PROFESSOR: We got here finally in terms of position and in terms of momentum. So this was not an accident that it worked for position and wave number. It works with position and for momentum. And remember, this phi of p now has interpretation of the weight that is associated with a plane wave of momentum p, and you're summing over P in here. So we'll do the natural thing that we did with x. We'll interpret phi of p squared-- phi of p squared-- dp is the probability.

Find the particle with momentum in the range p, p plus dp. Just the same way as we would say that psi squared of x, dx is the probability to find the particle between x and x plus dx. So this is allowed now by the conservation of probability and this, therefore, makes sense. It's a postulate, though. It's not something that can derive. I can just argue that it's consistent to think in that way, and that's the way we finally promote this phi of p, which did encode psi of x. Phi of b has the same information as phi of x. phi of p is the weight of the superposition but, finally, it's given a probabilistic interpretation. It represents a probability to find the particle with some momentum.

So this is what is going to allow us to do expectation values in a minute. But I want to close off this discussion by writing for you the three-dimensional versions of these equations. 3D version of Fourier transform. So this is what we want to rewrite. So what would it be? It's psi of the vector x. Since you're going to have three integrals-- because you're going into it over three components of momentum-- this factor appears three times. So actually, it's 2 pi h bar to the three halfs integral phi of vector p into the i vector p dot product vector x h bar d cube p, and phi of p vector the inverse theorem-- same factor, we keep the nice symmetry between x and p-- psi of x vector negative exponent same dot product but negative exponent d cube x.

So these are the three dimensional versions of your x versus p. And there is a three-dimensional version of Parseval. So oh there's a three dimensional version of the delta function. Just like we had a delta function here-- a delta function in three dimensional space would be delta cubed x minus x prime would be one over 2 pi cubed integral d cube k e to the i k vector x minus x prime vector.

It's all quite analogous. I think you should appreciate that you don't have to memorize them or anything like that. They won't be in any formula sheet, but they are very analogous expressions. Parseval also works in the same way. And you have-- just as you would imagine-- that the integral all over three dimensional space of psi of x squared is equal to the integral over three dimensional momentum space of phi of p squared. So the three results-- the Fourier theorem the delta function and Parseval hold equally well.