stewart)

Soft - Collinear Effective Theory (SCET)

For this part we'll switch sign convention for g

 $= i g T^A \gamma^\mu$ to agree with literature

Outline

Class 1: Intro, Degrees of Freedom, Scales
Expansion of Spinors, Propagators,
Power Counting see ②, ③

Class 2: Construction of Currents, Lagrangian
Multipole Expansion, Labels, Grid in detail
see ②, ③, ⑩ (not in notes)

Class 3: SCET_{I} Lagrangian, Gauge Symmetry, ③, ④, ⑥
Reparameterization Invariance (RPI)

Class 4: More RPI, Ultrasoft - Collinear Fact.
Hard-Collinear Factorization, IR divs,
Matching, Running see ④, ①, ②, ③

Class 5: DIS see ⑧
Soft - Collinear Interactions ④

Class 6: SCET_{II} ④, ⑦, ⑩
Power Counting Formulae ⑤
eg. $\gamma^* \gamma \to \pi^0$ ⑧, eg $B \to D\pi$ ⑨
eg. $B \to X_s \gamma$. Define a Jet ④
(Jets in e^+e^-, see ⑪)

Refs I used

① hep-ph/0005275
② hep-ph/0011336
③ hep-ph/0107001
④ hep-ph/0109045
Gauge Inv.
Power Counting
⑤ hep-ph/0205289
⑥ hep-ph/0204229
RPI
Gauge Inv. at λ^2
⑦ hep-ph/0303156
⑧ hep-ph/0202088
Hard-Scattering
⑨ hep-ph/0107002
$B \to D\pi$
⑩ hep-ph/0605001
0-bin
⑪ hep-ph/0212255
hep-ph/0603066

Intro, Degrees of Freedom, Coordinates

Want an EFT for energetic hadrons, $E_H \simeq Q \gg \Lambda_{QCD}$

Why? • Many processes have large regions of phase space where the hadrons are energetic, $E_H \gg M_H$

eg B-decays $B \to \pi e \nu$, $B \to K^* \gamma$, $B \to \pi\pi$, $B \to X_u e \nu$, $B \to X_s \gamma$, $B \to D^* \pi$, ...

$M_B = 5.279$ GeV $\gg \Lambda_{QCD}$

eg. Hard Scattering

$e^- p \to e^- X$ (DIS), $p\bar{p} \to X \ell^+ \ell^-$ (Drell Yan),

$\gamma^* \gamma \to \pi^0$, $\gamma^* p \to \gamma^{(*)} p'$ (Deeply Virtual Compton Scattering)

• Need to separate perturbative, $\alpha_s(Q)$ & non-perturbative "$\alpha_s(\Lambda_{QCD})$" effects ➡ factorization

What are the low energy degrees of freedom?

eg 1 $B \to D\pi$

in B-rest frame $p_\pi^\mu = (2.310 \text{ GeV}, 0, 0, -2.306 \text{ GeV})$

$\simeq Q\, n^\mu$ to good aprox.

$Q \gg \Lambda$, $n^\mu \equiv (1, 0, 0, -1)$, $n^2 = 0$ light-like

$\uparrow$ in 0,1,2,3 basis

Basis vectors $n^\mu, \bar{n}^\mu$

Use Light-Cone coordinates : $n^2 = 0$, $\bar{n}^2 = 0$, $n \cdot \bar{n} = 2$

vectors $p^\mu = \frac{n^\mu}{2}\, \bar{n}\cdot p + \frac{\bar{n}^\mu}{2}\, n \cdot p + \underset{\updownarrow \text{ orthogonal } n^\mu, \bar{n}^\mu}{p_\perp^\mu}$

metric $g^{\mu\nu} = \frac{n^\mu \bar{n}^\nu}{2} + \frac{\bar{n}^\mu n^\nu}{2} + g_\perp^{\mu\nu}$

epsilon $\epsilon_\perp^{\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta}\, \frac{\bar{n}_\alpha n_\beta}{2}$

Define

$p^+ \equiv n \cdot p$

$p^- \equiv \bar{n} \cdot p$

- since $n^2 = 0$ we needed to define complementary vector $\bar{n}$
- choice $n^\mu = (1,0,0,-1)$, $\bar{n}^\mu = (1,0,0,1)$ is possible, but other choices also work eg. $n^\mu = (1,0,0,-1)$
  $\bar{n}^\mu = (3,2,2,1)$

(more on this later)

In $B \to D\pi$ the B, D are soft $E_H \sim M_H$
& we can use HQET for their constituents
ie quarks & gluons with $p^\mu \sim \Lambda$

But pion is "collinear", $E_H \gg M_H$

In rest frame $\textcircled{\pi}$ has quark & gluon constituents $p^\mu \sim (\overset{+}{\Lambda}, \overset{-}{\Lambda}, \overset{\perp}{\Lambda})$

boosting for $B \to D\pi$ $\textcircled{\pi}\!\rightarrow$ has constituents $p^\mu \sim \left(\frac{\Lambda^2}{Q}, Q, \Lambda\right)$
$\equiv$ collinear fluctuations around $(0, Q, 0) = p_\pi^\mu$

Note : Boost in direction orthogonal to $\perp$ directions changes p^+, p^- multiplicatively

$p^+ \to a\, p^+$

$p^- \to \frac{1}{a}\, p^-$

Generically

$$(p^+, p^-, p^\perp) \sim Q\,(\lambda^2, 1, \lambda) \quad \text{is collinear}$$

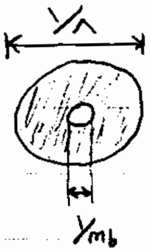
where $\lambda \ll 1$ is small parameter. (above eg. $\lambda = \frac{\Lambda}{Q}$)

What makes this EFT different?

- usually we separate scales $m_1 \gg m_2$ and have

$$\sum_{i=1}^{n} \underset{\substack{\uparrow \\ \text{short distance} \\ \text{Wilson Coeffs}}}{C_i(\mu, m_1)} \;\; \underset{\substack{\uparrow \\ \text{long distance} \\ \text{operators}}}{\mathcal{O}_i(\mu, m_2)}$$

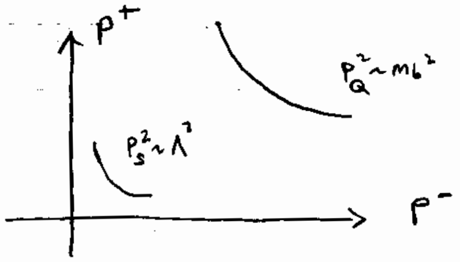
eg in HQET the B-meson



$m_b \gg \Lambda$

$p_Q^\mu \sim m_b$
$p_s^\mu \sim \Lambda$

picture momenta



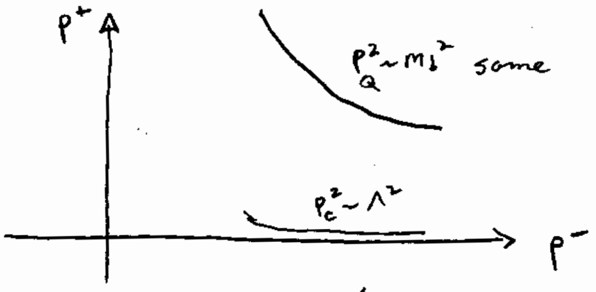
well separated in all components

- now we have overlap between perturbative & non-perturbative momenta in p^- component

for collinear pion

$E_\pi \sim m_b$

$p_c \sim (\frac{\Lambda^2}{m_b}, m_b, \Lambda)$



↑ overlap in p^-, but $p_c^2 \ll p_Q^2$ still

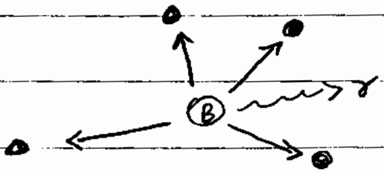
a. inclusive decay $B \to X_s \gamma$ from $b \to s\gamma$

$\uparrow \geq 1$ hadron, summed over

in general $E_\gamma = \dfrac{m_B^2 - m_{X_s}^2}{2m_B} \in \left[0 , \dfrac{m_B^2 - m_{K^*}^2}{2m_B} \right]$

for $m_X \in [m_B, m_{K^*}]$

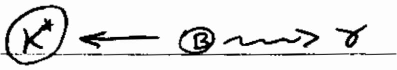
For $m_X^2 \sim m_B^2$



standard OPE just like we did for $B \to X_c e \bar{\nu}$

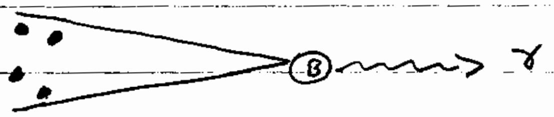
X has hadrons in all directions

For $m_X^2 \sim \Lambda^2$



exclusive decay (~~not~~ inclusive)

For $m_X^2 \sim m_B \Lambda$



jet of hadrons in X

jet constituents $(p^+, p^-, p_\perp) \sim (\Lambda, Q , \sqrt{\Lambda Q}) \sim Q (\lambda^2, 1, \lambda)$

collinear again

this time $\lambda = \sqrt{\frac{\Lambda}{Q}} \ll 1$

Infrared Degrees of Freedom

have $p^2 \lesssim Q^2\lambda^2$

modes	$p^\mu = (+, -, \perp)$	p^2
collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$
ultrasoft (usoft)	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$

Off shell modes have $p^2 \gg Q^2\lambda^2$ and are integrated out into wilson coefficients $C(\mu)$

eg $p^\mu \sim Q(1,1,1)$

Two useful cases

				examples
$SCET_I$	$\lambda = \sqrt{\frac{\Lambda}{Q}}$	collinear usoft	$p_c^2 \sim Q\Lambda$ $p_u^2 \sim \Lambda^2$	$B \to X_s \gamma$, * DIS, ...
$SCET_{II}$	$\lambda = \frac{\Lambda}{Q}$	collinear soft	$p_c^2 \sim \Lambda^2$ $p_s^2 \sim \Lambda^2$	$B \to D\pi$, $\gamma^*\gamma \to \pi^0$, ...

The theory $SCET_{II}$ can be derived from $SCET_I$ so we'll study I first

Factorization : $\sum_i C_i\, \mathcal{O}_i$ becomes continuous

$$\int d\xi \quad C(\xi) \quad \mathcal{O}(\xi)$$

since p^- were same size

Collinear Spinors u_n : labelled by direction n

(recall HQET spinors u_v)

massless QCD spinors
(Dirac Rep)

$$u(p) = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ \frac{\bar{\sigma}\cdot\bar{p}}{p^0}\, u \end{pmatrix} , \quad v(p) = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\bar{p}\cdot\bar{\sigma}}{p^0}\, v \\ v \end{pmatrix}$$

let $n^\mu = (1,0,0,1)$ and expand , $\bar{n}\cdot p = p^0 + p^3 = \frac{Q}{2} + \frac{Q}{2}$

$\bar{n}^\mu = (1,0,0,-1)$ $n\cdot p \ll Q$, $p_\perp \ll Q$

$$\frac{\bar{\sigma}\cdot\bar{p}}{p^0} = \sigma^3$$

$$u_n = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ \sigma^3 u \end{pmatrix} = \left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} , \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \right\} \quad \text{particles}$$

$$v_n = \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma^3 v \\ v \end{pmatrix} = \left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} , \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} \right\} \quad \text{antiparticles}$$

$$\not{n} = \begin{pmatrix} \mathbb{1} & -\sigma^3 \\ \sigma^3 & -\mathbb{1} \end{pmatrix} \qquad \not{n}\, u_n = \not{n}\, v_n = 0$$

$$\frac{\not{n}\not{\bar{n}}}{4} = \frac{1}{2} \begin{pmatrix} \mathbb{1} & \sigma^3 \\ \sigma^3 & \mathbb{1} \end{pmatrix} \qquad \frac{\not{n}\not{\bar{n}}}{4}\, u_n = u_n \, , \quad \frac{\not{n}\not{\bar{n}}}{4}\, v_n = v_n$$

$\nearrow$

Projection Operator , $\mathbb{1} = \frac{\not{n}\not{\bar{n}}}{4} + \frac{\not{\bar{n}}\not{n}}{4}$

field $\psi^{QCD} = \xi_n + \xi_{\bar{n}}$

we'll integrate out "small" component $\xi_{\bar{n}}$

Collinear Propagators $\quad p^2 + i\epsilon = \bar{n}\cdot p\; n\cdot p + p_\perp^2 + i\epsilon$

$\sim \lambda^0 * \lambda^2 + \lambda * \lambda$ same size

Fermions

$$\frac{i\not{p}}{p^2+i\epsilon} = \frac{i\not{n}}{2}\,\frac{\bar{n}\cdot p}{p^2+i\epsilon} + \dots$$

$$= \frac{i\not{n}}{2}\,\frac{1}{n\cdot p + \frac{p_\perp^2}{\bar{n}\cdot p} + i\epsilon\,\mathrm{sign}(\bar{n}\cdot p)} + \dots$$

↑ from $T\{\xi_n(x), \bar{\xi}_n(0)\}$

Gluons $\quad \frac{-i g^{\mu\nu}}{p^2+i\epsilon}$ stays same as QCD $\quad g^{\mu\nu}\sim\lambda^0$

(true in any gauge)

↑ (eg Feyn. Gauge)

Power counting for collinear fields

$$\mathcal{L} = \int d^4x \;\; \bar{\xi}_n \;\; \frac{\not{\bar{n}}}{2}\left[i n\cdot\partial + \dots \right] \;\; \xi_n$$

$$\lambda^{-4} \qquad \lambda^{a} \qquad \lambda^{2} \qquad \lambda^{a} \qquad = \lambda^{2a-2}$$

set $\mathcal{L}\sim\lambda^0$ ie normalize Kinetic term so no λ's

then $\boxed{\xi_n \sim \lambda}$

For gluons find $A_n^\mu = (A_n^+, A_n^-, A_n^\perp) \sim (\lambda^2, 1, \lambda)$

just like collinear momenta

ie have $P^\mu + A^\mu = iD^\mu$ homogeneous covariant derivative