MASSACHUSETTS INSTITUTE OF TECHNOLOGY
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Astronomy 8.282J-12.402J   April 21, 2006

Problem Set 10

Due: Friday, April 28

Reading: Zeilik & Gregory Chapter 18; start Chapter 19.

Problem 1

"Collapsing White Dwarf"

A white dwarf accretes matter until its mass exceeds the Chandrasekhar limiting mass of 1.4 $M_\odot$, after which it collapses to become a neutron star that is 10 km in radius. Estimate the energy (including all forms, e.g., photons, neutrinos, kinetic energy of any ejected matter, etc.) that is released in the ensuing supernova explosion.

Problem 2 **Optional**

"Maximum Distances for Applying Standard Candles"

Zeilik & Gregory, Chapter 18, Problem 4, page 378.
Refer to Tables 18–1, 18–2, and 18–3 in Zeilik & Gregory.

Problem 3

"Distance to a Nova"

Zeilik & Gregory, Chapter 18, Problem 5, page 378.

Problem 4

"Optical Luminosity of a Supernova at Maximum"

Zeilik & Gregory, Chapter 18, Problem 6, page 378.

Problem 5

"Cepheid Variables as Distance Indicators"

The table below gives some basic data on 25 Cepheid variables in the Small Magellanic Cloud (a neighbor galaxy) as derived by Henrietta Leavitt in 1912. For our purposes, the important quantities are contained in columns two, three and five, which represent the minimum and maximum apparent magnitudes during the brightness cycle, and the oscillation period (in days), respectively.

a. Make a plot of mean *apparent* magnitude, $\langle m \rangle = (m_{\max} + m_{\min})/2$, vs. $\log_{10}(P)$, where P is in days. About 10 points covering the complete period range should suffice to make a good plot.

b. Draw an "eyeball" estimate of a best–fit straight line through the data points. Interpret your straight line fit as an analytic function of the form:

$$\langle m \rangle = A \log_{10}(P) + B$$

Find the constants A and B. Estimate the uncertainty in B from the scatter of the data points about your straight line.

c. The distance to the Small Magellanic Cloud is about 55 kiloparsec (kpc). Use this fact to convert the above relation into one that relates the mean *absolute* magnitude $\langle M \rangle$ to the pulsation period.

d. The following table lists data for five Cepheids in the more distant Andromeda galaxy (M31). Use these data and your relation from part (c) to compute the distance to Andromeda.

e. Also shown is a plot of mean magnitude vs. $\log_{10}(P)$ for 11 Cepheids discovered with the Hubble Space Telescope in an even more distant galaxy, NGC 5253. Use these data and your relation from part (c) to compute the distance to NGC 5253.

Table 57.1 Periods of variable stars in the small Magellanic Cloud

H.	Max.	Min.	Epoch d	Period d	Res. M	Res. m.
1,505	14.8	16.1	0.02	1.25336	− 0.6	− 0.5
1,436	14.8	16.4	0.02	1.6637	− 0.3	+ 0.1
1,446	14.8	16.4	1.38	1.7620	− 0.3	+ 0.1
1,506	15.1	16.3	1.08	1.87502	+ 0.1	+ 0.1
1,413	14.7	15.6	0.35	2.17352	− 0.2	− 0.5
1,460	14.4	15.7	0.00	2.913	− 0.3	− 0.1
1,422	14.7	15.9	0.6	3.501	+ 0.2	+ 0.2
842	14.6	16.1	2.61	4.2897	+ 0.3	+ 0.6
1,425	14.3	15.3	2.8	4.547	0.0	− 0.1
1,742	14.3	15.5	0.95	4.9866	+ 0.1	+ 0.2
1,646	14.4	15.4	4.30	5.311	+ 0.3	+ 0.1
1,649	14.3	15.2	5.05	5.323	+ 0.2	− 0.1
1,492	13.8	14.8	0.6	6.2926	− 0.2	− 0.4
1,400	14.1	14.8	4.0	6.650	+ 0.2	− 0.3
1,355	14.0	14.8	4.8	7.483	+ 0.2	− 0.2
1,374	13.9	15.2	6.0	8.397	+ 0.2	− 0.3
818	13.6	14.7	4.0	10.336	0.0	0.0
1,610	13.4	14.6	11.0	11.645	0.0	0.0
1,365	13.8	14.8	9.6	12.417	+ 0.4	+ 0.2
1,351	13.4	14.4	4.0	13.08	+ 0.1	− 0.1
827	13.4	14.3	11.6	13.47	+ 0.1	− 0.2
822	13.0	14.6	13.0	16.75	− 0.1	+ 0.3
823	12.2	14.1	2.9	31.94	− 0.3	+ 0.4
824	11.4	12.8	4.0	65.8	− 0.4	− 0.2
821	11.2	12.1	97.0	127.0	− 0.1	− 0.4

Cepheids in M31

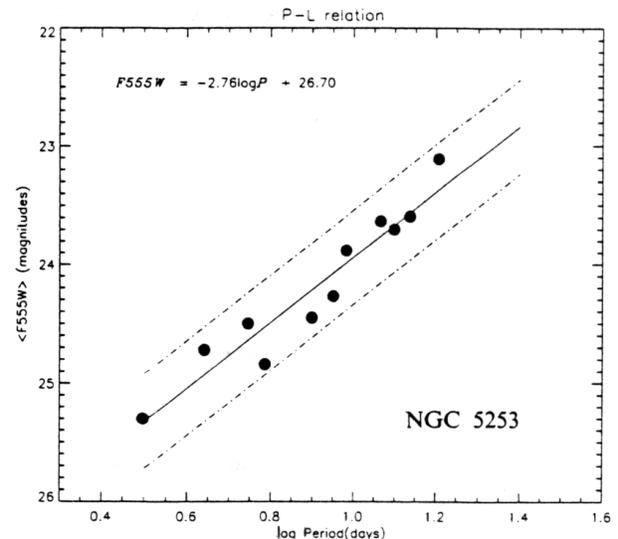| Period (in days)	Mean apparent magnitude $\langle m \rangle$
45.0	18.80
38.0	18.95
31.4	18.85
22.0	19.65
18.0	19.75

FIG. 3.—The $P − L$ relation in F555W magnitudes for the 11 Cepheids with definitive periods.

Problem 6 **Optional**

"Variations in the Radius of Cepheid Variables"

Find the fractional change in the radius of δ Cephei as it undergoes its cyclic radial pulsations.

a. Use the diagram shown on the next page to find the pulsational period of δ Cephei.
b. Perform a crude numerical integration (based on only 5 to 10 points) of the radial velocity curve to find the difference in the radius of δ Cephei between its largest size and its smallest size during a pulsation cycle. Note: Consider only velocity *differences* from the mean velocity of -15 km/s (the velocity with which the entire star is moving toward the Earth).
c. Use the period-absolute magnitude diagram (also given on the next page) to find the mean absolute bolometric magnitude of δ Cephei based on its pulsation period. Assume that δ Cephei is a type I Cepheid and that the photographic and bolometric magnitudes are nearly the same for a star of its spectral type.
d. Assume that the mean effective temperature of δ Cephei is 6000 K in order to deduce its mean radius. By comparison, the Sun has $T_{\text{eff}} = 5700$ K and an absolute magnitude of $+4.7$.
e. By what percentage does the radius of δ Cephei expand and contract?

Figure 31.4 Radial-velocity curve *(top)* and light curve *(bottom)* of the star δ Cephei.
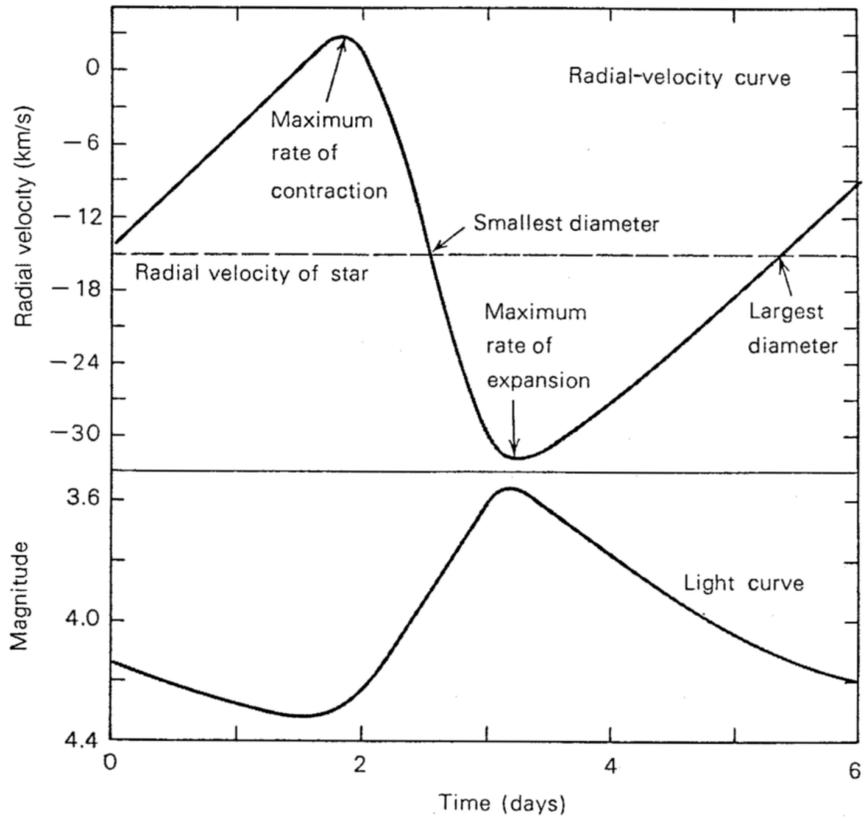

Figure 31.5 Period-luminosity relation for type I cepheids, type II cepheids, and RR Lyrae stars.