PROFESSOR: Great. So I will begin with phase shifts and do the introduction of how to make sure we can really-- so this is the important part of this. Just like when we added the reflected and transmitted wave we could find the solution I'm going to try to explain why with this things we can find solutions in general.

So this is the subject of partial waves, and it's a nice subject, a little technical. There might seem to be a lot of formulas here, but the ideas are relatively simple once one keeps in mind the one dimensional analogies. The one dimensional analogies are very valuable here, and we will emphasize them a lot. So we will discuss partial waves and face shifts.

So it's time to simplify this matters a little bit. And to do that I will assume from now on that the potential is central so v of r is equal to v of r. That will simplify the azimuthal dependence. There will be no azimuthal dependencies. You see, the thing is spherical is symmetric, but still you're coming from a particular direction, the z. So you can expect now that the scatter wave depends on the angle of the particle with respect to z because it's spherically symmetrical. But it shouldn't depend on five, the angle five, should just depend on theta. So expect f of theta.

Now, a free particle is something we all know how to solve, e to the ikx. Why do we bother with the free particle in so many ways? Because free particle is very important. Part of the solution is free particles. To some degree far away it is free particles as well. And we need to understand free particles in spherical coordinates. So it's something we've done in 805 and sometimes in 804, and we look at the radial equation which is associated to spherical coordinates for a free particle.

So we'll consider free particle and we'd say, well, that's very simple but it's not all that simple in spherical coordinates, and you'd say, OK, if it's not simple, it's spherical coordinates, why do we bother? We bother because scattering is happening in spherical coordinates. So we can't escape having to do the free particle in spherical coordinates. It is something you have to do.

So what are solutions in spherical coordinates? We'll have solution SI of r. Remember the language with coordinates was a U of r divided by r and of Ylm of omega. That was a typical solution, a single solution of the showing our equation will-- the U only depends on l, the m disappears, so this is r. This r's are r's without the vector because you're already talking about the radial equations, and depend on the energy and depend on the value of the l quantum number.

So what is the Schrodinger equation? The radial equation is minus h squared over 2m, the second the r squared plus. h squared over 2m l times l plus 1 over r squared. Remember the potential centrifugal barrier in the effective potential, then you would have v of r here, but it's free particle, so v of r is equal to 0. So if nothing else, U of El of little r is equal to the energy, which is h squared k squared over 2m UEl. And that's a parliamentary session of the energy in terms of the k squared, like that.

Well, there's lots of h squared, k squared, and 2m's, so we can get rid of them. Cancel the h squared over 2m. You get minus d second dr squared plus l times l plus 1 over r. UEl is equal to k squared UEl. It's a nice equation. It's the equation of the free particle in spherical coordinates.

Now, this is like the Schrodinger equation. And I think when you look at that you could get puzzled whether or not the value of k squared or the energy might end up being quantized. With the Schrodinger equation many times quantized is the energy, but here it shouldn't happen. This is a free particle. All values of k should be allowed, so there should be no quantization. This is an r squared here.

You can see one reason, at least analytically, that there is no quantization is that you can define a new variable row equal kr and then this whole differential equation becomes minus the second the row squared plus l times l plus 1 over row squared. Well, I can put the other number in there as well, or should I not? No, it's not done here. UEl is equal to UEl, and the k squared disappeared completely.

That tells you that the case will kind of get quantized. If there is a solution of this differential equation it holds for all values of k. And these are going to be like plane waves, and maybe that's another reason you can think that k doesn't get quantized because these solutions are not normalizable anyway, so it shouldn't get quantized.

So with this equation in here we get the two main solutions. The solutions of this differential equation are vessel functions, spherical vessel functions. UEl is equal to a constant Al times row times the vessel function lowercase j of row. There's a row times that function. That's the way it shows up. It's kind of interesting. It's because in fact you have to divide U by r, so that would mean dividing U by row, and it means that the radial function is just the vessel function without anything else.

And then there's the other vessel function, the n of l a row times of n of l of row. So those are spherical vessel functions. As you're familiar from the notation that j is the one that this healthy at row equals 0 doesn't diverge the n is the solution that diverges at the origin. And both of them behave nicely far away. So Jl of x goes like 1 over x sine of x minus l pi over 2, and ADA l of x behaves like minus 1 over x cosine of x minus l pi over 2. This is for x big, x much greater than 1, you have this behavior.

So these are our solutions, and here is the thing that we have to do. We have to rewrite our solutions in terms of spherical waves because this was the spherical wave so we should even write this part as a spherical wave. And this is a very interesting and in some way strange representation of E to the ikz You have E to the ikz that you have an intuition for it as a plane wave in the z direction represent it as an infinite sum of incoming and outgoing spherical waves. That's what's going to happen.

So this is the last thing we need do here. We have that e to the ikz is a plane wave solution, so it's a solution of a free particle, so I should be able to write the superpositions of the solutions that we have found. So it should be a superposition of solutions of this type. So it could be a sum of coefficients al times, well, alm you think of some a's times solutions. Remember, we're writing a full solution, so a full solution you divide by r. So you divide by this quantity. So you could have an alm Jl of row plus Blm ATA l of row times Ylm.

So this should be a general solution, and that would be a sum over l's and m's of all those quantities. But that's a lot more than what you need. First, this does not diverge near r equals 0. It has no divergence anywhere and the ATAs or the n's, I think they're n such and not ATAs, the n's diverge for row equal to 0. So none of this are necessary, so I can erase those.

l and m. But there is more. This function is invariant and there are some beautiful rotations. If you have your axis here, here's the z, and you have a point here and you rotate that the value of z doesn't change. It's independent of phi for a given theta, z just depends on r of cosine theta. So there's no phi dependence but all the Ylm's with m difference from 0 have phi dependent. So m cannot be here either. m must be 0. So you must be down to sum over l, al some coefficient, Jl of row, Yl0.

And all of those would be perfectly good plane wave solutions. Whatever numbers you choose for the little al's, those are good solutions because we've build them by taking linear combinations of exact solutions of this equation. But to represent this quantity the al's must take particular values.

So what is that formula? That formula is quite famous, and perhaps even you could discuss this in recitation. e to the ikz, which is e to the ikr cosine theta, is the sum 4 pi. Now you have to get all the constants right. Square root of 4 pi, sum from l equals 0 to infinity, square root of 2l plus 1. Coefficients are pretty funny. They get worse very fast. Now you have of i to the l, i to the l, Yl0 of theta doesn't depend on phi, Jl of kr.

This is the expansion that we need. There's no way we can make problems with this problem unless we have this expansion. But now if Y the intuition that I was telling you of these waves coming in and out, well, you have e to the ikz, you sum an infinite sum over partial waves. A partial wave is a different value of l. These are partial waves. As I was saying, any solution is a sum of partial waves is a sum over l.

And where are the waves? Well, the Jl of kr far away is a sine, and the sine of x is an exponential ix minus e to the minus ix over 2. So here you have exponentials of e to the ikr and exponentials of e to the minus ikr, which are waves that are here like outgoing waves and incoming waves. So the E to the ikz's are sum of ingoing and outgoing spherical waves. And that's an intuition that we will exploit very clearly to solve this problem. So we will do that next.