8.044 SOLUTIONS EXAM #2

1. $\rho = \rho(T,\varepsilon) \Rightarrow d\rho = \left.\frac{\partial \rho}{\partial T}\right|_\varepsilon dT + \left.\frac{\partial \rho}{\partial \varepsilon}\right|_T d\varepsilon$

$$\left.\frac{\partial \rho}{\partial \varepsilon}\right|_T = \left(a + \frac{b}{T}\right)N + 3cN\varepsilon^2 \quad \{\text{GIVEN}\}$$

$$\left.\frac{\partial \rho}{\partial T}\right|_\varepsilon = \frac{-1}{\left.\frac{\partial T}{\partial \varepsilon}\right|_\rho \left.\frac{\partial \varepsilon}{\partial \rho}\right|_T} = -\frac{\left.\frac{\partial \rho}{\partial \varepsilon}\right|_T}{\left.\frac{\partial T}{\partial \varepsilon}\right|_\rho} \longleftarrow \{\text{GIVEN}\}$$

$$= \frac{\left[\left(a+\frac{b}{T}\right)N + 3cN\varepsilon^2\right]\left[dT^2 - b\varepsilon\right]}{aT^2 + bT + 3cT^2\varepsilon^2} = \frac{N}{T^2}\left[dT^2 - b\varepsilon\right]$$

$$= Nd - \frac{bN\varepsilon}{T}$$

$$\rho = \left(a + \frac{b}{T}\right)N\varepsilon + cN\varepsilon^3 + f(T) \quad \left\{\begin{array}{l}\text{INTEGRATING FIRST} \\ \text{WITH RESPECT TO } \varepsilon\end{array}\right\}$$

$$\left.\frac{\partial \rho}{\partial T}\right|_\varepsilon = -\frac{bN\varepsilon}{T^2} + f'(T) = Nd - \frac{bN\varepsilon}{T^2} \Rightarrow f'(T) = Nd$$

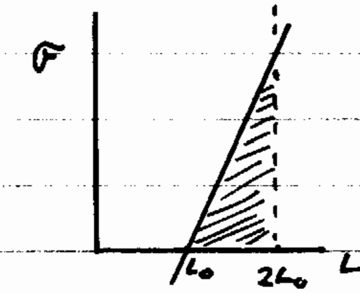
$$f(T) = NdT + K$$

$$\rho(T_0, \varepsilon = 0) = \rho_0 = NdT_0 + K \Rightarrow K = \rho_0 - NdT_0$$

$$\underline{\underline{\rho = \left(a + \frac{b}{T}\right)N\varepsilon + cN\varepsilon^3 + Nd(T - T_0) + \rho_0}}$$

2. a) $F(L_0,T)=0 \qquad F(2L_0,T)=(a+bT)L_0$

$$\Delta W = \int_{L_0}^{2L_0} F\,dL$$



$$= \frac{1}{2}F(2L_0,T)L_0 = \frac{1}{2}(a+bT)L_0^2$$

b) $\Delta U = U(2L_0,T) - U(L_0,T) = \frac{a}{2}L_0^2$

$$\Delta Q = \Delta U - \Delta W = \frac{a}{2}L_0^2 - \frac{1}{2}(a+bT)L_0^2 = -\frac{bT}{2}L_0^2$$

NOTE THAT HEAT FLOWS OUT OF THE ROD

c) $dU = \left.\frac{\partial U}{\partial T}\right|_L dT + \left.\frac{\partial U}{\partial L}\right|_T dL$ {EXPAND}

$= đQ + F\,dL$ {1^{ST} LAW}

$$đQ = \underbrace{\left.\frac{\partial U}{\partial T}\right|_L}_{cT^3} dT + \underbrace{\left(\left.\frac{\partial U}{\partial L}\right|_T - F\right)}_{\cancel{a(L-L_0)} - \cancel{a(L-L_0)} - bT(L-L_0)} dL$$ {REAR} {FROM GIVEN}

$= 0 \Rightarrow cT^3\,dT = bT(L-L_0)\,dL$

$$\frac{dL}{dT} = \frac{cT^2}{b}\frac{1}{(L-L_0)}$$

∞ WHEN $L=L_0$, CHANGES SIGN ABOVE + BELOW $L=L_0$

THUS AGREES WITH SUPPLIED SKETCH.

3. a) R GOES FROM 0 TO N.

MINIMUM $E = -NJ$ (ALL PARALLEL)

MAXIMUM $E = NJ$ (COMPLETELY STAGGERED)

b) IF THE FIRST SPIN IS UP, THERE ARE $\frac{N!}{(N-R)!\,R!}$ DIFFERENT ARRANGEMENTS FOR A GIVEN R (ONE REVERSAL IS INDISTINGUISHABLE FROM ANOTHER). IF THE FIRST SPIN IS DOWN THERE ARE AN EQUAL NUMBER OF DIFFERENT ARRANGEMENTS.

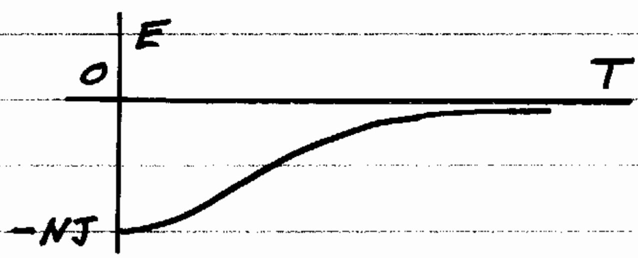
$$\underline{\underline{\Omega(R) = \frac{2N!}{(N-R)!\,R!}}}$$

c) $S = k\ln\Omega \approx \underline{\underline{k\left[\ln 2 + N\ln N - (N-R)\ln(N-R) - R\ln R\right]}}$

d) $\frac{1}{T} = \frac{\partial S}{\partial E}\Big|_N = \frac{\partial S}{\partial R}\Big|_N \underbrace{\frac{\partial R}{\partial E}\Big|_N}_{1/2J} = \frac{k}{2J}\left[\ln(N-R) - \ln R\right]$

$$\frac{2J}{kT} = \ln\frac{N-R}{R} = \ln\left(\frac{N}{R}-1\right) \Rightarrow \frac{N}{R} = e^{\frac{2J}{kT}} + 1$$

$$R = \frac{N}{1+e^{2J/kT}} \qquad E = NJ\left(\frac{2}{1+e^{2J/kT}} - 1\right) = \underline{\underline{NJ\,\frac{1-e^{2J/kT}}{1+e^{2J/kT}}}}$$



e) $\underline{\underline{\lim_{T\to\infty} R = \frac{N}{2}}}$

NOTE THAT THIS IS TRUE FOR BOTH POSITIVE & NEGATIVE J.