PROFESSOR: Square well. So what is this problem? This is the problem of having a particle that can actually just move on a segment, like it can move on this eraser, just from the left to the right. It cannot escape here.

So the way we represent it is the interval 0 to a on the x-axis. And there's going to be two walls, one wall to the left and one wall to the right, and no potential in between. That is, I write the potential V of x as 0, for x in between a and 0, and infinity for x less than or equal to 0, and x greater than or equal to a. So basically the particle can move from 0 to a, and nowhere else. The potential is infinity.

Now, this problem, meaning that the wave function-- the particle cannot be outside the interval, means that the wave function must vanish outside the interval. And you could say, how do you know? Well, if the potential is close to infinite amount of energy to be there, so the particle cannot really be there if it's really infinite energy that you need. You will see in the finite square well that the particle has probability to be in regions where it classically cannot be. But that probability will go to 0 if the potential is infinite.

So we can think of it as a limit and we will reconfirm that. But in fact, if the potential is infinity, we will take it to mean that psi of x is equal to 0 for x less than 0 and for x greater than a. I am putting this equals or-- there are many ways of doing this. If this function, as this continues, you have a wall at a, is the potential 0 at a or is it infinity? Well it doesn't quite matter.

The issue is that the wave function is 0 here, is 0 there, and we've said that the wave function must be continuous. So it should be 0 by that time you're at 0 or at a. So therefore we will take psi of 0 to be 0, and psi of a to be 0 by continuity.

So we discuss why the wave function has to be continuous. If the wave function is not continuous, the second derivative of the wave function is terribly singular. It's like a derivative of a delta function, which is an impossible situation.

So the wave function, we will take it to vanish at these two places, and this is what is called a hard wall. So what is the Schrodinger equation? The Schrodinger equation is, again, a free Schrodinger equation.

Nothing, no potential here, so it's the same Schrodinger equation we had there, psi double prime equals minus 2mE over h squared, psi of x. Or, again, minus k squared psi of x. Let's solve this. So how do we do it?

Well it's, again, a very simple equation, but this time it's conveniences-- we don't have a circle or periodicity to use sines and cosines. So I'll take psi of x to be c1 cosine of kx plus c2 sine of kx. But the wave function must vanish at 0. And at 0, the cosine is 1, so you get c1. And the sine is 0, so this must be 0, so c1 is gone. There's no c1 contribution to the solution.

So psi of x is c2 sine of kx. But we're not done. We need this function to vanish at the other side. So psi of x equals a must be 0, and that c2 sine of ka must be 0.

And therefore we realize that ka must be equal to a multiple of pi because sine vanishes for 0, pi, 2 pi, 3 pi, minus pi, minus 2 pi, minus 3 pi, all the multiples of pi. And therefore we will write kn equals 2 pi n-- not 2 pi n. Pi n over a.

OK, well, let me ask you, what should we take for n? All integers? Should we skip some? We took all integers for the circle, but should we take all integers here?

So what happens here, n equals 0. What's the problem with n equals 0? n equals 0, k equals 0, the wave function vanishes. Well, wave function vanishing is really bad because there is no particle then. There is nowhere in the probability to find the particle.

So n equals 0 is not allowed, for sure. n equals 0, no. So why did we allow it, n equals 0, in the circle? In the circle for n equals 0, exponential doesn't vanish. It's a constant and that constant is a fine wave function.

0 is not fine, but the constant is good. But n equals 0 is not. So how about positive ends or negative ends. And here comes the problem, see we're getting to it. For n equals minus 2 or for n equals 2.

So in one case, k is a number. And in the other case, k is the opposite sign number. And sine of a number, or minus a number, that number goes out. So if you have a sine of minus kx, that's minus sine of kx. And two wave functions that differ by a sign are the same wave function, physically. There's nothing different. They could differ by an i and other things.

So when you pick negative n minus 1, or pick n equals plus 1, you get the same wave function, but just different by a sign. So it's not new. So in this case, it's very interesting that we must restrict ourselves. We can correct all this and just say n equals 1, 2, 3, all the way to infinity.

The wave function, then, is psi n of x, is proportional to sine of n pi x over a. And you look at it and you say, yes, that looks nice. For x equals 0, it vanishes. For x equals a, it vanishes. n and minus n would give me the same wave function up to a sine.

So this is good. I just have to normalize it. And normalizing it would be done by putting an n here. And then the integral psi n squared dx from 0 to a only would be n squared integral from 0 to a dx of sine squared n pi x over a.

Now, you can do this integral by calculation. And our sine squared is written in terms of a double angle cosine of double angle plus a 1/2. The intuition with these things are that if you're integrating over the right interval that contains an integer number of cycles of the sine squared, then the sine squared has average 1/2. Because sine squared plus cosine squared is equal to 1. So you don't have to do the interval in general. This is n squared times 1/2 times the length of the interval, which is a.

And therefore n squared, this is equal to 1, and therefore n is equal to square root of 2/a and we can write now our solutions. Our solutions are m psi n of x equals the square root of 2/a sine n pi x over a. And n equals 1, 2, up to infinity. And En is equal to h bar squared, k squared, so pi squared, n squared, a squared, to m.

That's it for the solutions of-- are there degeneracies? No. Every energy state is different because there's any single 1, 2, 3, infinity, each one has more energy than the next. No, I'm sorry, the energy increases as you increase n. The energy levels actually become more and more spaced out.

And the last thing I want to do with this box is to look at the states and see how they look and gather some important properties that are going to be very relevant soon.