

8.08 Problem Set # 3

Feb. 16, 2005
Due Feb. 23, 2005

Problems:

1. S of a spin-1 spin can take three different values $S = 0$ and $S = \pm 1$.
 - (a) Consider one spin-1 spin in a magnetic field. The energy is given by $E = -hS$. Find the average spin $\langle S \rangle$ at temperature T .
 - (b) Consider a 1D Ising model formed by spin-1 spins

$$E = -J \sum_{i=1}^N S_i S_{i+1}$$

Using the mean-field theory to find the critical temperature T_c , below which the system spontaneously generate magnetization.

2. Water-vapor transition can be described by the following Ginzberg-Landau free energy (or more precisely Gibbs potential) near the critical point

$$G = (h_1 T - h_2 P) \delta n + (-a_1 + a_2 T + a_3 P) \delta n^2 + b \delta n^4$$

All the constant coefficients $h_{1,2}$, $a_{1,2,3}$, and b are positive. Here $\delta n = n - n_c$. n is the density of water molecules and n_c is the density at the critical point (T_c, P_c) .

- (a) Determine P_c and T_c of the critical point from the constants $h_{1,2}$, $a_{1,2}$ and b .
 - (b) Determine first order phase transition line between water and vapor the T - P space. Sketch such a line in the T - P space.
 - (c) Calculate the change of the density Δn across the first order transition line. (ie $\Delta n = n_{water} - n_{vapor}$ on the two sides of the first phase transition.)
3. Problem 14.6 in K. Huang's book. (20 pts)
(It may be less confusing to rename the order parameter from S to m so not to be confused with entropy.)