PROFESSOR: So in quantum mechanics, you see this i appearing here, and it's a complex number-- the square root of minus 1. And that shows that somehow complex numbers are very important. Well it's difficult to overemphasize the importance of i-- is the square root of minus 1 was invented by people in order to solve equations. Equations like x squared equals minus 1. And it so happens that once you invent i you need to invent more numbers, and you can solve every polynomial equation with just i. And square root of i-- well square root of i can be written in terms of i and other numbers.

So if you have a complex number z-- we sometimes write it this way, and we say it belongs to the complex numbers, and with a and b belonging to the real numbers. And we say that the real part of z is a, the imaginary part of z is b. We also define the complex conjugate of z, which is a minus i b and we picture the complex number z by putting a on the x-axis b on the y-axis, and we think of the complex number z here-- kind of like putting the real numbers here and the imaginary parts here. So you can think of this as ib or b, but this is the complex number-- maybe ib would be a better way to write it here.

So with complex numbers, there is one more useful identity. You define the norm of the complex number to be square root of a squared plus b squared and then this results in the norm squared being a squared plus b squared. And it's actually equal to z times z star. A very fundamental equation-- z times z star-- if you multiply z times z star, you get a squared plus b squared. So the norm squared-- the norm of this thing is a real number. And that's pretty important.

So there is one other identity that is very useful and I might well mention it here as we're going to be working with complex numbers. And for more practice on complex numbers, you'll see the homework. So suppose I have in the complex plane an angle theta, and I want to figure out what is this complex number z here at unit radius. So I would know that it's real part would be cosine theta. And its imaginary part would be sine theta. It's a circle of radius 1. So that must be the complex number. z must be equal to cosine theta plus i sine theta.

Because the real part of it is cosine theta. It's in that horizontal part's projection. And the imaginary part is the vertical projection. Well the thing that is very amazing is that this is equal to e to the i theta. And that is very non-trivial. To prove it, you have to work a bit, but it's a very famous result and we'll use it.

So that is complex numbers. So complex numbers you use them in electromagnetism. You sometimes use them in classical mechanics, but you always use it in an auxiliary way. It was not directly relevant because the electric field is real, the position is, real the velocity is real-- everything is real and the equations are real. On the other hand, in quantum mechanics, the equation already has an i. So in quantum mechanics, psi is a complex number necessarily. It has to be.

In fact, if it would be real, you would have a contradiction because if psi is real, turns out for all physical systems we're interested in, H on psi real gives you a real thing. And here, if psi is real then the relative is real, and this is imaginary and you have a contradiction. So there are no solutions that are real. So you need complex numbers. They're not auxiliary. On the other hand, you can never measure a complex number. You measure real numbers-- ammeter, position, weight, anything that you really measure at the end of the day is a real number.

So if the wave function was a complex number, it was the issue of what is the physical interpretation. And Max Born had the idea that you have to calculate the real number called the norm of this square, and this is proportional to probabilities. So that was a great discovery and had a lot to do with the development of quantum mechanics. Many people hated this. In fact, Schrodinger himself hated it, and his invention of the Schrodinger cat was an attempt to show how ridiculous was the idea of thinking of these things as probabilities. But he was wrong, and Einstein was wrong in that way.

But when very good physicists are wrong, they are not wrong for silly reasons, they are wrong for good reasons, and we can learn a lot from their thinking. And this EPR are things that we will discuss at some moment in your quantum sequence at MIT. Einstein-Podolski-Rosen was an attempt to show that quantum mechanics was wrong and led to amazing discoveries. It was the EPR paper itself was wrong, but it brought up ideas that turned out to be very important.