8.851 Homework 1

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Problem 1) Matching with Massive Electrons

Consider QED with electrons and photons. For photon momenta q^μ much less than m_e we can integrate out the electrons.

a) Calculate the one-loop photon vacuum polarization diagram with dimensional regularization and $\overline{\rm MS}$, and expand $\Pi(q^2)$ in q^2/m_e^2.
[Note: The first term in the expansion motivates matching onto a theory without electrons at a scale $\mu \sim m_e$ rather than $\mu \sim 1\,{\rm TeV}$ so that a large logarithm does not upset the perturbative expansion in α.]

b) Write down a Lagrangian with a gauge invariant photon operator that reproduces the second term in the expansion. Use your calculation from part a) to determine the Wilson coefficient of the operator at this order in α.

c) What QED symmetry(s) forbid dimension-6 operators with three field strengths from ever appearing?

d) At dimension-8, operators are generated which give light-by-light scattering. Determine the number of α's in their coefficients. Then use dimensional analysis in the low energy effective theory to numerically estimate the size of the $\gamma\gamma \to \gamma\gamma$ cross section for $10\,{\rm keV}$ photons.

Problem 2) Right Handed Neutrinos

Consider adding three right-handed singlet neutrinos N_R to the standard model. A Majorana mass term is allowed, so

$$\mathcal{L}_N = \bar{N}_R\, i\,\partial\!\!\!/\, N_R - \frac{1}{2}\bar{N}_R^c M N_R - \frac{1}{2}\bar{N}_R M^* N_R^c\,, \qquad (1)$$

where $N_R^c = C\bar{N}^T$ is the charge conjugate field, $C = i\gamma_2\gamma_0$ (in the Dirac representation), and M is a complex symmetric Majorana mass matrix.

a) Use gauge symmetry to determine the most general dimension-4 operators that couple N_R to the other fields in the standard model.

b) Starting with Eq. (1) transform the N_R fields to three Majorana mass eigenstates that satisfy $N_i = N_i^c$, $i = 1, 2, 3$ with real masses M_i. For the diagonalization of the Majorana mass matrix feel free to simply quote the relevant linear algebra theorem.

c) Count the total number of physical parameters in M and the coefficients of the operators in part a).
[Hint: Consider the $G = U(3) \times U(3) \times U(3)$ flavor symmetry of the free L_L, e_R, and N_R kinetic terms. This symmetry is broken by the mass and Yukawa matrices, so the number of physical parameters can be obtained by subtracting the number of parameters in G from the number in the original Yukawa and Majorana matrices. For the ambitious, repeat the counting for n families of light leptons and n' right-handed neutrinos. How many of the parameters are CP-odd phases? The case $n = 3$, $n' = 2$ should agree with the 14 parameters mentioned in class. In this case 3 parameters are CP-odd phases.]

d) Take the masses M_i large compared to the electroweak scale and integrate out the right handed neutrinos at tree level. Show that the leading term reduces to the form of the dimension-5 standard model operator we discussed in class.

Problem 3) X-Decay

You observe a very heavy ($m > 1\,\mathrm{TeV}$) particle X of unknown origin which decays to well known light hadrons and/or leptons. You observe it decaying to two light particles $X \to Y_1 Y_2$ and to three $X \to Y_1 Y_2 Y_3$. Assume you have a model where the couplings for these two transitions are the same size. What can you estimate for the size of the ratio of decay rates $R = \Gamma(X \to Y_1 Y_2 Y_3)/\Gamma(X \to Y_1 Y_2)$?