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PROFESSOR: So good afternoon. Let's have an on-time departure. Last week, we talked about quantum states of light. So we're talking about the photon, the quantum description of the electromagnetic field. And I introduced for you the most classical quantum state of the electromagnetic field, namely the coherent state. I don't want to go through the formalism again, but the coherent state has a simple definition, simple but subtle. It's an eigenstate of the annihilation operator, and it has a complex eigenvalue α . And from that, you can pretty much derive everything you want to know about coherent states. Colin.

AUDIENCE: What was the purpose of-- whoever discovered eigenstates-- what was the purpose of trying to find the eigenvalues of destruction?

PROFESSOR: Yeah, isn't that subtle? Isn't that weird? How does somebody want to find the eigenfunctions of an annihilation operator, because the annihilation operator is non-Hermitian? And why the annihilation operator? Why not the creation operator? Well, if you try to find the eigenstates of the creation operator, you run into problems. The eigenstates are non-normalizable. It doesn't make any sense. So here, creation and annihilation are not symmetric. You have to pick the annihilation operator.

Colin, my gut feeling is that somebody found it in a different way. The person who popularized those states was [INAUDIBLE], and he got amply rewarded for that. And I think you define coherent state as some form of superposition state. Maybe people have figured out if you take a superposition state of number states that this has special properties, that this is a minimum uncertainty state, which is as close as we can go in quantum physics to obtain a classical electric field.

So I think maybe people found it like that. And then they say, hey, but there is

another operator definition eigenstate of the annihilation operator. I'm 99 percent certain. But maybe you can ask [INAUDIBLE] when he comes to a CUA seminar if he discovered coherent state as eigenstates of the annihilation operator. I don't think so. Other questions?

So what I want to do is-- what the theme of the end of last lecture and today is that the coherent state is a complex number. The coherent state has a time evolution. It moves in a circle. And this is really the phasor of the electric field associated with it. So coherent state, the alpha value is directly related to an electric field. That's why this state is closely related to the classical limit of the electromagnetic field.

We discussed some properties of the coherent state. We looked at the fluctuations and showed that it's a Poissonian statistic. And then we use now-- once you define something, you can use it as a tool. We are now using the coherent states to look at any other quantum state of the electromagnetic field, any statistical operator which describes photons by forming the diagram matrix element of the statistical operator with alpha.

If you just want to sort of think about it intuitively, alpha is like the best description of an electromagnetic field. So if you write down this, we ask, What is the probability for an arbitrary quantum state described by an arbitrary statistical operator-- what is the probability that the electric field is alpha in the complex description? So that's what those quasi-probabilities are-- Q of alpha.

And we immediately looked at some examples, of course, of probabilities. We showed that the vacuum state is sort of an area. It's a Gaussian. And the area is-- it's an area. I don't know. I forgot the [? prefect. ?] It's on the order of 1 or $1/2$. We realize that thermal states are also a Gaussian centered at the origin. But it is a much wider distribution. So thermal states have much more uncertainty in the electric field than the vacuum state.

And then we looked at the coherent state. And now, of course, when we looked at the coherent state, we realize that coherent states are maybe not as wonderful as I tried to make you believe. They have some nasty properties. And that is when you

ask, What is the probability? The quasi-probability of the coherent state would be beta.

Now, if you would expect a delta function, because when you ask, What is the probability of a momentum state to be p naught? If you have a momentum state p naught, it has a delta function at p naught. But here, it's not a delta function. It's a Gaussian of the standard deviation. I think the standard deviation here is-- what is it, square root 1 over square root 2? Because the Gaussian has usually 2 times sigma squared. And since the denominator is unity-- so the standard deviation of 1 over square root 2. And that showed us that the coherent states are not orthogonal. You have to be a little bit careful. It's not your standard basis set. It's an overcomplete basis.

So therefore the coherent state is not a delta function in the causal probability. It's a little bit blurred circle with an area on the order of unity. And you have some homework assignment to look at it. So in some way, how you should look at it is that if you see some quasi-probability distribution, the distance from the origin-- this is the absolute value of the electric field. The phase here is almost 45 degrees. But if you look at this uncertainty, you would also say, Well, the phase is uncertain to within this angle. So you can pretty much read the uncertainty in the total electric field, the uncertainty in the phase. You can read pretty much everything you want from this diagram.

Now, when we look at a number state-- Well, you often know in quantum mechanics, number and phase are complimentary. If the number of photons is fixed, you know nothing about the phase. And indeed the quasi-probability of a number state is a ring. It has no phase. It has completely random phase over the 2π circle. The energy is sharp of a number state, since the energy is e squared. You may have expected a delta function in the radial coordinate. But what you get is also something blurred on the order of unity. And I want to say something about that in a second.

Finally, we discuss the time dependence. And the time dependence is very easy.

After all, we're dealing with an harmonic oscillator. And in an harmonic oscillator, if you have a plane of x and p , symmetric and anti-symmetric combination of a and a^\dagger , in this plane in an harmonic oscillator, the quantum state is just rotating circle, a rotation with ω . And indeed we showed that when we apply the time evolution operator-- and some of you were right, of course, with a minus sign-- it moves with e to the minus $i \omega t$. And therefore everything rotates in a clockwise way.

Now, we discussed the operator of the electric field. And in this quasi-probability-- Sorry, we discussed the operator of the electric field. And I hope you remember that in the analogy with the harmonic oscillator, the electric field was a minus a^\dagger . And this is the momentum operator.

In those quasi-probabilities-- and we will see more about it-- something which is sharp in momentum is a sliver parallel to the x -axis. So therefore, you can regard the vertical axis, which is the imaginary part of α , as the momentum axis. And you can regard this as-- the horizontal axis-- as the x -axis.

So therefore, since momentum is electric field, you always get the electric field by projecting onto the vertical axis. And if you project this fuzzy ball, you get a value 0 with some uncertainty. And if this quasi-probability starts to rotate due to the time evolution, we get an oscillating electromagnetic field, almost classical except for that fuzziness.

So that's where we want to continue. Any questions about that? Yes?

AUDIENCE: Why is there no phase fuzziness in the $E(t)$?

PROFESSOR: No phase fuzziness. There is a phase fuzziness. For instance, if you would say the phase is determined by the 0 crossing, you don't know exactly when the 0 crossing happened, and that corresponds to an uncertainty in the phase. Trust me, everything is in this diagram.

Now, there are two things we want to continue. One is I want to show you that the coherent state is a minimum uncertainty state. The product of $\Delta x \Delta p$ is just--

is it \hbar or $\hbar/2$? One of the two. So it's a minimum uncertainty state. And therefore, you can never have a quantum state which is less fuzzy than the coherent state. So this fuzziness here is the intrinsic uncertainty of quantum physics. So that's what we want to discuss today.

But then we will immediately start with non-classical states. And that is, well, if this area is determined by Heisenberg's uncertainty relation, what can be maybe deform the circle into an ellipse, and these are three states of light. So that's an outlook. That's what you're going to do in the second half of the class.

But before I do that, I want to be a little bit more accurate about quasi-probabilities. And this is almost like a disclaimer now. Just to give you the bigger picture. What we want to achieve with those quasi-probabilities, we want to do what phase space densities in classical physics. We have a coordinate which is x , a coordinate which is p .

And you can often describe a classical system if you know the phase space distribution, if you know the probability that a particle has position x and momentum p . So all this is about phase space densities. Probability of x and p . Of course, writing it down, you immediately see the problem. In quantum mechanics, you cannot measure x and p simultaneously. These are non-commuting variables.

So therefore, what happens is, if you now define a phase space function, which is done in quantum mechanics textbooks, you can actually do it in three different ways. And the three different ways are Q , P , and W . The definition-- I don't want to go through the mathematical subtleties-- the definition of those functions involves the operator definition, a and a^\dagger , or, which is equivalent, the x and p operator. If you define something in units of x and p or a and a^\dagger , you can have a product which is fully symmetric in the ordering of x p , which is anti-normal or normal.

So in other words, the order matters. And you have three choices. In an operator product, you can have symmetric ordering, which means not $x p$, but $x p + p x$, then it's over 2. That's symmetric. You can have an ordering which is called normal

and one which is called anti-normal. Ordering of operators in the operator base definition. Of course, if you have three choices, you would say, Which one is the best? Which one is the winner? But the fact is all three have their advantages and disadvantages. So they all have pluses and minuses.

The reason why I picked for the course Q of α is that it's a real probability, it's always positive. It is a diagonal matrix element of a statistical operator, and this has to be positive. So it's a real probability. The other guys, P of α , can be positive or negative. And also, W of α can be positive or negative. So if you use a P α distribution statistical operator, it can be written like this.

And as a result, the coherent state is now not this Gaussian. It doesn't have this Gaussian distribution as a course of probability. It's what you want-- what maybe some of you wanted to see-- oh, by the way, it's a delta function. The probability of the coherent state α has a delta function peak at α , which is sort of nice. And the number state is not a ring of a finite radius.

I just mentioned to you the number state. You would naively expect the energy is sharp. The square root of the energy's electric field, shouldn't it be sharp? And indeed, it is sharp. It's actually worse than a delta function. It's a derivative of a delta function. But at least here, in the probability P , which is also called the Glauber-Sudarshan P representation, you get the delta function, which may be very natural for certain purposes.

AUDIENCE: Since the [INAUDIBLE] and you can express one in terms of the other, is that unique?

PROFESSOR: It is unique by some symmetry choice here. So that's the advantage of it, that by-- I'm not going into the mathematics. I'm not giving you the definition but the way how it's defined. It's unique. It can be written in such a way. And the way how the quasi-probabilities P -- the P representation-- is defined, you get a delta function at the coherent state.

And finally, W stands for Wigner distribution. And the Wigner distribution is

something you actually find in most quantum mechanics textbooks. The Q and P distribution are more common in quantum optics. But the Wigner distribution has the advantage that the projection on the x- and y-axes are indeed ψ of x squared, ψ of p squared. So you get actually the x wave function and the p wave function.

So now, as a full disclaimer, if you want the electric field, which is the momentum of the harmonic oscillator, which is the electromagnetic field, what you really want to project is the W function, because for the W function, the Wigner function, the projection is exactly the momentum distribution in the electric field. So the Wigner distribution is closest to the classic phase space distribution, as close as you come without violating commutators. But of course it has a disadvantage that it has negative values. And try to explain to your next neighbor what is a negative probability.

Some people actually see negative probabilities spring out the non-classical character. And I know in our field, in AMO physics, a few years ago, in my lifetime, there were really attempts to use quantum state tomography and measure for the first time a negative Wigner distribution and show that to the world. So that meant something to a lot of people.

Anyway, you can read about it in quantum physics textbooks. All I want you to know is to know about it, but then also just relax. In the bigger picture, all the three distributions are the same. It's more sort of on the level of whether something is a delta function or has widths unity. So on the small scale, it matters. But if you map out something on a bigger scale, they are all related to each other.

And for the rest off today and the next class, when I show you those phase space distribution, when I say I project onto the vertical axis to get the electric field, I'm not completely rigorous which of the three functions I've really chosen. Any questions?

OK. So I promised you-- and this is what we're going to do now is we're want to understand in more depth the fluctuations. And in particular, I want to show you that coherent states are minimum uncertainty states.

So by identifying the vertical axis with p , the horizontal axis with x , we immediately expect that we find a result related the Heisenberg uncertainty principle, which says the widths in P and the widths in Q has to be larger than $\hbar/2$. And we are mainly talking with light about the photon creation and annihilation operator. But, just as a reminder, the momentum and position operator are symmetric and anti-symmetric combinations.

So let me just call those [? prefactors ?] Q naught and this one P naught. Well, we can then immediately, by just using elementary commutator, calculate what are the expectation values in a coherent state for P , P squared, Q , and Q squared. For P , it is-- the P operator is a dagger minus a .

If we act with a on α , we get α , because α -- the coherent state-- is an eigenstate of a . Now, with a dagger we act on the left hand side. And we get α^* . Well, with P squared and Q squared, you have to use one or two more steps to get rid of the products. But ultimately, you can express space all that just in powers of α , α squared, α^* , and such.

So what we want is we want to find out what are the fluctuations ΔQ , ΔP . Just as a reminder, ΔQ squared is of course Q squared average minus Q average squared. And Q squared average and Q squared is what we have just calculated. Actually, let me leave it in as a reminder for you-- Q squared minus Q average squared, the square root of it.

And what we find is that it's square root $\hbar/2\omega$ and, for ΔP , it is square root $\hbar\omega/2$. And what we verify, that indeed, for a coherent state, the product of the 2 is Heisenberg uncertainty limit. So therefore, coherent states are minimum uncertainty states.

And maybe, Colin, to address your question, I could imagine that coherent states were maybe invented by simply saying, We have an harmonic oscillator data. We want to find the minimum uncertainty states for which the uncertainty in x and p , when expressed in natural units, is the same.

In other words, the coherent state is the solution to the following question. If you plot the quasi-probability distribution, you give yourself an uncertainty area. One horizontal uncertainty is ΔQ . The vertical uncertainty is ΔP . And the minimum uncertainty state means that the area of the 2 uncertainties, the area of the shaded region, ΔP times ΔQ is $\hbar/2$. Or if I want to rewrite that in-- Yes, in the real part of α times the uncertainty in the imaginary part of α for the quasi-probability, then this is $1/4$.

OK. So what we have learned is that-- just one second. We have learned about one way to characterize uncertainties-- the quantumness of the electromagnetic field-- by looking at the uncertainties in the x and p variable. And that led us to minimum uncertainty states. Now, I want to now introduce to you two other ways of characterizing the uncertainty of quantum states of light.

One is we can ask, What are the fluctuations in the photon number? Right now, we have asked, What are the fluctuations in the electric field? But the next question is, What are the fluctuations in the photon number? Or we can ask, what are fluctuations in the intensity when we measure the intensity of the electromagnetic field? So let's do that.

The fluctuations-- It seems I've lost my lines. OK. So the fluctuations of the intensity are usually expressed by the second order temporal coherence function. That's what we want to introduce now. Yes, this is the second order temporal correlation or coherence function.

What I'm always encountering in this course is I would just like to immediately tell you how it is defined in terms of a 's and a^\dagger 's. It's simple. It's quantum mechanical. It's exact. But I always feel that if you want to really appreciate the quantum character, you have to know the classic description first. So I want to first tell you what is the second order coherence function for classical light, which has a classic description. And then you'll see what is the difference for quantum states of light.

So the classical description is you measure the intensity of light. You're just sitting

here. You receive light from a light bulb. You measure the intensity at time t . You measure the intensity at time τ later. And then you form the product. And you normalize it by the average intensity squared. So if τ equals 0, it's nothing else than the intensity squared average divided by the average of the intensity squared.

So this is the classical definition, g_2 of τ . And I've left the proof to the homework to show that the classical g_2 of τ is always larger than 1. But it's pretty much what you show is for τ equal 0, it means I squared average is larger than I average squared, which is, of course, always the case. But you can show that this is also the case for finite τ .

So quantum mechanically, we will see that the g_2 function is not necessarily larger than 1, it can be smaller than 1. And that's actually an interesting-- you can see-- litmus test for the quantumness. If you generate states of the electromagnetic field-- Fock states, photon number states-- we see that in a moment. And they have a g_2 of τ which is smaller than 1. You know it is not possible in any way to associate an intensity of the electromagnetic field with that photon state, because whenever you can associate a classical intensity with it and use a classical intensity to calculate the second order correlation function, you get something which is larger than 1.

So often therefore again, when people want to show we really have now non-classical photon states, they show g_2 of τ is smaller than 1. This is similar to what I said before when you want to show that you have non-classical light, you do quantum state homography, measure the Wigner distribution, and show that the Wigner distribution has negative quasi-probabilities. That's again something which is classically not possible. It's only possible if you have a truly non-classical state.

AUDIENCE: So in the definition of g_2 classical, the expectation values would need averaging over time, not over ensembles of I of t ?

PROFESSOR: OK. You can average over ensembles. So you can have 1,000 light bulbs, switch them all on, and then measure at a certain time z . But what we assume here is that the light bulb is on continuously. So things don't really depend on t . I've implicitly taken care of it by defining the g_2 function only of τ and not of t and τ . You

assume any time is the same, because it's a distribution, it's this ensemble in steady state.

But it's sort of the same story again and again. In classical physics, you can determine an ensemble average by taking an ergodic system and observing it at many, many times. And then the idea is that one system as time goes by will sample all possible states. Or you can prepare many, many identical systems and do more of what is an ensemble average. So in other words, you would actually think, if you switch on a light bulb with a stable power supply, that the light emitted by the light bulb will go through all possible quantum states as time evolves. And therefore, the temporal average is equal to the ensemble average.

So how do we generalize that to quantum mechanics? Well, one possibility would be that-- OK. First of all, for quantum mechanics, we have to use operators. And one choice would be that, well, you say the intensity of the electromagnetic wave is proportional to the electric field squared. And you want to use the operator for the electric field squared.

Now, there is a little bit of a problem, because what we really mean quantum mechanically by $\langle E^2 \rangle$ is we measure the intensity now and a little bit later. But measuring the intensity really means absorbing photons, because the only way how you can measure the intensity of light is with a photomultiplier. It makes click, the photon is absorbed. And this is not fully described by the electric field.

Just assume you have no photon, you are in the zero-point state of your harmonic oscillator, and your E^2 has zero-point fluctuations. So what is more closely related to an experiment how you measure the correlation function is you want to look at something else, namely at the probability of absorbing 2 photons.

So you start with an initial state. You annihilate 2 photons. And then you have a final state. But if you're only interested in what is the probability you want to characterize your initial state, you may want to sum over all final state. And this is your total probability that you can absorb 2 photons out of an initial state.

But since you're in sum now over all final state, this turns into-- Now, I could put for you as in the final state is a complete basis. But I can also take it out. So this is what we get. So this suggests that experiments where we look at two subsequent clicks of a photomultiplier where we determine the photon correlation, that this is measuring a correlation function, which, for quantum states of light, should be defined as the expectation value of a dagger, a dagger, a, a. And now we have to normalize by the probability squared of absorbing 1 photon, which is a dagger a expectation value squared.

I just tried to be a little bit too motivated. In many, many textbooks, the discussion would start with this expression. You ask, Where does it come from? And you realize yes, for some measurement with photomultipliers, that's what you measure. But I wanted to show you how it is related to the intensity correlation function defined classically. Cody.

AUDIENCE: But this looks like it's constant in tau, which doesn't really make sense intuitively to me, because at least classically, it shouldn't be constant in tau.

PROFESSOR: OK. That's the next thing I wanted to say is I've swept here-- I just wanted to give you the structure of the operators and not get distracted by discussing time. I've dropped the time argument here. But the fact is that as long as we limit ourselves to a single mode of the electromagnetic field, a single harmonic oscillator, things are independent of tau.

In other words, g_2 of tau equals g_2 of 0, as long as we're dealing with single mode light. You can actually say-- and that sort of tells you where the fluctuations come-- if g_2 of tau changes, it comes because you have several modes of the electromagnetic field, which, as a function of time, constructively and destructively interfere. But if you have a single mode, in a single mode-- and what is a single mode? It's just a sine wave. And nothing happens as a function of time. It's constant.

AUDIENCE: So both modes go inside.

PROFESSOR: I'm squeezing a textbook of quantum optics into two classes. And I want to give you the ideas and the concepts. What I've sort of mixed up here deliberately is I've given you the classic definition of the intensity correlation function, which is the famous Hanbury Brown Twiss experiment, and used correlations as a function of time. Then I've motivated how this should be defined for quantum states of light.

But when I transitioned to quantum states of light, I decided to deal with only one mode of the light. We should now sum over-- I should put double or triple indices on all the alphas for polarization, for spatial modes, for different frequencies, and we sum over all of them. But instead what I did is I wanted to just show you the simple case-- and I think you will be thankful for that in your homework, that you only have to deal with the simple case-- that you only have to look at a's and a daggers in the operator algebra for single mode, for single harmonic oscillator.

But what we lose, so to speak here, is there is nothing interesting going on in time. I've already told you that, for a single mode, all of these quasi-probabilities, they just rotate in a circle. So the time evolution of the system you're describing right now is completely boring. It's really a rotation. And if you would rotate your head at omega, nothing will happen. And this is exactly what you see here.

So you find everything you want to for coherence time. Coherence time is the time for 2 modes to get out of phase. But if you have 1 mode, there is no coherence time. And when you find, for classical light, that the g_2 function has a peak which decays with the time, it is the time for modes to get out of phase. But in a single mode picture, this is absent.

Anyway, what is important now for the discussion of quantum character of light is really, we find that in a single mode picture. So I want to show you now-- or at least give you the summary of the results, which can be very easily derived, because the math is very simple of those operators, that we are now with that definition. We have g_2 functions, which are no longer following the classic constraint that g_2 has to be larger than 1. We find g_2 functions which are smaller than 1. And this sort of tells us now where do we find the most non-classical behavior, namely when g_2 of tau is

as small as possible.

OK. In your homework, you will show immediately that the g_2 function is related to number fluctuations. It's related to an average and an n -squared average. Let me just write that down. It's independent of τ . And the reason is we have now limited ourselves to just one mode of the electromagnetic field. Fano factor. Just give me one--

OK So we are back to-- We started with intensity fluctuations. But for a single mode of the electromagnetic field, we are back to photon numbers. So what we are now expressing with the g_2 function are, in other words, just fluctuations of photon numbers. I want to in a minute draw you a table. What are the fluctuations in the photon number for the states we have encountered-- the number state, the Fock state, the coherent state, and the thermal state? And we want to characterize those quantum states of light by the g_2 function.

And actually, I can drop to time argument or set it to 0. But then we first make a reference that g_2 can now be smaller than 1. So g_2 for a single mode light is nothing else than a function. When you know what is an average and n squared average, you know your g_2 function.

There is another quantity, which we often use to characterize the fluctuations in the photon number. And this is called the Fano factor. The Fano factor is-- wants to compare the fluctuations. n square average minus n average squared, these are the fluctuations.

The classical fluctuation-- well, I say classical, classical in the simplest case-- are Poissonian fluctuations. So maybe we want to normalize the fluctuations by Poissonian fluctuation. So for Poissonian statistics, what I just wrote down would be 1. And well, if you now subtract 1, we have the situation that the final factor, which is positive, is super-Poissonian-- more fluctuation than Poissonian-- and the negative final factor is sub-Poissonian.

So with those definitions, we can now compare the different states of light we have

introduced. We started out with black-body radiation, thermal radiation. We defined coherent states. And the harmonic oscillator description naturally gave us harmonic oscillator eigenstates, the number of states, or Fock states.

So there are three ways to characterize it. They are all useful. One is we can look at n squared. We can calculate the Fano factor. Or we can calculate g_2 of 0. For the coherent state-- remember, the coherent state is as close as we can come quantum mechanically to the ideal of a pure electromagnetic wave. It has a Fano factor of 0. This means it's Poissonian. The g_2 function is simply 1, which is the lowest classical limit.

So those two tell you that a coherent state is sometimes what you think-- what comes out of a laser is an ideal electromagnetic wave, which has no temporal fluctuations in the intensity. Therefore, g_2 is 1. And the photon number is Poissonian distributed. And that means that n squared is an average times 1 plus an average. We actually discussed it earlier.

The thermal state is quite different. If you use-- kind of pluck together the results we have obtained, it has a Fano factor of n -bar. So this is super-Poissonian. If the occupation number n is large, you have fluctuations which are much, much larger than Poissonian fluctuation. The g_2 function is 2. It's sometimes called thermal light. Chaotic light has a g_2 function of 2. Laser light or coherent state has a g_2 function of one. And n square average is n -bar 1 plus 2 n -bar.

OK. But now finally, maybe the most interesting state from the perspective of non-classical light-- of quantum light-- is the photon number state. Well, for the photon number state, the number of photons is an eigenvalue. Therefore, n squared average is n average squared. The Fano factor is minus 1. Sub-Poissonian distribution. And the g_2 function, which classically cannot go below 1, is now n minus 1 over n . It is smaller than 1. And you see immediately that the biggest violation for g_2 is to go to minus 1 for the case of a single photon state. Any questions?

AUDIENCE: Shouldn't it be 0 for [INAUDIBLE]?

PROFESSOR: The g_2 function? No, if you put in-- wait. Yes. Gosh. I'll double-check. I'm a little bit confused. But I-- At least it's consistent now. I'm not sure if it's right, but it's consistent. By the way, if there is a question, I sometimes make a question mark in my notes. And when I post the notes-- maybe not when I post the notes on the next day, but a few days later-- the question marks are eliminated.

Just as an example, last class, there was some relation for which I didn't remember how the derivation was done. When I post those notes today, the derivation is now in the notes. I'm not announcing it separately. But whenever I have a question mark in class and I don't think it's worth an extra announcement, it is fixed in the posted notes and you find the information there.

OK. So this is now drawing our attention to the single photon. And this is our next subsection. This shouldn't come unexpected. If you want to emphasize the difference between quantum light and classic light, where does it come from? Well, it comes from the quantization of light. It comes from the photon character. It comes from the effect that light is not a continuous stream of energy, it comes quantized in photons. So the granularity of light due to the photon character is, of course, most pronounced for a single photon.

For instance, when we define the g_2 function as a correlation of detecting a photon and detecting another photon, well, if you've only one photon, you find one photon, and for the next photon, there is no photon to be detected. So the probability of detecting two photons is 0. And that only happens when you go down to similar photons. So this is when certain fluctuations are most pronounced, because the energy is dependant on a singular photon.

Let me first address one misconception. You can say, Well, let's just use a coherent state. And we talk about the attenuator-- the quantum attenuator with all its operator beauty-- probably not today but it the next lecture. But let me already sort of prepare you for that. When you have a coherent state, when you have a laser beam, you can put an attenuator. And your laser beam gets weaker and weaker and weaker.

But it stays a coherent state. And I will prove that to you very soon. So you can now say that you take your coherent state and you attenuate it down that there is only one photon left. Is that a single photon state? The answer is no. It is an attenuated coherent state. Coherent states, as I've just shown you, are very classical. They've always Poissonian distribution. They've always a g_2 function of 1. And attenuation is not changing it. Attenuation is preserving that.

So now I want to show you explicitly why an attenuated coherent state may have an average photon number of 1 but it shares nothing else with a quantum state of 1 photon or n equals 1 Fock state. So a coherent state with an expectation value of 1 photon is not a single photon. And this can be, for instance, expressed by looking what is actually the probability, if you have such a coherent state, to find 1, 2, or 3 photons. Well, the probability to find 0 photons-- no photon at all-- is actually $1/3$.

So the probability to find 1 photon or to find 0 photons is the same. The probability to find 2 photons is 0.18. So here you have a probability of finding a photon and correlating it with the next photon click. And you have even, on first sight, a surprisingly large probability to find 3 and 4 photons. So in 2 percent of the cases, you will find 4 photons. Whereas in contrast, the Fock state with quantum number n equals 1 is an eigenstate of the number operator \hat{n} with eigenvalue n equals 1.

So that tells you, if you want to get n equals 1, if you want to get a single photon state, you cannot just use a strong laser beam or a strong light source and attenuate it. You have to work with something which genuinely creates only 1 photon without any ambiguity, without any fluctuations, without any possibility of creating 2 photons. And I would actually say over the last 10 or 15 years, the creation of single photons has been sort of a small cottage industry, because single photons are often needed for protocols in quantum computation-- for experiments which really require accurate quantum state preparation of light-- and in particular, non-classical light.

But of course, there are ways how you can get single photons. And this is, well, you start with single atoms. If you have a single atom in the excited state, it can emit

only one photon. So in other words, you cannot-- Usually, we don't have the tools to prepare a single photon-- to take a single photon out of many, many photons-- and store it separately. But what we can control is single atoms. We can prepare single atoms. And then we can make sure that single atoms create single photons.

It's a little bit a way that we cannot control the bullets which are fired. But we can control the guns. And we make sure that each gun can emit exactly one bullet. So that's a way how we can create non-classical states of light.

Yes. So let's now look for those single photons at the quasi-probability distribution. So we obtain the quasi-probability distribution by taking the single photon state and projecting it on a coherent state.

You remember-- it's higher up in the notes-- what the coherent state α is in the Fock state basis, in a number basis. And so we're just picking out n equals 1. And this was nothing else than α squared times e to the minus α squared. So therefore, in these diagrams with the real part and imaginary part, if I plot the quasi-probability distribution, we get a ring.

The ring immediately tells us that there is no phase defined. All the phases are equally probable. And that also means, if you don't have a phase, the average value of the electric field is 0.

Since the equation of single photons is essential for studying non-classical light, but also since it's a very active frontier of our field-- actually, one of the leaders in this field is Professor Vladan Vuletic here at MIT-- let me at least give you a taste of how to create single photons.

I already gave you the major ingredient. Namely, it's about-- it involves single atoms. But it's a little bit more demanding like that. So you want to take 1 atom home or 1 ion. But the problem is if the atom or ion emits a light, it can emit the light into all directions. And therefore, you have a single photon afterwards. But you have many, many different spacial modes. And in any given mode, it will not have a single photon.

So therefore, what you have to add to the single atom or single ion is-- you have to put it in a cavity. And then, in the limit of a very, very high finesse cavity, the probability will be very, very high that your photon is emitted into the mode of the cavity and it is not emitted perpendicular into other modes. But that itself is not yet sufficient, because you have a single photon, but you also want to know when does the single photon arrive. You want to do experiments. You want single photons on demand.

And so one option is that you prepare your atoms in the ground state. You take a pi pulse, which, with 100 percent probability, excites the atom to the excited state. And then within an actual lifetime, or maybe even in resonator-enhanced inverse natural line widths, the single photon is emitted. So therefore, you pump the system and then you know within the next few nanoseconds your cavity mode will have a single photon.

The problem here is that you're dealing-- You have to prepare single atoms, which is difficult. You have to couple them to a high finesse cavity. There is technically another approach, where you use many atoms. So you could say that many atoms, you no longer have a single gun. So therefore, you can get several photons. But the singleness in photons comes because the photons are now heralded-- they are announced.

And the idea is the following. If you start with the state 1, and you have an excited state, and you have a pump pulse. Now, if you have many atoms, you have a much higher efficiency of pumping atoms comes to the excited state, because if you have n atoms, it's n times more efficient. But now you take the following situation. You wait until there is a Raman transition-- until you detect the photon for Raman transition, where the excited state decays to the state 2. At this moment, you know I have now 1 atom in state 2. So in other words, you're not starting with single atoms, which is sometimes more demanding. How can you prepare the system? It has other disadvantages that n atoms have.

I'm not sure if I should mention it, but with n atoms, they have a super-radiant factor

n with n atoms. You can get an n-times enhancement of emission into a single mode. So there are real, massive advantages in working with many atoms. But now you know that one atom is in state 2 when you detect the first photon.

And then you have the same situation which we had above. You can now take your system-- let me just redraw the level diagram. Excited state, state 1, state 2. You know now that you have 1 single atom here. And by using a laser pulse, you can excite it to the excited state. And then you observe the single photon 2.

So in other words, the observation of the first photon tells you that your system is prepared with 1 atom in state 2. And then you can get a single photon out of it, which is the photon for the inverse Raman transition when you pump the system back to state 1.

AUDIENCE: Doesn't this rely on you being able to reliably prepare a single atom in another quantum state? Won't there always be some uncertainty as to how many atoms you can prepare in one sitting?

PROFESSOR: There are some uncertainties. And what you're saying is correct. We don't have perfect single photon sources. And people characterize the fidelity of the single photon source. For instance, if you detect the single photon, you would say, Now my system is ready to emit a single photon triggered by this pump pulse. And if you can get now a single photon in 90 percent of the cases, you publish a wonderful paper, because you've set a new record for the fidelity of a single photon source. So people are really struggling with some of those uncertainties.

But to involve, for instance, three levels is sort of an advantage, because you're not limited by the preparation of a single atom, for instance. You can have many atoms. The atoms are always there. And the moment one atom is prepared in state 2, this atom announces itself with a single photon. So it takes a lot of uncertainty out that the system says, with the first photon, I'm ready now. I can emit a single photon. And then you get your single photon.

Then you can gate your whole experiment to a time following the detection of the

first photon. And for your gated time afterwards, you have a very, very high probability of finding this photon. Or if you want, you can now do experiments with 2 photons.

If you keep this laser on and pump the atom immediately back, you have actually a single photon here, followed by a single photon here. It's sort of click clack. And now you can do correlations between two different single photon states and such. It's a very rich frontier of our field. You have another question?

AUDIENCE: Yeah. Is this also within a cavity now, like the single photon [? reflection ?] in the first test?

PROFESSOR: Yes. Actually, these photons-- the wavy lines-- are emitted spontaneously. Spontaneous emission, pretty much by definition, goes into all possible spatial modes. And the only way to control the spatial mode is with a cavity.

What I don't want to go into details-- but those who have an understanding of that-- is the following. If you n atoms and you prepare one atom here in state 2, you do not know which of your n atoms is prepared. So you have n indistinguishable possibilities. And if you have n indistinguishable probabilities which atom you have prepared, the emission of the photon back is n times enhanced.

So therefore, you have actually a system which has an n times stronger coupling to the cavity. So having n atoms makes it much, much easier to construct a high finesse cavity. You get this super radiance increase of the strong coupling for free.

And this is why there are many reasons why you do not want to work with the single ion or a single atom. It's possible. For very fundamental experiments, people have shown that it can be done. But technologically, it's much easier and much more robust to work with many atoms. But then you need the many atoms have to tell you when 1 atom is prepared to emit now a single photon.

If you're interested in this subject, talk to the students in [INAUDIBLE] school. They're really the world experts in that.

OK. Finally, we're not really getting to squeeze light. We start with squeezing the light tomorrow. The last thing in this major section is the famous Hanbury Brown Twiss experiment. This was a landmark experiment done in the 1950s. And it was the first experiment which really looked at g_2 functions correlations, which one could say was the beginning of quantum optics and modern experiments with light.

Probably until then, light was just an electromagnetic wave people regarded as boring. But by measuring now correlations, people realize that it's an interesting object to study. I want to just go over the basic scheme. You will look at it a little bit more closely in your homework assignment.

So the idea behind an Hanbury Brown Twiss experiment is that you have a light source and you want to characterize it. So the light source emits light. And now, this is important. Whenever you want to detect 2 photons, you want to figure out it's 1 photon followed by another photon.

Technically, you cannot do it with a photomultiplier, because when a photomultiplier clicks, it needs many, many nanoseconds for the photomultiplier to read out the signal and recover to recharge up its electrodes and be ready to detect the next photon. So therefore, when you want to find click click-- double clicks in the stream of light-- you have to involve a beam splitter and involve two photodetectors.

Sure, you now need an adequate description. But in principle, you can now find 2 photons which are only a few picoseconds apart, because the first photon is observed by the first detector, and the second photon is detected by the second detector.

Sure, you would say, What happens if you're in the beam splitter, both photons go to detector one? Well, that's tough luck. You take 1 chance in 2 that the 2 photons go to the 2 different detectors. But the beam splitter has to be part of your experiment, has to become part of your description. And this is what you do in homework number one.

So what we are asking here when we measure the g_2 function. Let's assume that

tau equals 0. We are reading out a signal. And what we are using is a coincidence detector. We are looking that in a very small temporal window, 2 photons are detected simultaneously. Well, this is the quantum version. The classical version is the g_2 function is the product of the intensity at time t times the product of the intensity at time t plus tau.

So what you would do here in this circuit, you would take the signal from this detector, the signal from that, and multiply the two. This is how you determine I of t times I of t plus tau. So whether you do it in the classical domain or whether you do it in the quantum domain, this is the way how you experimentally measure the second order correlation function.

In the classic limit, you don't have to worry about it. The classical limit is always a limit of high intensity. So at any given time, you have a ton of photons. And then, at beam splitter, half of the photons go left, half of the photons go right. You have an equal splitting. You have exactly half the intensity in the left arm and the right arm after the beam splitter. So there is no problem at all with the quantization, because this is the classical limit.

So classically you can say-- actually, I shouldn't say photon, I should actually say intensity. The intensity splits equally. And if you do the measurement with the coincidence detector, you find the difference between the g_2 function, which I discussed earlier. That the g_2 function is 1 for coherent light or laser light, and the g_2 function is 2 for thermal light.

That's actually the only way how you can distinguish a light bulb from a laser beam. If you put a light bulb into a cavity or couple the light from a light bulb into a fiber, the light becomes spatially a single mode. If you put a very narrow spectral filter or Fabry-Perot cavity, the light becomes single mode in frequency domain. So when you take a light bulb and spatially and spectrally filter it that it is single mode, in terms of the mode, it is single mode as a single mode laser.

But it is the correlation experimental which shows you that you started with a thermal source. It has a g_2 function of 2, and you can never get rid of it, whereas

the laser beam has a g_2 function of 1.

OK. So this is the classical version. In the quantum version, especially when we have a single photon, the photon can go to only one detector. And that means, for this extreme case of a single photon, the g_2 function is 0. Anyway, you will look at those situations in more detail in your homework assignment. Yes?

AUDIENCE: Sir, if you know that different for τ non-zero, then even the single mode in the function would have a final g_2 [INAUDIBLE], because there would be some probability [INAUDIBLE].

PROFESSOR: We have to be now a little bit careful. If you have a single mode Fock state, there's only 1 photon. And once this photon is detected, we have vacuum, we have no photon left. So what may be confusing here is, on first sight, is that the way how the experiment is done and has been historically done, it's not done with Fock states. It's done with the light source in a continuous stream of photons.

So Hanbury Brown Twiss experiment, he looked at light bulb, or he looked at star light and such, and determined the intensity correlation. He couldn't do the experiment with lasers, but now we would with a laser beam. And we find that g_2 is 1. But in those situations, we actually do not have the way-- we have actually a beam, which is a stream of photons. And this requires a little bit different description.

In other words, when I have a coherent state, a laser beam is always replenishing the coherent state. The laser beam preserves the electric field, whereas, strictly speaking, the way how I described it for pedagogical reasons is you have a cavity, which is filled with a coherent state α . And then you start analyzing that. But in other words, what we have focused in the simple description is that we have a quantum state which is prepared-- it's a closed system. And now we do our detection.

The experiment how it is done is often done as in an open system, where you couple the system to a light source, which is always replenishing your experiment.

So to be careful, especially with a single photon. I think, in essence, the experiment would be done with a single photon light source.

But the single photon light source has a high repetition rate. I showed you how to generate single photons. You have an heralded single photon. You know the single photon comes now. You do the experiment. And then you repeat it again.

And then what you have is you only look at one single photon bunch at a time. And then you find indeed, that during that temporal window, you will never find a second photon. So this may happen in a few nanoseconds. Then you wait a microsecond, and the next photon arrives.

Of course, if you would now describe your light source, what you is the g_2 of τ , and τ is a microsecond, and the microsecond is the time between the first single photon burst and the next single photon burst, now you will find that your g_2 function is not 0. But this is then related to the repetition rate of a single photon and not to the single photon itself. I think you've got the taste.

Your homework is a really simple. You just deal with a closed system. You look at a quantum state at a time. But if you map it onto a beam experiment, you have to think about what does it mean to have a replenishment of the quantum state. Any question? I know I have to stop, but.

AUDIENCE: In the beam splitter, the other port of the beam splitter should do some stuff, right?

PROFESSOR: Yes. Actually, this will keep us busy for a while, that even if you do not put in any light, we put in the vacuum state. And you will find that the description of the beam splitter in the quantum state is incomplete, unless you specify that your beam splitter is coupling in from a different mode the vacuum state. Yes, this will be very important. And we will cover it in its full glory.

AUDIENCE: I just have a quick question. The way we can distinguish from a thermal state and a coherent state is through the g_2 . So from an experimental perspective, could we do most of experiments if we just had thermal sources that were very single mode, so to speak?

PROFESSOR: Well, yes. All the laser cooling, all the [INAUDIBLE], all the absorption imaging-- all that would work if you had a single mode thermal source. Let's face it, we say we use lasers for everything we're doing. But the only property which distinguishes a laser from the thermal light source-- we're not taking advantage of it.

Of course practically, if you take a thermal source and filter it down to a single mode, you will be left with only a few photons. You cannot create an intense enough single mode light source unless you use stimulated emission into a single mode. And that's a laser. Colin.

AUDIENCE: The correlation function for, for example, an LED light-- that's not really a thermal source. It's not described by a Maxwell-Boltzmann distribution. And that would be closer to something for a laser, even though there is no cavity?

PROFESSOR: Well, LEDs in some limit are quantum objects. Actually, do you know what the g_2 function of LED is? Does it become laser-like, or does it become even antibunched? Because if it's a relaxation mechanism-- So anyway, what Colin says is there are actually more different light sources than just the laser and the thermal light source. There are LEDs or semiconductor devices, which provide photons with interesting statistical properties. I've heard about it-- that LED light sources have some special properties, but I don't remember which was the one. OK. We have to stop. I'll see you on Wednesday.