Review :

Last lecture :

* Thin film interference

Explained why soap bubbles are colorful

* We will learn about :

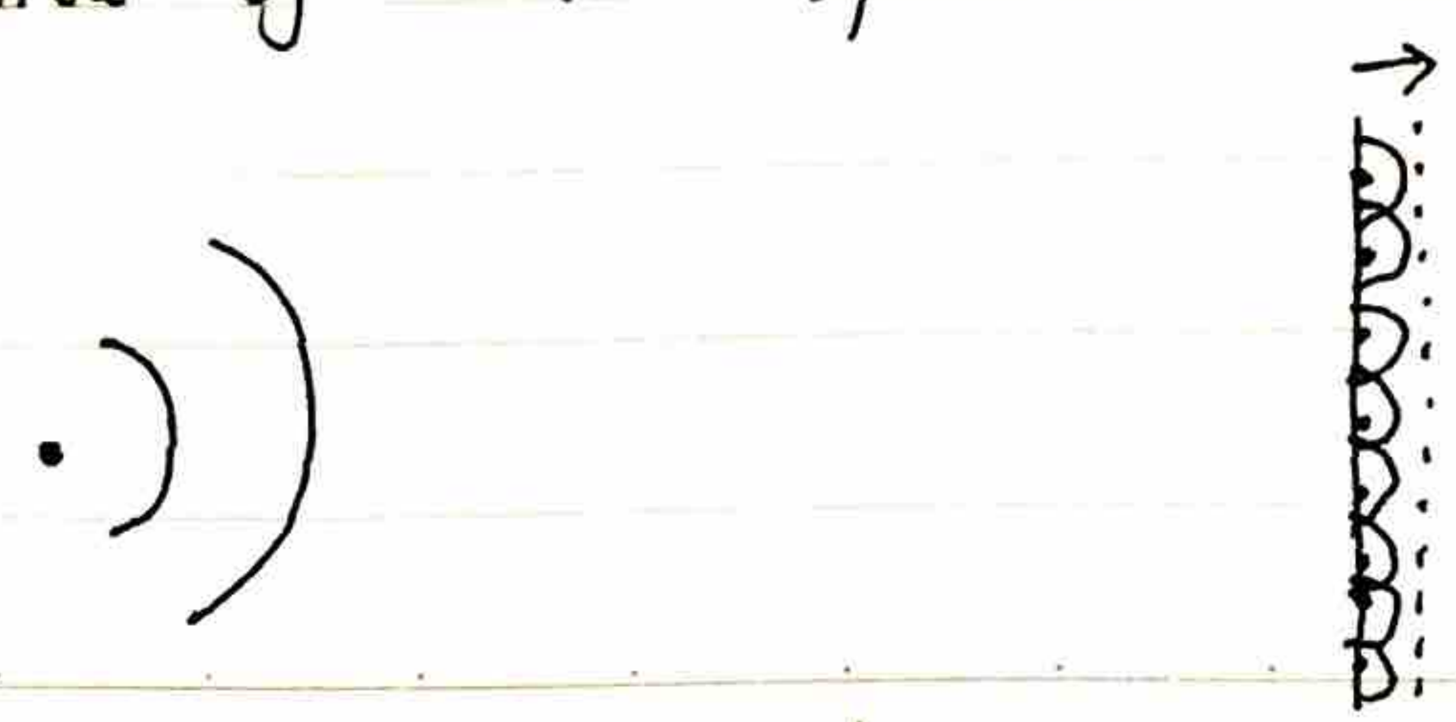
(1) Interference phenomenon with double - slit experiment : laser , water ripple

(2) How phased radar works. (radio waves , 3 kHz ~ 300 GHz

(3) Connection to Quantum Mechanics.

* Reminder : Huygens Principle :

All point on a wave front becomes a source of a spherical wave .

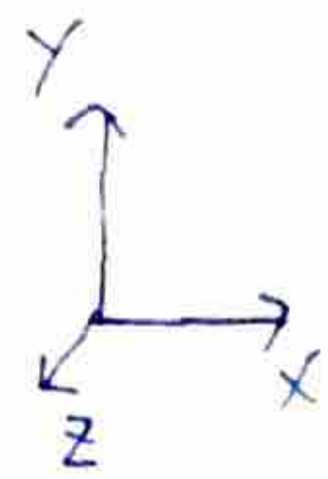


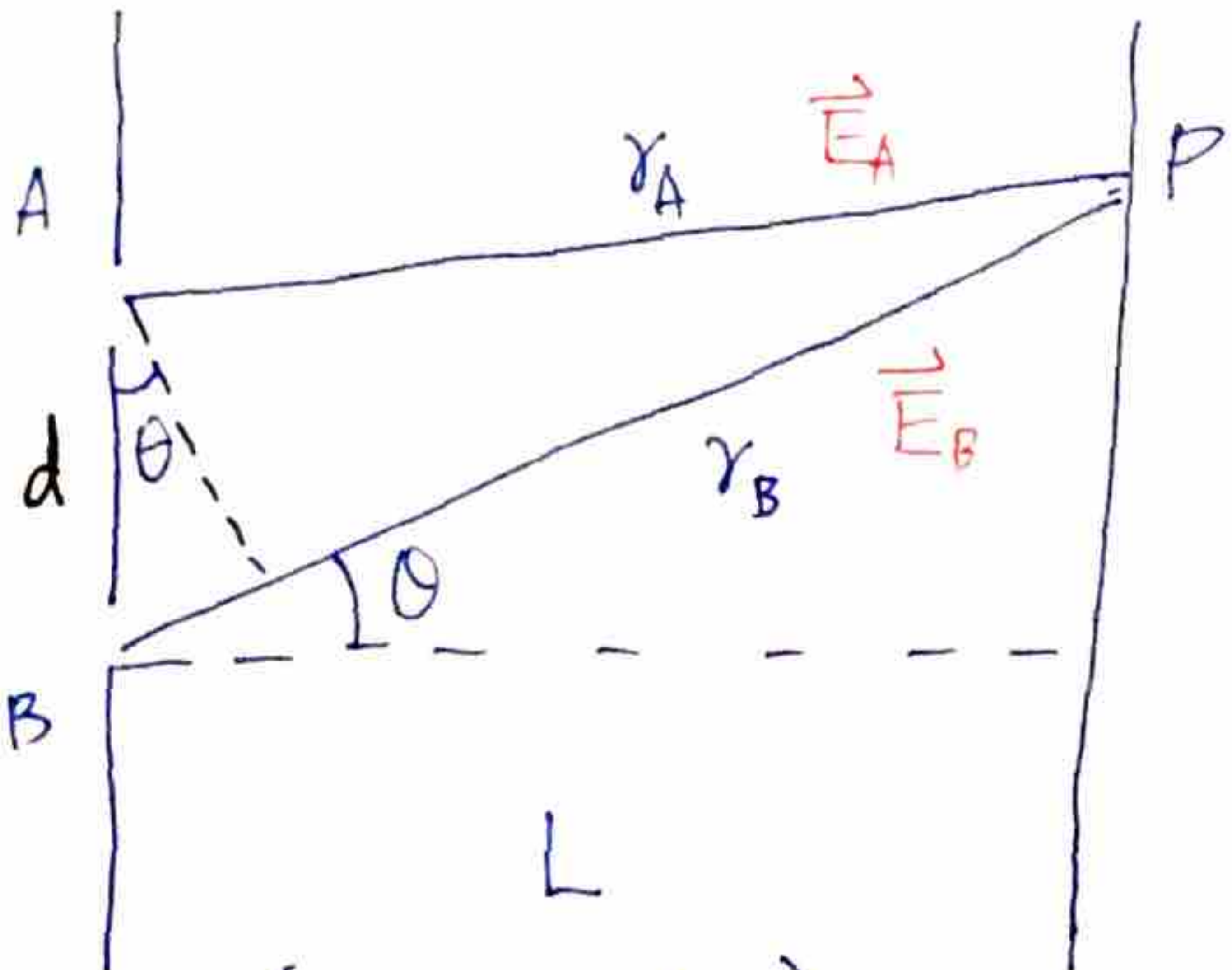
(works for odd spatial dimension , can be derived from Maxwell's Eqs)

Last time:

DEMO

Double-Slit Experiment:





$(L \gg d)$

Optical Pathlength Difference:

$$r_B - r_A = d\sin\theta$$

$\Rightarrow$ Phase difference:

$$\delta = \frac{d\sin\theta}{\lambda}\cdot 2\pi$$
$$= (d\sin\theta)\cdot k$$

What is the resulting intensity?

First: write down the electric field (in the complex notation)

$$\vec{E} = \vec{E}_A + \vec{E}_B = \left(E_0 e^{i(\omega t - k r_A)} + E_0 e^{i(\omega t - k r_B)} \right) \hat{z}$$

$$= E_0 e^{i(\omega t - k r_A)} \left[1 + e^{-i\delta} \right] \hat{z}$$

$$= E_0 e^{i(\omega t - k r_A)} e^{-i\frac{\delta}{2}} \underbrace{\left[e^{+i\frac{\delta}{2}} + e^{-i\frac{\delta}{2}} \right]}_{= 2\cos\frac{\delta}{2}} \hat{z}$$

$$\langle I \rangle \propto |\vec{E}|^2 = E\cdot E^* \propto \cos^2\frac{\delta}{2}$$

$$\Rightarrow \langle I \rangle = A \cos^2\frac{\delta}{2}$$

$\uparrow$ the intensity at $\delta = 0$

$$\sin\theta = \frac{\lambda}{2\pi d}\,\delta$$

$\delta = 0$ $\quad \vec{E}_B \; \vec{E}_A$

$\delta = 2\pi$ $\quad \vec{E}_B \; \vec{E}_A$

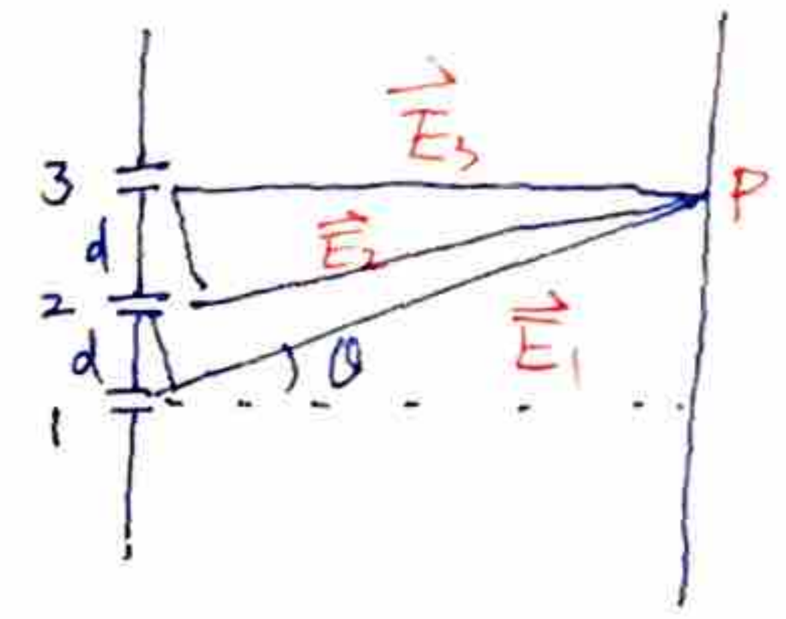
$\delta = \pi$ $\quad \vec{E}_B$, $\vec{E}_A$

DEMO

slides 6-8.

Now we have the knowledge we need to understand how radars work!!

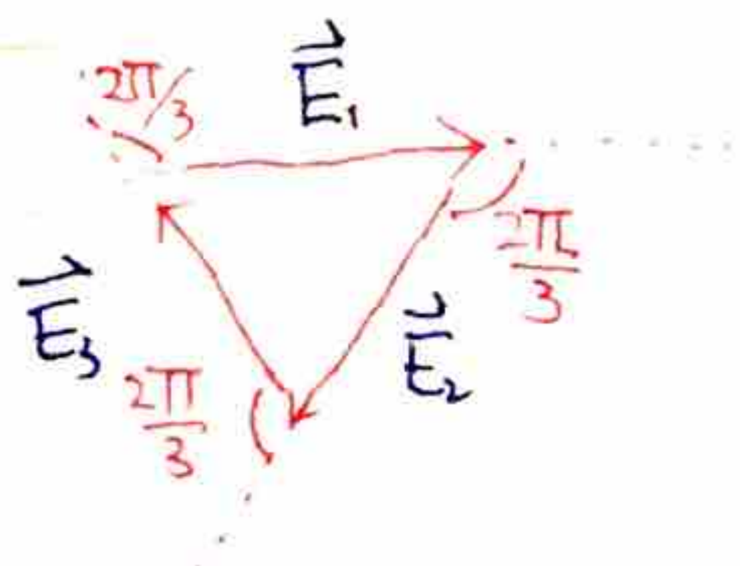
Consider a three-slit interference exp:



$$\delta_{12} = \delta_{23} = d\sin\theta = \delta$$

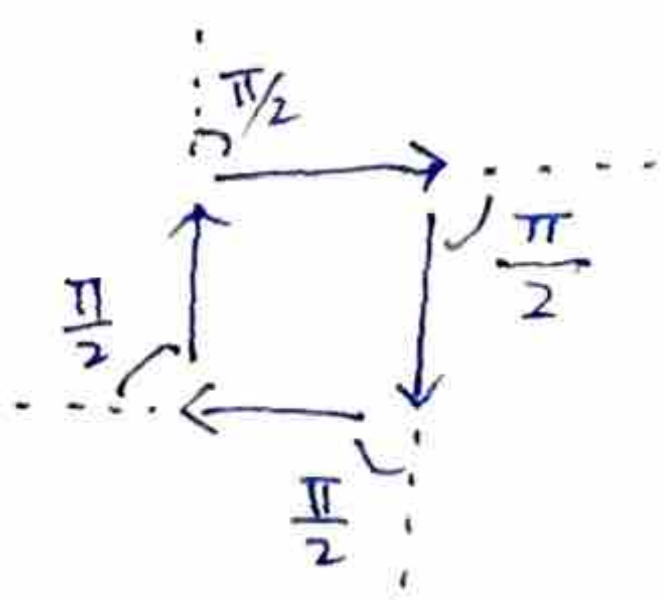
(minimum)

What is the required δ to have destructive interference?



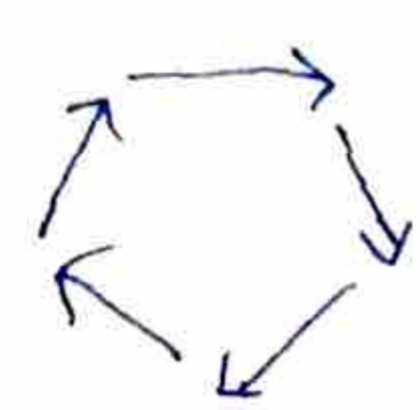
$$\Rightarrow \quad \delta = \frac{2\pi}{3}$$

How about 4-slit?



$$\delta = \frac{2\pi}{4} = \frac{\pi}{2}$$

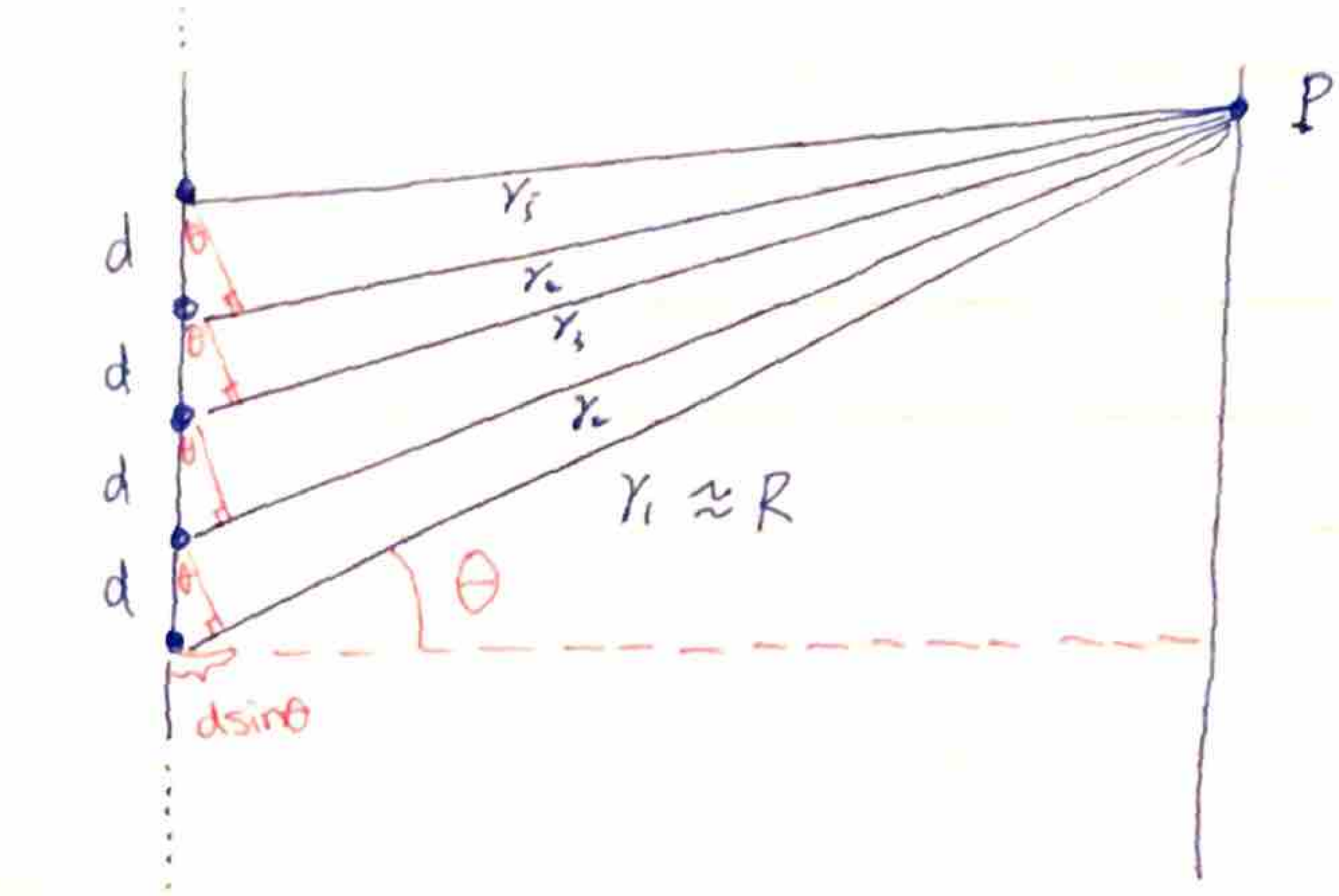
5-slit?



$$\delta = \frac{2\pi}{5}$$

$\Rightarrow$ You can see that the width of the intensity peak is DECREASING as we increase the number of slits!!

N-slit (N source) interference :



$\delta = d\sin\theta \cdot k$

$$E_{total} = E_0 \left[e^{i(\omega t - kR)} + e^{i(\omega t - kR - \delta)} + e^{i(\omega t - kR - 2\delta)} + \cdots + e^{i(\omega t - kR - (N-1)\delta)} \right]$$

$$= E_0 e^{i(\omega t - kR)} \underbrace{\left[1 + e^{-i\delta} + e^{-2i\delta} + \cdots e^{-(N-1)\delta} \right]}_{= \sum_{m=0}^{N-1} (e^{-i\delta})^m}$$

$$= E_0 e^{i(\omega t - kR)} \underbrace{\left(\frac{1 - e^{-i\delta N}}{1 - e^{-i\delta}} \right)}_{=}$$

$$\boxed{\sum_{m=0}^{N-1} r^m = \frac{1 - r^N}{1 - r}}$$

$$\frac{e^{-i\frac{\delta N}{2}} \left(e^{i\frac{\delta N}{2}} - e^{-i\frac{\delta N}{2}} \right)}{e^{-i\frac{\delta}{2}} \left(e^{i\frac{\delta}{2}} - e^{-i\frac{\delta}{2}} \right)} = e^{-i(\delta(N-1)/2)} \frac{\sin(N\delta/2)}{\sin(\delta/2)}$$

Therefore $\vec{E}_{Total} = E_0 e^{i(\omega t - KR)} e^{-i\left(\frac{\delta(N-1)}{2}\right)} \dfrac{\sin(N\delta/2)}{\sin(\delta/2)}$

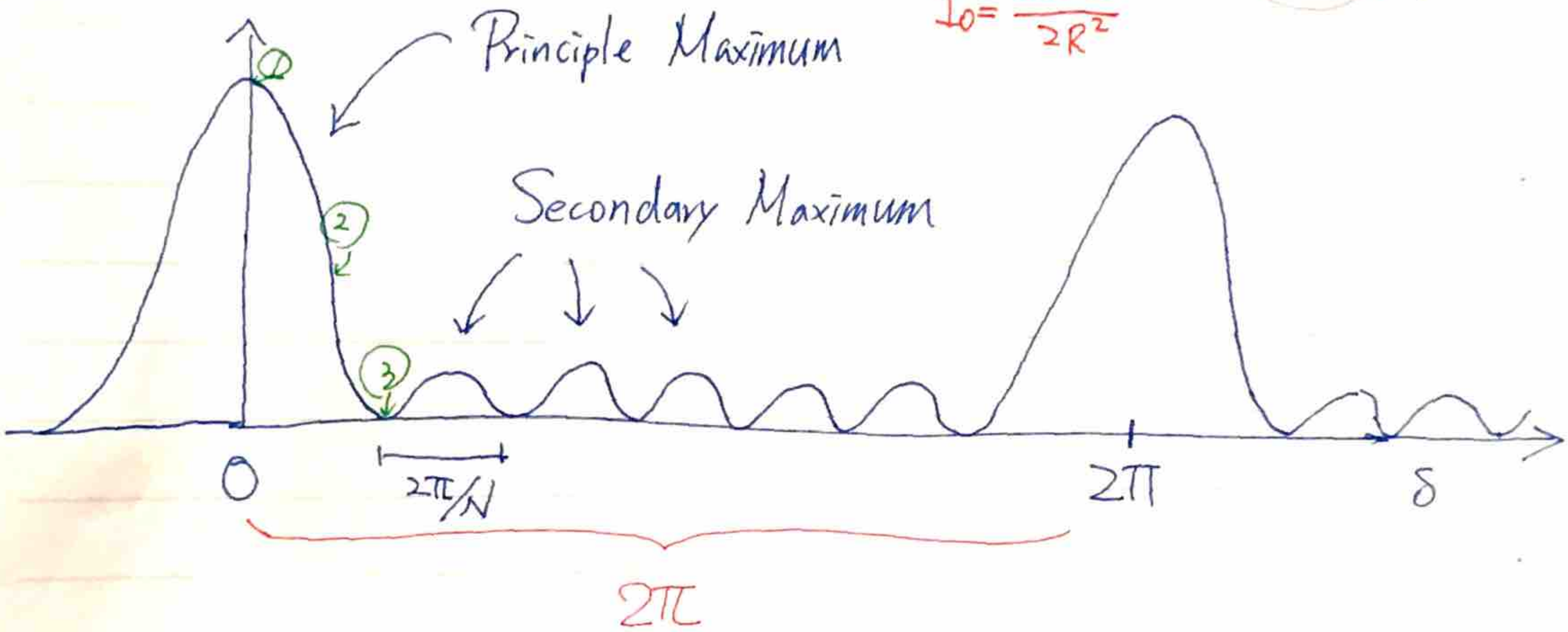
$\langle I \rangle \propto |\vec{E}|^2 = \vec{E} \cdot \vec{E}^*$

$$\Rightarrow \langle I \rangle = I_0 \left[\frac{\sin\left(\frac{N\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)} \right]^2$$

Ex: $N = 7$

$I_0 = \dfrac{\varepsilon_0 c E_0^2}{2R^2}$

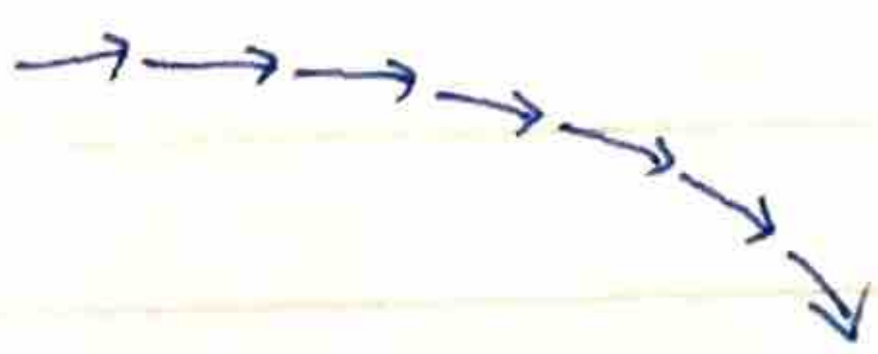
Slides 9



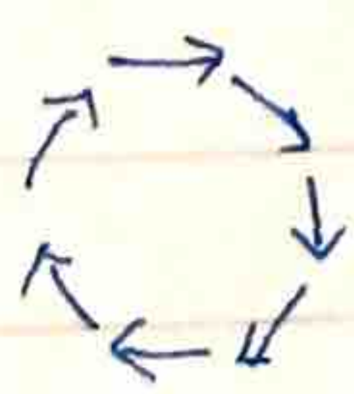
1 At $\delta = 0$

2 increase δ

δ



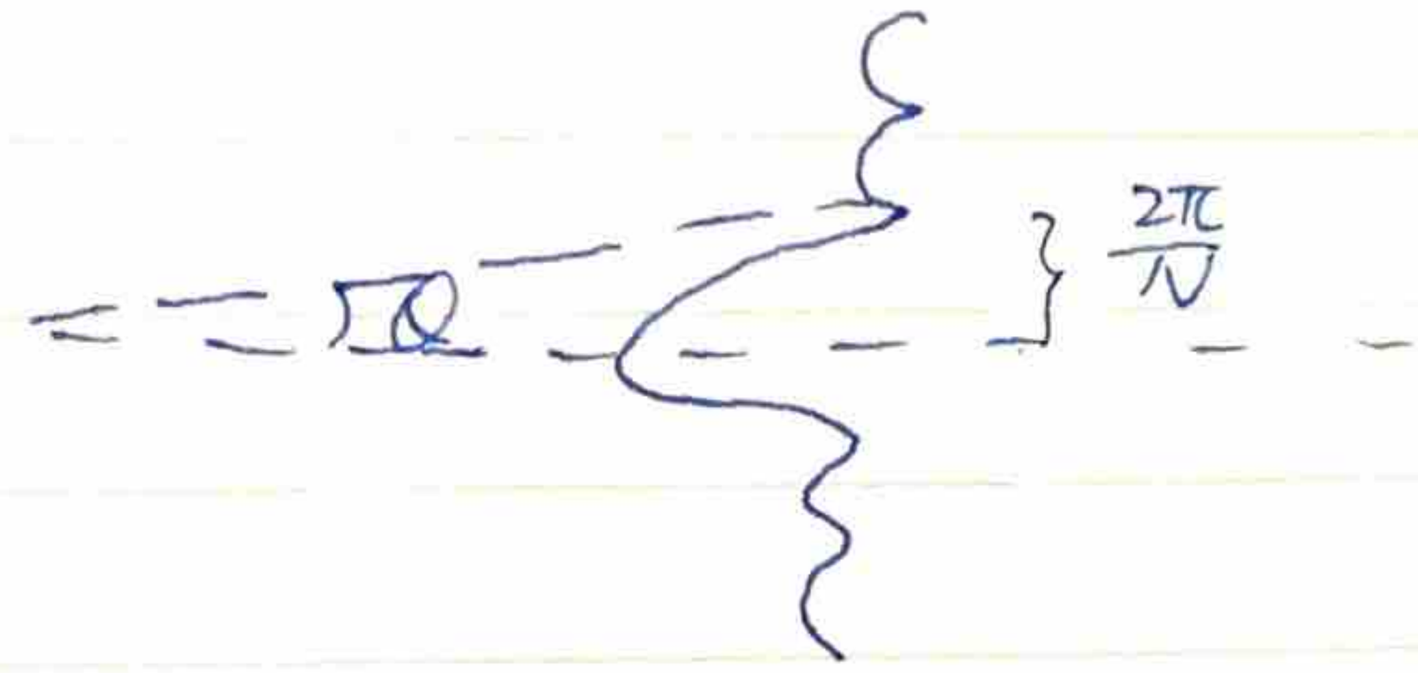
3 $\delta = \dfrac{2\pi}{N}$



N-radiators $\Rightarrow$ $N-2$ secondary maximum

Width of principle maximum: $\dfrac{4\pi}{N} \propto \dfrac{1}{N}$

Corresponding resolution:



$$\frac{d\sin\theta}{\lambda} = \frac{2\pi}{N}$$

$$\sin\theta = \frac{2\pi\lambda}{Nd}$$

We learn that: to get high resolution (small θ)

(1) Use small λ

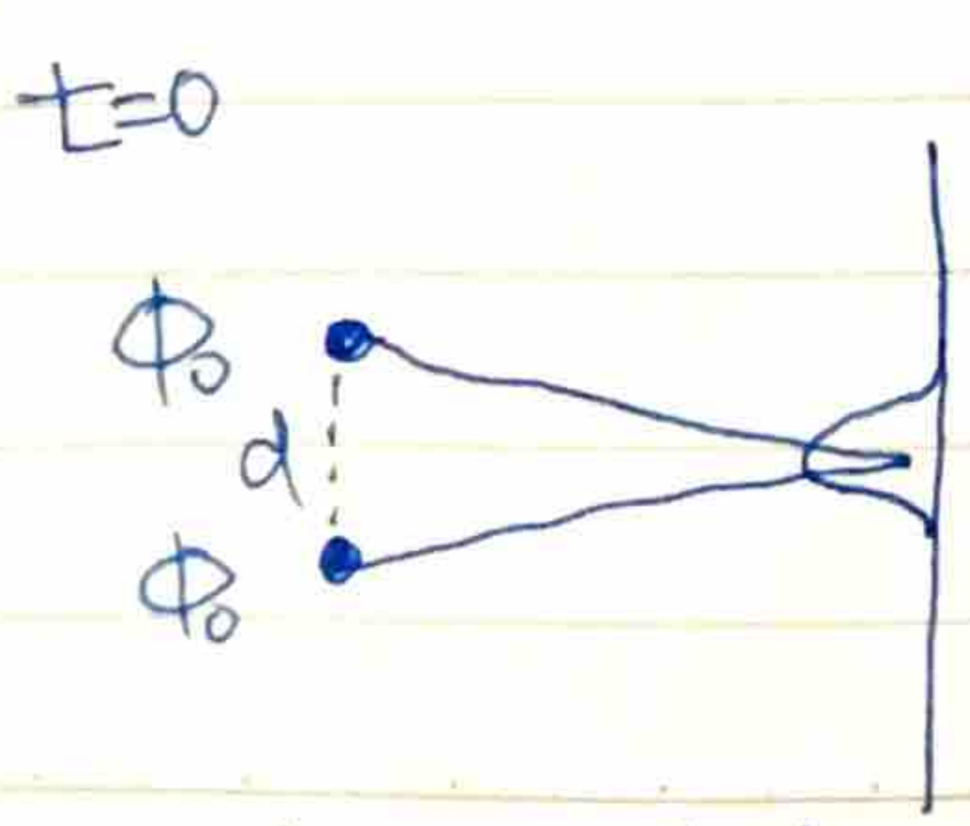
(2) Large d

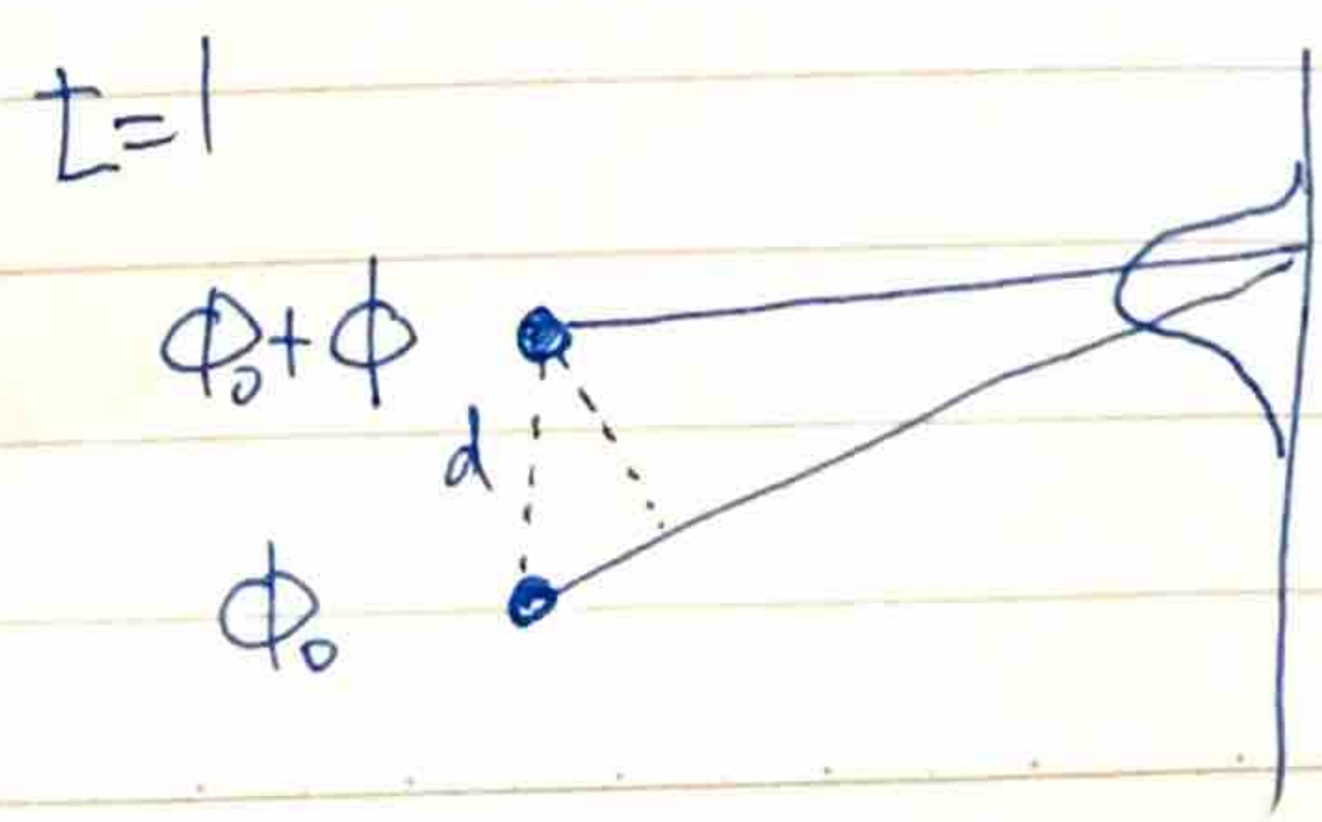
(3) Large N

Sweep?

If we want a sweep frequency ϕ

$\Rightarrow$ Add additional phase difference between the sources $\Delta\phi = \phi \cdot t$





Φ_0: original phase

$$\delta = \underbrace{\frac{2\pi}{\lambda} d\sin\theta}_{} - \underbrace{\Phi \cdot t}_{}$$

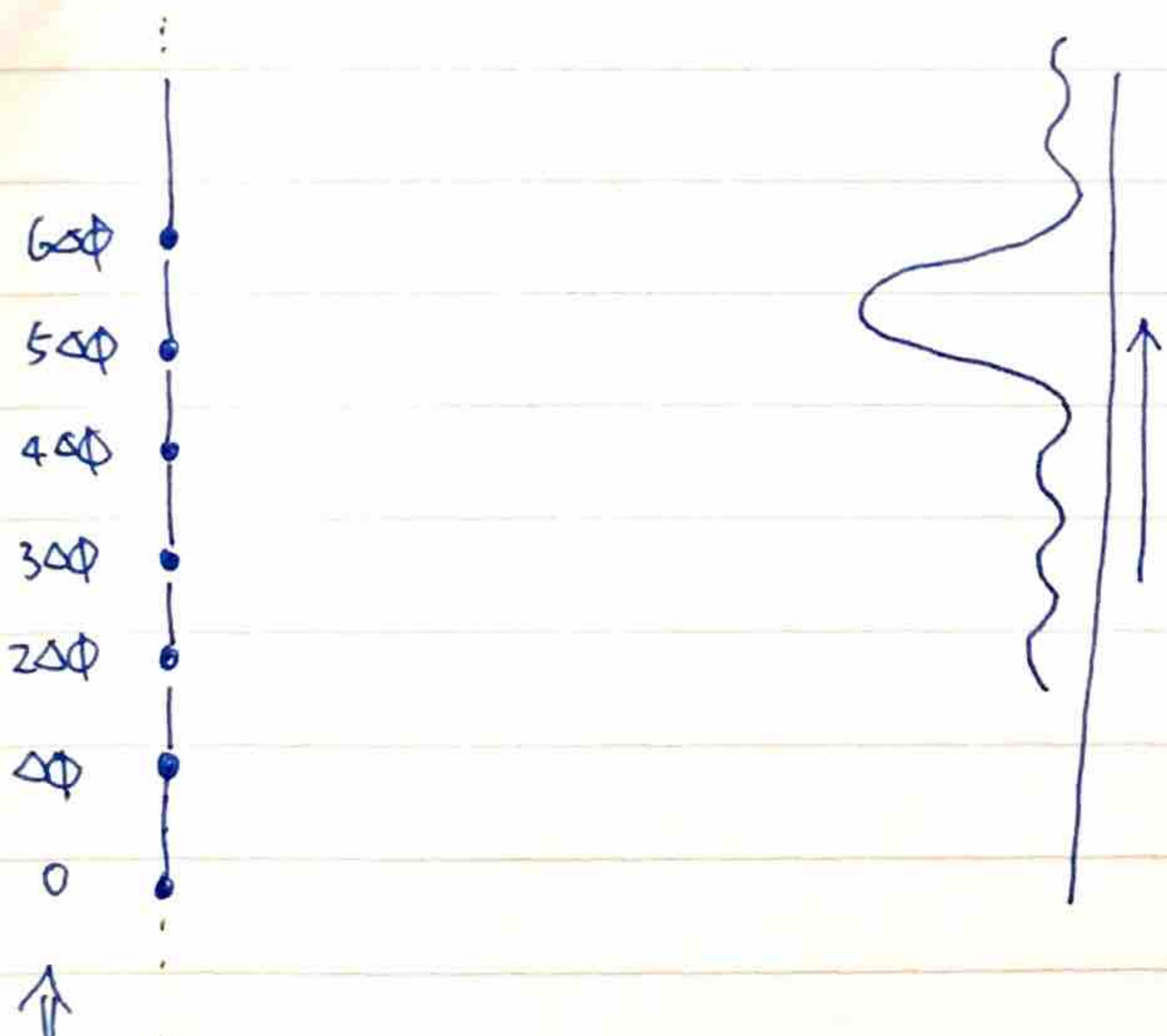
Phase difference from optical path length difference.

Additional Phase difference from the source

$\Rightarrow$ Principle maximum : $\delta = 0$

$$\Rightarrow \quad \sin\theta = \frac{\Phi t \lambda}{2\pi}$$

N source Phased Radar :



$\Uparrow$

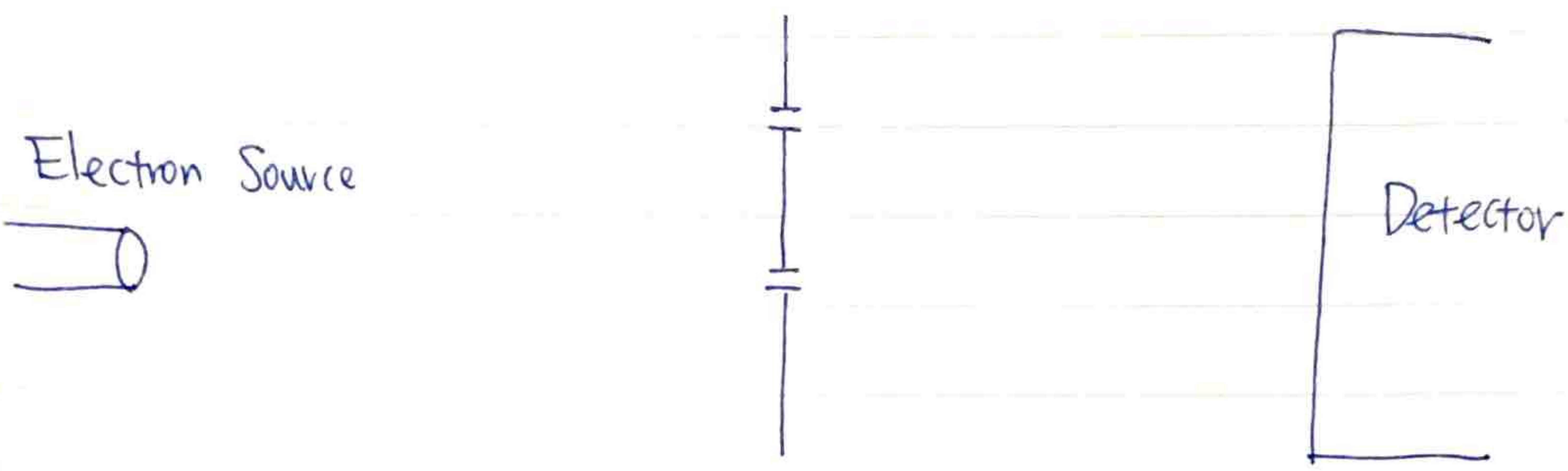
additional phase difference

$\Rightarrow$ change the direction of the principle maximum

Break?

We see interference : light, water, sound, ...

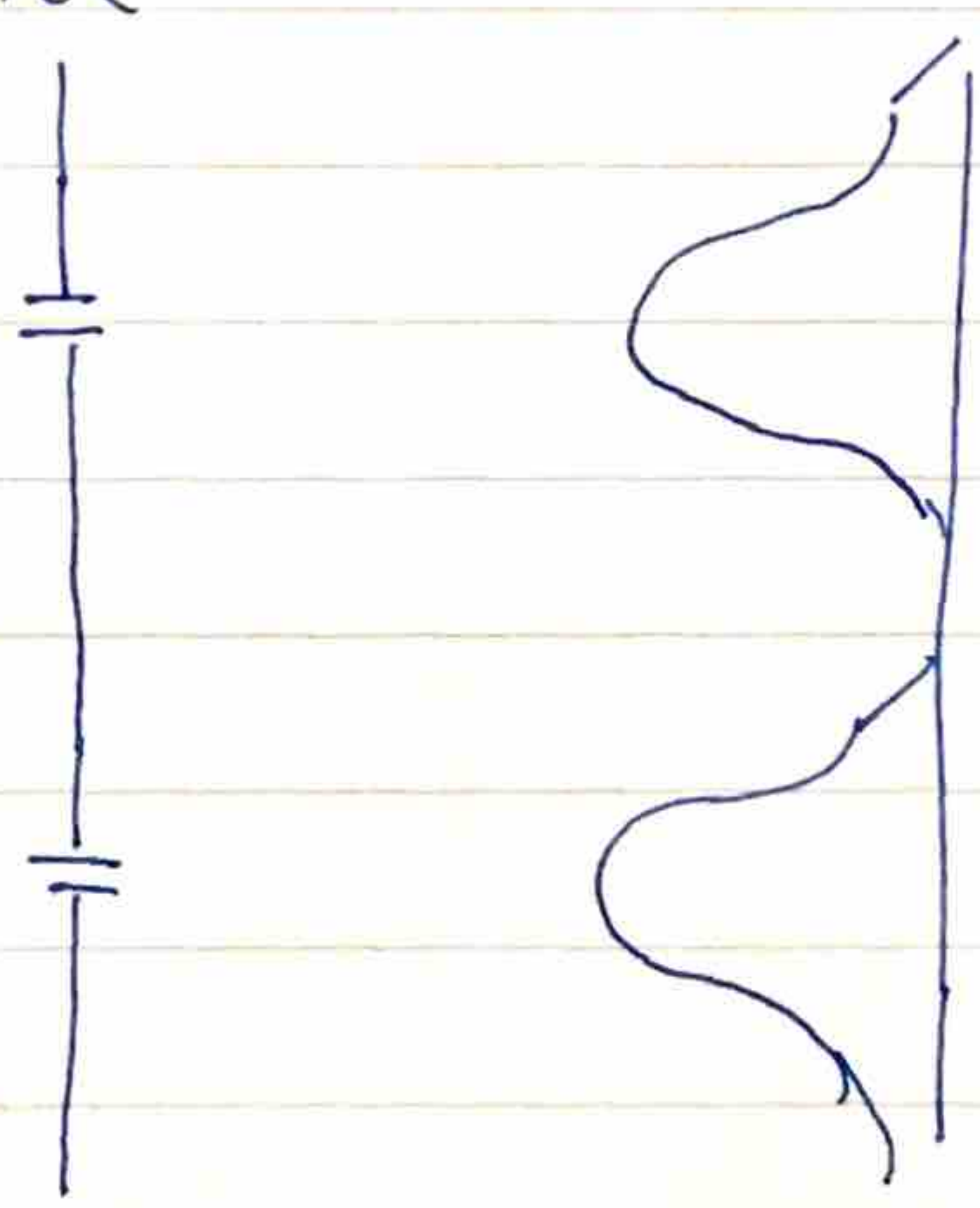
Single Electron Experiment :



Emit one electron every time.

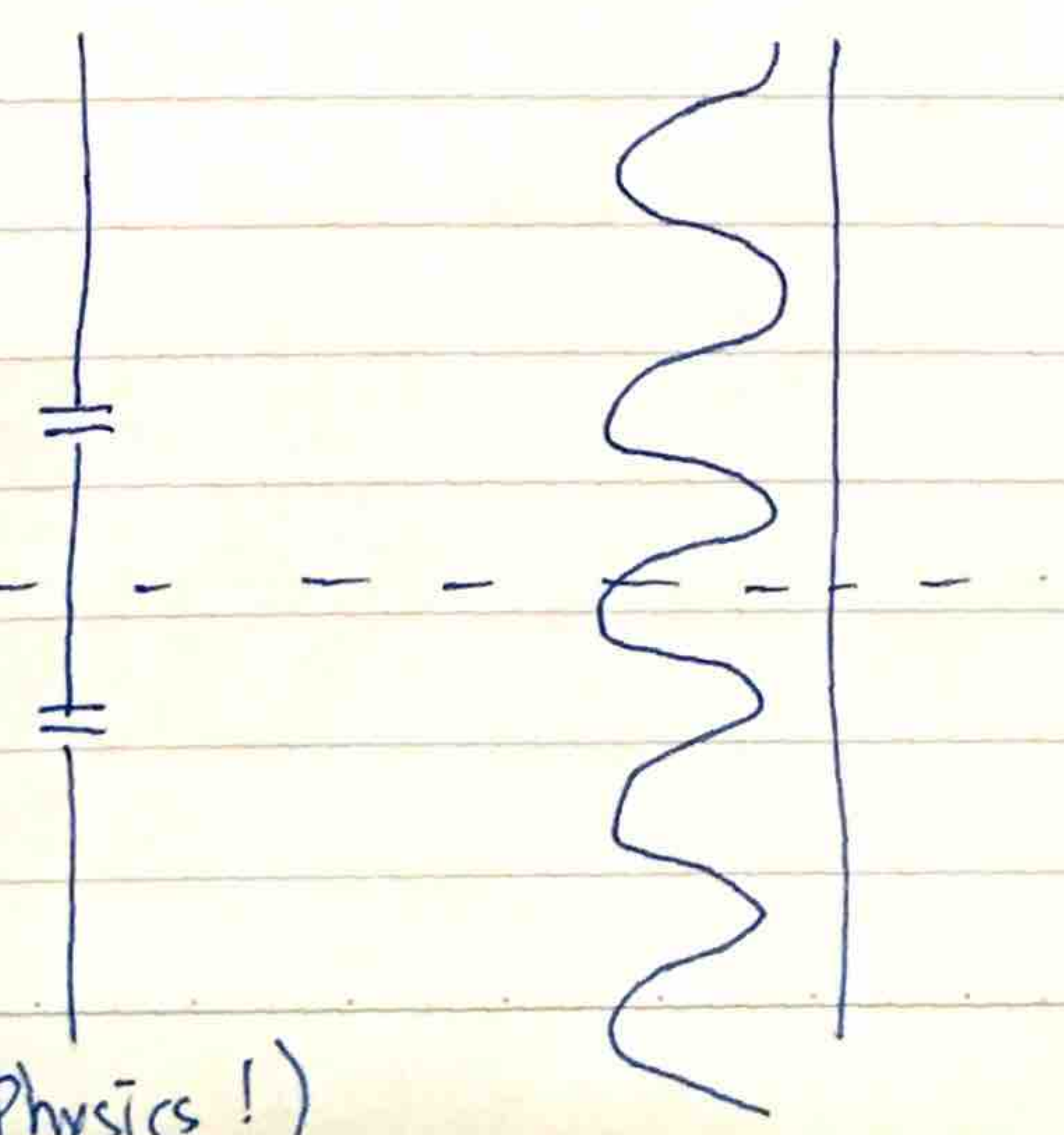
(1) No interference

X



(2) Interference

O



An electron interfers with itself

(Predicted by Quantum Physics !)

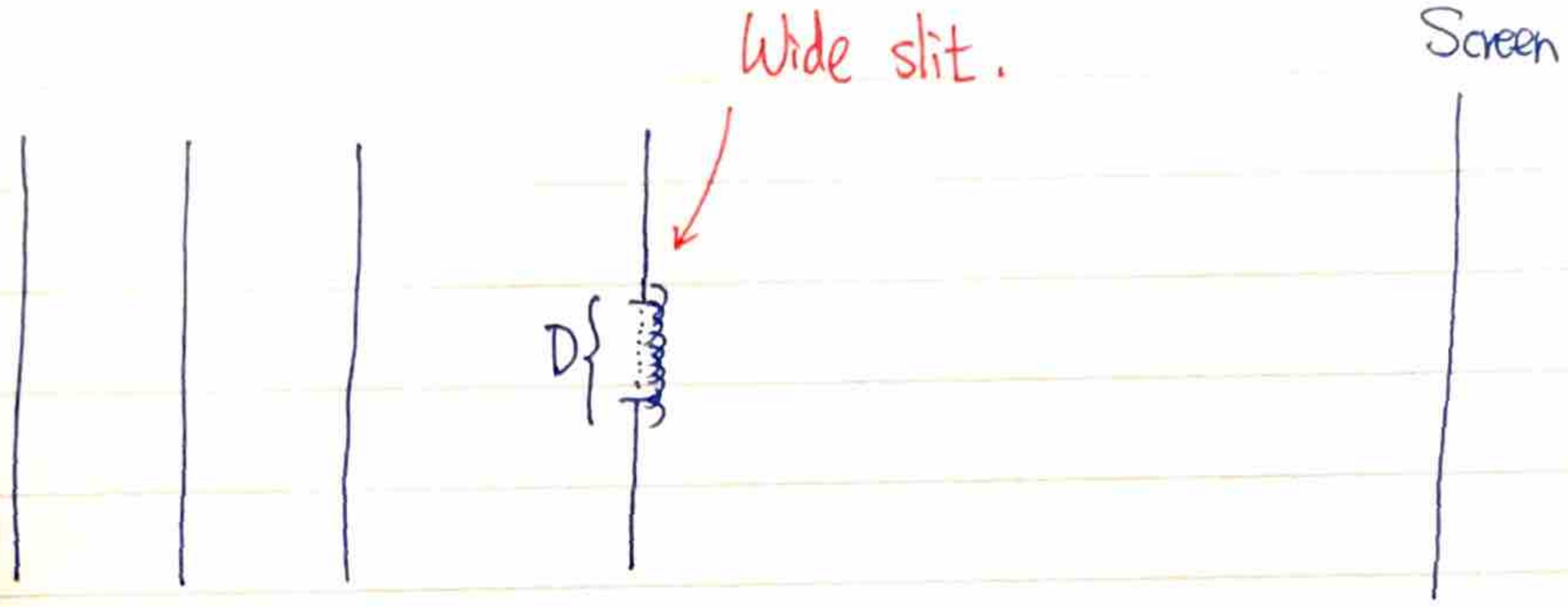
Break?

We learned the interference of 2 EM waves
↓ N EM waves
↓

$\boxed{\text{EM}} \to \boxed{\text{2EM}} \to \boxed{\text{N-EM}} \to \boxed{\infty\text{-EM}}$

Interference of infinite number of EM waves.

"Diffraction"



We have ∞ point like sperical EM wave sources.

This situation: we will see the "interference" between all the spherical wave sources.

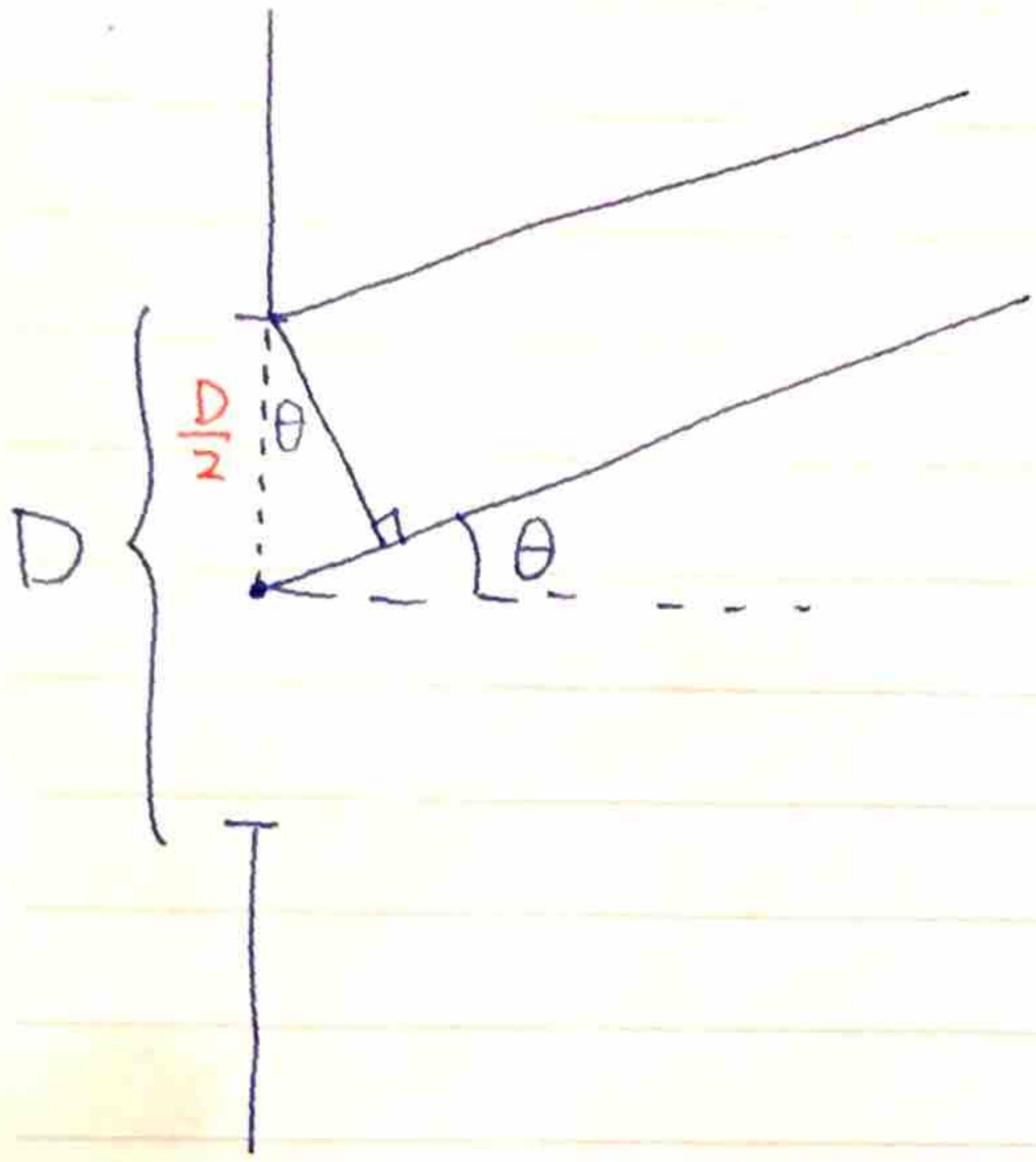
We call it "diffraction"

Feynman: No one has ever been able to define the difference between interference and diffraction satisfactorily

It is just a question of usage.

Usually: Interference: a few sources

Diffraction: many sources

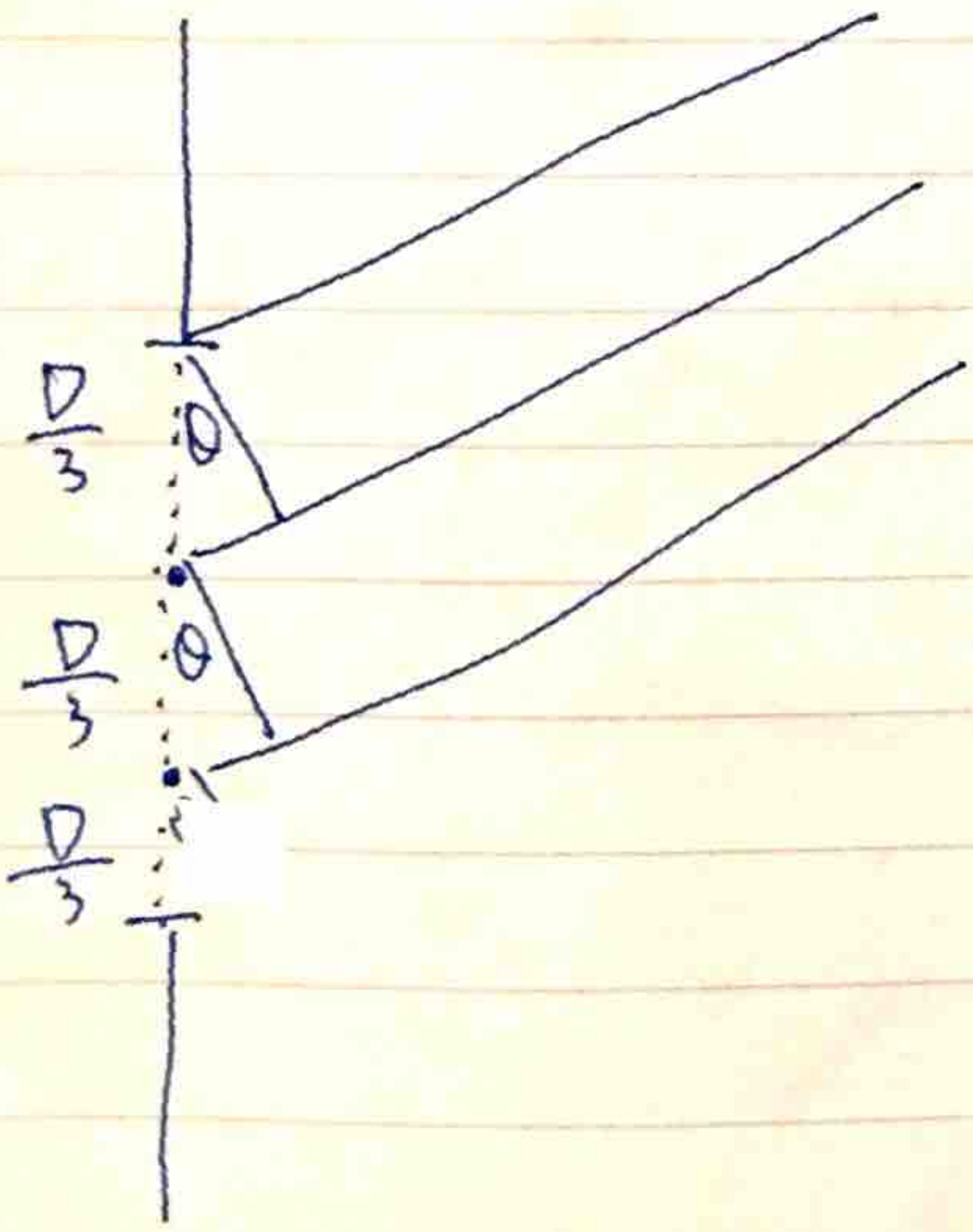


$$\delta = \frac{D}{2}\sin\theta\,\frac{2\pi}{\lambda}$$

Destructive Int: $\delta = \pi$

$$\Rightarrow \sin\theta = \frac{\lambda}{D} \quad \cdots\cdots \text{minimum!}$$

We can also divide the slit into 3 pieces



Destructive

$$\delta = \frac{D}{3}\sin\theta\,\frac{2\pi}{\lambda} = \frac{2\pi}{3},\ \frac{4\pi}{3}$$

$$\Rightarrow \sin\theta = \frac{\lambda}{D},\ \frac{2\lambda}{D}$$

⋮

divide into N pieces

$$\Rightarrow \sin\theta = \frac{\lambda}{D},\ \frac{2\lambda}{D},\ \frac{3\lambda}{D},\ \cdots \frac{(N-1)\lambda}{D}$$

Next time: Intensity..