8.311: Electromagnetic Theory   Problem Set # 8   Due: **4/7/04**

Special relativity review. Relativistic dynamics.

Reading: Schwinger, Chap. 9, 10 (a good review of special relativity can be found in Jackson, Chap. 11)

1. *(Jackson, problem 11.1)* A possible clock is shown in the figure. It consists of a flashtube F and a photocell P shielded so that each views only the mirror M, located a distance d away, and mounted rigidly with respect to the flashtube-photocell assembly. The electronics of the box is such that, when the photocell responds to a light flash from the mirror, the flashtube is triggered with a negligible delay and emits a short flash towards the mirror. The clock thus ticks every $(2d/c)$ seconds when at rest.

a) Suppose that the clock moves with a uniform velocity v, perpendicular to the line from PF to M, relative to an observer. Using the special relativity postulate of the constancy of the speed of light, show by explicit geometrical or algebraic construction that the observer sees the relativistic time dilation as the clock moves by.

b) Suppose that the clock moves with a uniform velocity v parallel to the line from PF to M. Verify that here, too, the clock is observed to tick more slowly, by the same time dilation factor.

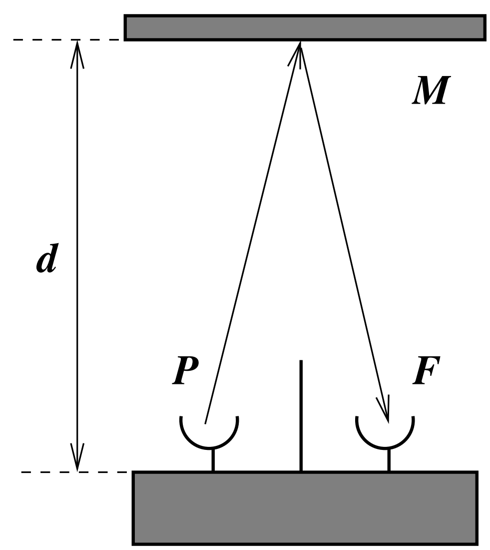

Figure 1: A clock model.

2. Relativistic velocity addition. Show explicitly that two successive Lorentz transformations in the same direction are equivalent to a single Lorentz transformation with a velocity $v = (v_1 + v_2)/(1 + v_1 v_2/c^2)$.

3. The twins. *(Jackson, problem 11.4)* Assume that a rocket ship leaves the earth in the year 2020. One of a set of twins born in 2000 remains on earth; the other rides on the rocket. The rocket ship is so constructed that it has an acceleration g in its own rest frame (this makes the occupants feel at home). It accelerates in a straight line path for 5 years (by its own clocks), decelerates at the same rate for 5 more years, turns around, accelerates for 5 years, decelerates for 5 years, and lands on earth. The twin in the rocket is 40 years old.

(a) What year is it on earth?

(b) How far from the earth did the rocket ship travel? (Hint: see Problem 7 below)

4. Relativity of simultaneity. *(Jackson, problem 11.5)* In the reference frame K two very evenly matched sprinters are lined up a distance d apart on the y axis for a race parallel to the x axis. Two starters, one beside each man, will fire their starting pistols at slightly different times, giving a handicap to the better of the two runners. The time difference in K is T.

a) For what range of time differences will there be a reference frame K' in which there is no handicap, and for what range of time differences is there a frame K' in which there is a true (not apparent) handicap?

b) Determine explicitly the Lorentz transformation to the frame K' appropriate for each of the two possibilities in (a), finding the velocity of K' relative to K and the space-time positions of each sprinter in K'.

5. Nonrelativistic charge. *(Schwinger, problem 9.1)* Consider the Lagrangian of a particle in a given electromagnetic field,

$$L(\mathbf{r},\mathbf{p},\mathbf{v},t) = \mathbf{p}\cdot\left(\frac{d\mathbf{r}}{dt} - \mathbf{v}\right) + \frac{1}{2}mv^2 - e\phi(\mathbf{r},t) + \frac{e}{c}\mathbf{v}\cdot\mathbf{A}(\mathbf{r},t) \tag{1}$$

(a) Reduce to Lagrangian form and then derive the equations of motion;

(b) Reduce to Hamiltonian form and then derive the equations of motion;

(c) Show the equivalence between (a) and (b).

6. Relativistic charge. *(Schwinger, problems 10.1, 10.2)* The relativistic modification of the action principle involves the replacement

$$\frac{1}{2}mv^2 \to -m_0c^2\sqrt{1 - \frac{v^2}{c^2}} \tag{2}$$

in $L(\mathbf{r},\mathbf{p},\mathbf{v},t)$ of Problem 5, Eq.(1).

(a) Use the Lagrangian viewpoint and find the equations of motion;

(b) Repeat the above using the Hamiltonian viewpoint.

7. Charge in a uniform electric field

(a) For a charge moving in a constant and uniform electric field $\mathbf{E}$, derive the equations of motion

$$\frac{d\mathbf{p}}{dt} = e\mathbf{E}, \quad \frac{d\mathbf{r}}{dt} = \frac{c^2}{E}\mathbf{p} \tag{3}$$

with $E = \sqrt{m^2c^4 + p^2c^2}$ the kinetic energy. (Hint: Use Problem 6 (b).)

(b) The charge starts motion with zero initial velocity. Find the trajectory $\mathbf{r}(t)$ and the relation between the charge proper time and the time in the laboratory frame.