

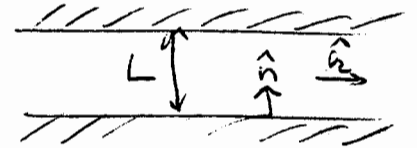
# CASIMIR EFFECT

(cf. S. Haroche  
Les Houches Summer School)

Attractive force between two parallel conducting plates  
due to vacuum fluctuations of the e.m. field

e.m. modes in cavity with  $L$ , cross section  $a^2$ ,  $a \gg L$

boundary conditions at  $z=0$  and  $z=L$



$$\sin kL = 0$$

$$k = m\pi/L \quad m=0, 1, 2, \dots$$

$$\omega^2 = k^2 c^2 + m^2 \pi^2 c^2 / L^2$$

$$\text{TE modes: } \vec{A}_{\vec{k}, m} = \sin\left(\frac{m\pi z}{L}\right) (\hat{k} \times \hat{n}) e^{i\vec{k} \cdot \hat{k} \vec{r}} + \text{c.c.} \quad m \geq 1$$

$$\text{TM modes } \vec{A}_{\vec{k}, m} = \left\{ \frac{ck}{\omega} \cos\left(\frac{m\pi z}{L}\right) \hat{n} - \frac{im\pi c}{L\omega} \sin\left(\frac{m\pi z}{L}\right) \hat{k} \right. \\ \left. \times e^{i\vec{k} \cdot \hat{k} \vec{r}} \right\} + \text{c.c.} \quad m \geq 0$$

Density of states

- number of modes with given  $m$  and  $k$  between  $k$  and  $k+d k$

$$\frac{d k_x}{(2\pi/a)} \frac{d k_y}{(2\pi/a)} = \frac{a^2 k dk}{2\pi} = \frac{a^2}{4\pi} d(k^2) = \frac{a^2}{c^2} \frac{\omega d\omega}{2\pi}$$

- for a given  $\omega$ :  $m = 0, \dots, \text{Int}\left(\frac{\omega L}{\pi c}\right)$

$$g(\omega) d\omega = \frac{a^2}{2\pi c^2} \left[ 1 + 2 \text{Int}\left(\frac{\omega L}{\pi c}\right) \right] \omega d\omega$$

$$= \frac{a^2}{2\pi c^2} \left[ 1 + 2 \sum_{m=1}^{\infty} \Theta\left(\omega - \frac{m\pi c}{L}\right) \right] \omega d\omega$$

↑ Heaviside step function

Zero-point energy in the volume  $a^2 \times L$

$$W(L) = \int_0^{\infty} \frac{\hbar \omega}{2} \rho(\omega) d\omega$$

$$= \frac{a^2 \hbar}{4\pi c^2} \left[ \int_0^{\infty} \omega^2 d\omega + 2 \sum_{m=1}^{\infty} \int_{\frac{m\pi c}{L}}^{\infty} \omega^2 d\omega \right]$$

add convergence term  $e^{-\lambda \omega/c}$   
[cutoff at high frequencies  $\omega \gg \lambda/c$ ]

$$W(L) = \text{const.} + \frac{a^2 \hbar}{2\pi c^2} \sum_m I_m$$

with  $I_m = \int_{\frac{m\pi c}{L}}^{\infty} \omega^2 e^{-\lambda \omega/c} d\omega$

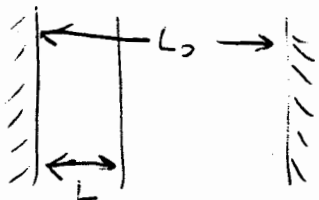
$$I_m = c^2 \frac{\partial^2}{\partial \lambda^2} \int_{\frac{m\pi c}{L}}^{\infty} e^{-\lambda \omega/c} d\omega = c^3 \frac{\partial^2}{\partial \lambda^2} \left[ -\frac{e^{-m\pi \lambda/L}}{\lambda} \right]$$

$$\sum_m I_m = \frac{c^3}{L} \frac{\partial^2}{\partial \lambda^2} \left[ \frac{1}{\left(\frac{\pi \lambda}{L}\right)} \frac{1}{e^{(\pi \lambda/L)} - 1} \right]$$

Expansion around  $x=0$  ( $x = \pi \lambda/L$ , eventually  $\lambda \rightarrow 0$ )

$$\frac{1}{x(e^x - 1)} = \frac{1}{x^2} - \frac{1}{2x} + \frac{1}{12} - \frac{x^2}{720} + \dots$$

$$W(L) = W_0 + \frac{a^2 \hbar c}{2} \left[ \frac{6L}{\pi^2 \lambda^4} - \frac{1}{\pi \lambda^3} - \frac{2\pi^2}{720 L^3} + \dots \right]$$



$$W_T = W(L) + W(L_0 - L) = \text{indep. of } L$$

$$= \frac{a^2 \hbar c}{2} \left[ \frac{6L}{\pi^2 \lambda^4} + \frac{6(L_0 - L)}{\pi^2 \lambda^4} - \frac{2}{\pi \lambda^3} - \frac{2\pi^2}{720} \left( \frac{1}{L^3} + \frac{1}{(L_0 - L)^3} \right) + \dots \right]$$

$10^{-5}$  mbar @  $L = 1 \mu\text{m}$

$\lambda$  independent potential energy

$$P_{\text{vacuum}} = \frac{1}{a^2} \frac{\partial U}{\partial L} = \frac{\pi^2 \hbar c}{240} \frac{1}{L^4}$$

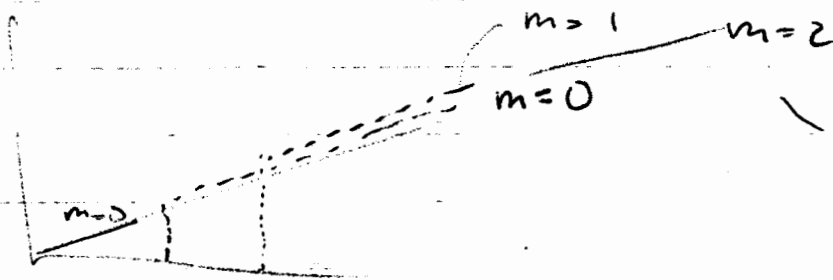
$$U(L) = -\frac{\pi^2 \hbar c}{720} a^2 \frac{1}{L^3}$$

[Casimir energy]

# Casimir force

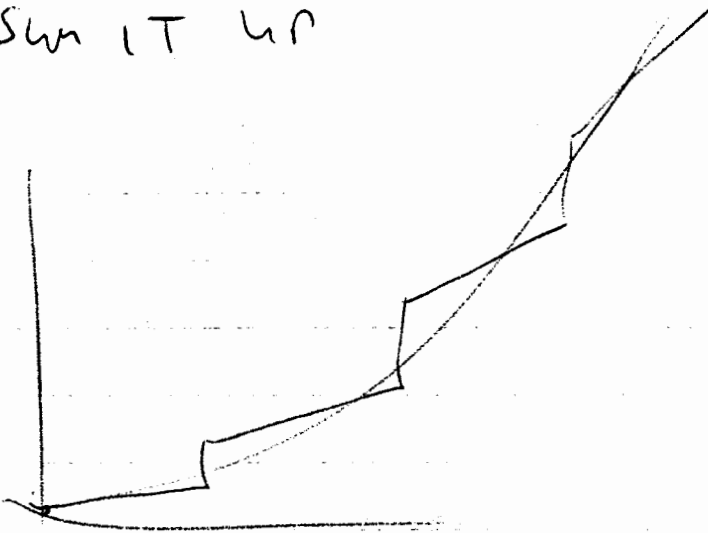
Density of states for a given  $m$

( )  $w \ll \lambda$



states with step function except for  $m=0$

Sum IT up



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