BARTON ZWIEBACH: After this long detour, you must think that one is just trying to avoid doing the real computation, so here comes, the real computation. The real computation is taking that right hand side on the top of the blackboard and trying to just calculate this right hand side. So back to the calculation. The calculation dN/dt is equal to this thing over there, integral dx i over h-bar. I'll still copy it here-- h psi-star psi minus psi-star h psi.

OK. Well, let's do this. This whole quantity is d-rho/dt, and let's see how much it is. Well, you would have of the following-- i over h-bar h psi-star. Well, h in detail is over there, so I'll put it here. Minus h squared over 2m d second dx squared of psi-star. So I'm beginning h psi star-- that's from the first term in the Hamiltonian-- times psi. And then from the other term in the Hamiltonian is the potential, so it would be plus V of x and t psi-star psi. V of x and t times psi-star times psi.

This other term would be minus psi-star h psi, so it's going to be opposite sign to here, so plus h squared over 2m psi-star d second dx squared psi, and then minus psi-star V of x and t psi. Here, there was a little thing that I probably should have said before is that the potential is real, that's why it didn't get complex conjugated here. H psi would have a term V psi and we just conjugate the psi.

OK, this is not so bad. In particular, you see that these two terms cancel. So that's neat. And now, this becomes the following-- this d-rho/dt has become minus ih over 2m d second psi-star dx squared times psi minus psi-star d second psi dx squared. OK. That's what d-rho/dt is and that's the thing that should be 0 when you integrate-- it doesn't look like anything equal to 0, and that was pretty much to be expected.

So what do we have to do with this? Well, we have to simplify it more, and what could save us is, and it's usually the same thing that saves you all the time when you want to show an integral vanishes, many times, what you show is that it is a total derivative. So remember, we're computing here d-rho/dt, which is all this thing circled here, and it's to be integrated over x. So if I could show this is a derivative with respect to x, the total x derivative, then the integral would go to the boundaries and I would have a chance to make it 0.

So what do we have? That derivative is indeed at boundaries, so d-rho/dt is equal to minus i h-bar bar over 2m. And look, this can be written as d/dx of something and what is that something? It's d psi-star dx times psi minus psi-star d second psi-- no not d second-- d first psi dx. The nice thing that happens here is that if you act with this d/dx, you get the second derivative terms that you had in there. But you also get derivatives acting here on d psi and here on d psi-star, but those will cancel. So it's a very lucky circumstance, it had better happen, but this is a total derivative with respect to x. And that's just very a good deal.

So we're going to rewrite it a little more. I'll write it as the following way-- this whole factor is h over 2im, that's with its sign, output the d/dx outside-- I'll put an extra minus sign, so I will flip the order of these two terms-- psi-star d psi dx minus psi d psi-star dx. OK.

Well, in many ways, the most difficult part of the calculation is over and it's now a matter of giving proper names to things. Why do I say that? Because look, want to see the finish line? It's here. We've shown this whole integrand is d/dx of that right hand side. Therefore, when you do the integral, you will have to go to the boundary with that thing, so you just need to see what happens to these quantities as x goes to infinity. And as x goes to infinity, we said that psi must go to 0 from the beginning. And d psi dx must not blow up, so if psi goes to 0 and d psi dx doesn't blow up, this whole thing goes to 0 and dN/dt is equal to 0 and you're done. So you're done with the conditions that we mention that the wave function must satisfy these conditions.

But let's clean up this, because we've actually discovered an important quantity over there that is going to play a role. So here you see that you have a complex number minus its complex conjugate. So this is like z minus z-star, which is equal to 2i I times the imaginary part of z. If you subtract from a complex number its complex conjugate, you get the imaginary part only survives, but it's twice of it. So from here, this whole thing is 2i times the imaginary part of psi-star d psi dx.

So d-rho/dt is equal to minus d/dx of what? Of 2i times the imaginary part of that, cancels the 2i, you get h-bar over m imaginary part of psi-star d psi dx. And this quantity is going to be called the current density. So the current density, you say, why the current density? We'll see in a minute. But let's write it here because it'll be very important. J of x and t is h-bar over m imaginary part of psi-star d psi dx.

So if this is called the current density, you would have an equation here d-rho/dt is equal to minus dJ/dx d/dx of J dx, or d-rho/dt plus dJ/dx is equal to 0. Now this is called current conservation. You've seen it before in electromagnetism and we'll review it here in a second as well.

So look what has happened. You began with the introduction of a charged density, which was a probability density, but you were led now to the existence of a current. And you've seen that in three dimensions, more than in one dimension-- I think probably in one dimension it doesn't look that familiar to you, but let me make sure you will recognize it in a few seconds.

So think units here first. Units. What are the units of the wave function? Well, the wave function, you integrate over x squared and it gives you 1. So the integral of psi squared dx is equal to 1, so this has units of length, this must have units of 1 over square root of length. And what are therefore the units of psi-star d/dx psi, which is part of the current formula ? Well, 1 of the square root of length-- 1 over square root of length is one over length and another 1 over length is 1 over length squared. OK.

And then you have h-bar, which has units of mL squared over T. Probably done that before already. And therefore, h over m has units of L squared over T. So the current has units of h over m-- the units of current has units of h over m, which is L squared over T-- times units of this whole thing, which is 1 over L squared, so at the end, 1 over T. And this means just probability per unit time. That's the units of current. Probability has no units, so we're dealing probability, those are pure numbers, but this is probability per unit time. So probability per unit time.