MITOCW | L13.2 Transition rates induced by thermal radiation (continued)

PROFESSOR: We have now expressed in terms of the energy density on this mode omega l the contribution to the transition amplitude. The fact that all these modes act incoherently means that each one has a shot in producing the transition. And the probabilities now must be added.

So the way to sum probabilities now is the following. If you have a sum over l of this frequency sum l of some energy density u of l times any function of omega l, you can replace it by an integral d omega. Instead of little omega l's that you're summing, you now integrate over omega the energy density of your radiation field times the function of omega.

So instead of having a sum of these things, you now integrate over a continuous variable. And this represents the energy density in the range. This whole thing is the energy density in the range d omega, which is in that range d omega, the energy range. The energy density is the sum of the energy densities of each of the contributions.

So that's what we're going to do here to express and to get our transition amplitude. So we'll say that we have the sum over l of Pab l of t equals 8 pi over h squared integral u of omega d omega dab and omega sine squared of 1/2 omega ba minus omega t. So all the omega l's have become omegas. That's the variable of integration.

And this is information about your radiation field. If it's a black body radiation, we gave the formula last time. So this is your transition amplitude. And we have to try to do the integral. This one l that remained here.

So what do we have to do now? Well, is it, again, the kind of useful story? We know that this factor from Fermi's golden rule tends to say, basically, you get only the contribution from omegas equal to omega ba. So whatever else is being integrated, nothing varies as fast as this. And you can take it out of the integral approximating omega for omega ba.

So that is a little more delicate here. So I'll just write an equality here. So it's 8 pi over h squared. I'll take the u at omega ba out of the integral, this factor.

And I want to take this one out as well. But this one is a little funny. As I change the omega, the polarization of this electric field is going to come in all directions. It's going to come at random. So even for a given frequency omega, there might be many modes of frequency omega that are coming at the particle at different directions.

And that's exactly what I would expect for thermal radiation, which is truly the case we're doing here, incoherence superposition of light. So when I have this, and I integrate over omegas, the various omegas, even for a given omega, there might be many lines that correspond to just a different direction. Because the field comes in all directions.

So the interpretation is that we can take this out, but we must do an average over all directions of omega. So this factor will go out as dab dot n. But here, we will average over all directions of n. And it's a square here.

So this is important because the field comes in all directions. We've taken care of this. Then, finally, we have the integral, the omega of the sine squared function a minus omega.

All right. So a couple of things still remain to be done. One thing that I don't think we need to do explicitly again is this integral. We've done it a couple of times. You do a change of variables. The t will go out linearly, and this becomes an integral of sine squared over x squared, which is equal to pi.

And so this integral has been done a couple of times. Let me just write the answer to this thing is 1/2 t times pi. It's linear in t. That we've observed. And it's pretty important that it's linear in t because that means that the probability-- you know, we've been writing this thing-- the probability of transition is linear in time. Therefore, you can divide by time to get that rate of transition.

So we're almost there. Let's put this together. We have this is the sum of P's. And there's this t there. So the transition rate, W from b to a is this probability divided by time. So it's going to cancel this time.

Then the 2 is going to give a 4. The pi is going to give a pi squared. There's going to be an h squared. There's going to be this factor dab dot n squared. And there's going to be the u of omega ba.

So this is going to be the rate. We've divided by t, the previous result. And there we have it. We're almost finished with the calculation. The rate is here.

The only difficulty here is this average, but it is not complicated, in fact, doing this average. It's actually kind of simple. So let's do it.

So what is it? It's all the matter of writing things properly here. There's the average of dab dot n squared. So this is the average of dab dot n complex conjugate times dab dot n.

So it's good to do this thing because, actually, you have to face as to what these symbols really mean. dab is a vector with complex components. Electric field is a vector, but it has real components. dab is a vector. Why is it a vector? Because it has a vector index here. It really came from this operator over there d is qr. So it's x, y, and z.

And each one has matrix elements between states a and b. And with matrix elements between states a and b that are complex wave functions, this can be complex numbers. So in general, this dab is a complex vector. And the star is necessary here.

So, this is a number. This, however, is a number. And I took the number star times the number and average. So the dot products can be written as sums. So I'll put here a sum over I and a sum over j. dab, the ith component times ni, the ith component. Here, this should be star. And that's the sum over i is the first dot product. dabj and j, the sum over j is the second dot product.

These d's are numbers, so they don't have anything to do with the average that we're doing over different directions. So we have i and j of di ab star dj ab times the average of ni nj.

This average, if you wish, you could simply do it. If you don't want to use symmetry arguments to do an average like that, you take a vector, parameterize it with theta and phi and just do the integral over solid angle. And divide by 4 pi. This should give you the same answer.

If you're uncomfortable with what we're going to say now, you should do that because that average just means take the vector n, integrate it over all directions, all solid angles, and average. So what is this ni nj? I claim this thing is 1/3 delta ij.

And it's based on the following idea, that the average between nx and ny, by the time you integrate over the sphere, is 0. The average of off diagonal things don't have averages. nx with nx, however, would have an average because it's always positive. ny with ny would have an average because it's always positive. And nz with nz would have an average.

And each one of these three would be the same because there's no real difference between the x, y, and z directions. And therefore, you have three things. And at the end of the day, this average of nx and x plus ny and y plus nz and z is this average of n squared, which is equal to 1. So it should be equal to 1.

So that each one of these nx, nx, or ny, ny, or nz, nz must add up to a total of 1. Therefore, this is the 1/3. So that's what this is. And if you find this a little funny, you should do the integral and then think about this argument again.

So anyway, that's the answer. And therefore, we get 1/3. And the delta ij establishes a dot product between these two things. So this is dab vector star dotted with the dab vector. This is a vector complex conjugated because we said the components can be complex. So that's the answer.

And most people like to just simply write it as dab vector squared like that. But this is not just the vector dotted with itself. It's a vector dotted with its complex conjugate.

So with this number, the whole formula is now finished. This is the final form. The rate for spontaneous transition triggered by electromagnetic fields, an incoherent superposition of electromagnetic fields is 4 pi squared. We get the 1/3 here over 3h squared. dab vector squared u omega ba.

And it's a transition rate per atom. We've considered a single atom. So it's transition rate per atom. All right. Long derivation, but a lot of physics in it.

A transition between discrete state got convoluted with an integral of the stimuli as it comes in and produce a nice result in the style of Fermi's golden rule. Griffiths called that Fermi golden rule. He works with Si units. And there's epsilon zeros and different numbers of pi's there.

I'm sorry. With checked. This is consistent, and we can't do much about that.