YEN-JIE LEE: So I hope you can hear me. Welcome back, everybody. I hope you have the full energy back from Thanksgiving vacation. Everybody, welcome back.

So before we start, we'll talk about where we are now. So this is actually the goal, which we said, for 8.03, at the beginning. So what we have been discussing is to learn about how to translate physical situations into mathematics, simple harmonic oscillator, coupled oscillators, et cetera.

And we tried to put together infinite number of oscillators. And we found waves from this interesting exercise. And of course, we learned about Fourier decompositions of waves and also learned about how to put together physical systems. In order to do that, you need to define boundary conditions. And those conditions need to be satisfied, so that you can describe multiple physical systems all together.

And the third part of the course, we have been focusing on many, many applications, for instance, for the phenomenon related to electromagnetic waves and also many practical applications in optics.

And we are pretty close to the discussion between wave and vibrations and the future course, which is 8.04, the connections to quantum mechanics. And if we have time, we will talk about gravitational waves if we manage to do that. It depends on how fast we progress.

So let's start the lecture today. So first, we will give you a short review. Before Thanksgiving, we were talking about polarizer filters. And we have been researching how to make a very good photo so that you can post it on Facebook, right? So that's essentially what we learned.

So if you want to take a picture of the sky, which is deep blue, then you need to use a polarizer. And the reason? We also understand that it's because, if you look at the sky, which is actually, roughly a 45 to 90 degree wave from the direction of the sun.

Basically, what you get is that all those light fronts scattering, between the sunlight and the molecules or little dust in the sky, polarize the light. Therefore, you can actual filter them using the polarizer. Of course, you have to tune your polarizer carefully, so that you can actually minimize the light from the sky, so that you get a sharper image in your photo.

And also, we discussed about, with a polarizer, polarization filter, we can actually filter out also the reflected light, for example, from the water or from the window of a car. And that is because something which is closely related to the boundary condition, which we learned about from Maxwell's equation in matter.

And this is actually the four equations, which we discussed last time. And in that issue, we were using that to explain the incident light, from air to something which is denser, for example lighter gas.

And then we found that, if we start with unpolarized light-- this incident wave unpolarized light-- what we found is that the transmitted wave, which is actually in the bottom of this diagram, is actually still pretty close to unpolarized light but slightly polarized because of the transmission and the boundary condition.

And the reflected light, something very interesting happens. Only the component, which is actually polarizing the direction, such that the electric field is oscillating in a direction perpendicular to the surface of this light, will survive. And that actually gives you polarized light, which is actually reflected from the surface.

And this interesting phenomenon reaches the maxima, where you get the fully polarized light, when you actually set the incident angle of the unpolarized light at so-called Brewster's angle. And this Brewster's angle is happening when the reflected light and this transmitted light direction actually are orthogonal to each other. And that actually gives you the maxima effect we are looking for.

So that's actually what we have learned from electromagnetic wave in matter and, also, matching the boundary conditions between the electromagnetic waves inside the material and in the air, such that we actually learn about all those interesting phenomena.

And basically, we have learned how to describe electromagnetic waves, how to add electromagnetic waves together, how they propagate from one position to the other position, and how the boundary condition works and your equation of motion, et cetera, and something related to dielectric material.

And today what we're going to do is to put all the things we have learned together and see if we can actually explain a very interesting phenomenon. So before I start, I will show you a demonstration, which I'm not sure if I will be successful. It's very difficult, actually. And of course, during the break, you're welcome to come over and play with all those demos.

And here, I have two sticks. And I am going to create a soap bubble from this soap and water. And let's see if I will make it or not. So basically, I put this into the soap water. And I will try to open it to see if I can create a bubble.

Yeah. You can see. You can see that there's a colorful soap bubble created. You can see that it is not always easy. Oh, it's getting very messy now. I'm trying to destroy the classroom. But it's OK, because we are MIT.

You can that it's really beautiful. It's colorful. It live for a while, then it breaks. And of course, during the break, you are welcome to do this. And it's actually non-trivial to create this size of bubble. So the success rate is like 50%.

So as you can see from this demonstration, we see something really beautiful. This bubble is colorful. And I didn't actually shine this soap bubble by all kinds of different preset colors. So it appears automatically and just shiny with all kinds of different-- whatever, wavelengths I get from the lights that are in here.

Pretty bright light, there, on my face. And you can see that it becomes colorful. And we are going to understand what is going on and where this color is coming from. And the good news is that, based on the knowledge we have learned, we are in a very good position to understand this phenomenon.

So before we start to explain this phenomena, I would like to talk about a phenomenon, so-called interference. So suppose we have two electromagnetic waves. We can actually add them together because of superposition principle.

So what we can do is that, suppose I have two electric fields. E1 is actually defined as A1 cosine omega t minus kz plus phi 1 in the x direction.

So by now, you should know that this is actually the electric field propagating at angular frequency omega, with a wave number k, going toward positive z direction. And the electric field is perpendicular to the direction of propagation in the x direction. And also, this electric field have a phase of phi 1.

So that's actually what we know already by now. What does this mean, this expression mean? And it's actually the harmonic oscillating electric field. And of course, since I am talking about interference, basically, I can add this, the first electric field and the second electric field together and see what will happen.

So now, if you define the second electric field to be A2, which is the amplitude, cosine omega t minus kz, basically, they have the same wavelengths and also the angular frequency-- plus phi 2. But they have different phase. And of course, in this setup, I asked them to be pointing to the same direction, which is the x direction.

So we were wondering, what is going to happen if I consider the superposition of these two electric fields. And the total electric field, which is called the E vector, is actually E1 plus E2.

So before that, I would like to remind you about pointing vector and also the so-called intensity. So pointing factor is actually defined as this S vector, pointing vector, is equal to 1 over mu 0 E cross B. So this is actually the directional flux of energy per unit area. So that should be the pointing vector which we have been using for a while.

And also, another reminder is that, given the electric field, which is actually a harmonic progressing wave, the corresponding B field would be equal to 1/v. v is actually the speed of the light in some specific material. And k hat is actually the direction of propagation across E. This will give you the magnitude and also the direction of the corresponding magnetic field for the electromagnetic waves.

So now I'm interested in what would be the resulting intensity, I, if I try to superimpose this, to try to put together these two electric fields. As you can see, these two electric field have different phase. The first one has phase phi 1. The second one, has phase phi 2.

So this means that they may reach maxima or minima at different position in space. In this case, it's in the z direction. And what I'm actually defining here is two plane waves. And so it really depends on what would be the relative phase for the first and second electromagnetic wave.

In order to quantify how much they cancel each other or how much they enhance each other, what I'm going to do is to evaluate the intensity of the resulting electromagnetic waves.

And what is actually intensity? Intensity is actually the amplitude of the pointing vector. So I write this as the length of the S vector, pointing vector. And I can, of course, calculate what will be the value of this or, say, the length of the pointing vector. This will be equal to 1 over mu 0, based on these equations here-- just a reminder.

And I would like to know what would be the length of the E cross B field. That will give you the length of the pointing vector. And basically, what you're going to get is 1 over mu 0 times-- since the B field is actually highly related to the electric field, and it takes a hit of 1/v, right, in terms of the size of the amplitude. So basically, you're going to get 1/v, E squared.

And this cross product actually is OK, because it become E squared, because B field and the E field are always orthogonal to each other. And I can now rewrite this v. v is actually the velocity of the speed of light in matter. So basically, what I can rewrite is that will become 1/v will become c/n, which is actually the refractive index of a specific material. And still I have E squared here.

Finally, I can rewrite this formula, since c is equal to 1 over square root mu 0 epsilon 0. Therefore, I can rewrite this expression in terms of epsilon 0. And what I'm going to get is c times n time epsilon 0, There E squared.

So basically, what I do is I multiply both numerator and the denominator by c, and also I actually use this expression. Then I can actually cancel the mu 0 and then write everything in terms of c, n, and epsilon 0.

So until here, there was basically no magic. Basically, it's just rewriting the length of the pointing vector in terms of n and also the electric field. So now, what I am going to do is to plug in this expression into that formula and see what we are going to get.

So let me evaluate what would be the E squared, the length of the E squared. So basically, the definition of the E is shown here. It's the superposition of E1 and E2. Therefore, I can now quickly write down what would be E squared here.

Basically, you are going to get A1 squared, cosine squared, omega t minus kz plus phi 1. Basically, that is coming from the E1 times E1. The second term, which I'm going to get, is actually E2 times E2. E2 times E3, you are going to get A2 squared, cosine squared, omega t minus kz plus phi 2. Basically based on this equation, and I square it, and then I get the second part here.

And finally, what I am going to get is the third term, which is actually E1 times E2. Basically, you are going to get 2 times A1 A2 cosine omega t minus kz plus phi 1 cosine omega t minus kz plus phi 2, which is actually the cross term of this E vector squared.

Any questions so far? I hope this is pretty straightforward to you. And of course, I can now rewrite this product with 2 cosine, cosine times cosine, right? Basically, I can rewrite this using the formula, which we have related to a cosine times a cosine. Basically, I can rewrite this as 1/2 cosine 2 omega t minus 2kz plus phi 1 plus phi 2 plus 1/2 cosine phi 1 minus phi 2.

So basically, the first term is actually collecting the content of the two cosine and their length together. The second term is actually calculating the difference between the content of the cosine function. And what I'm going to get is actually phi 1 minus phi 2.

Now, based on this definition, intensity, I is actually equal to the magnitude of that pointing vector. So remember, our goal is to evaluate what will be the resulting average intensity. So now I can calculate what would be the average intensity over one period.

So this will be equal to 1/T, integration over 0 to T, one period, and the instantaneous intensity, I, dt. And what I am going to get is that-- so we have three terms. The first term is here, which is actually A1 squared, cosine squared. It's actually related to omega t.

And the second term is here. It's also proportional to cosine squared omega t. And finally, we have two terms here, which is actually proportional to cosine 2 omega t. And finally, the last term is actually independent of time.

So what I'm going to do is to evaluate the individual terms. For the first term, basically, A1 squared, cosine squared, omega t. So by now, it should be pretty straightforward for you if I integrate cosine squared over one period of time, basically, what you are going to get is 1/2. So is actually done several times in the p set.

So basically, what you're going to get is-- I am going to collect all of those constants from here and copy here. So basically, you have c times n times epsilon 0, which is actually coming from the definition of intensity.

And then for the first term, what I am going to get is A1 squared divided by 2. This 1/2 is actually just an integral related to cosine squared. Similarly, you are going to get the same result, a very similar result, for the second term.

The second term is going to give you A2 squared divided by 2. Finally, you can have the third term. The third turn is going to give you what value? Can somebody help me?

AUDIENCE: 0.

YEN-JIE LEE: Yes, 0, right? Because this is actually cosine 2 omega t, right? So if you integrate over one period, you are going to get 0 plus 0. Each period will give you 0, right? So 0 plus 0 is 0, so therefore you get 0. Very good.

How about the last term, anybody can help me?

AUDIENCE: It should remain as it is.

YEN-JIE LEE: That's right. Because it's a constant. So the average of a constant is a constant, which is actually giving you 1/2 times 2 times A1 times A2 cosine phi 1 minus phi 2. Of course, I can cancel this 1/2, which is actually coming from here, and the 2, which is coming from here. And basically, you are getting A1 A2 cosine phi 1 minus phi 2. And I need to close this bracket.

Any questions so far? So what we have been doing is that I evaluated the total electric field. I basically calculated the superposition of the two fields. And then I am interested in what would be the average intensity coming from this field.

And I write down E squared explicitly. There are four terms and only three of them actually survive. And basically, the expression I'm getting is like this. Basically, you have some constant multiplied by A1 squared over 2 plus A2 squared over 2 plus A1 A2 cosine phi 1 minus phi 2.

You can see that the intensity depends on phi 1 and phi 2, right? So this actually would change the resulting intensity. So in order to get some idea about what does that mean and also how does the average intensity change as a function of phi 1 minus phi2, what I'm going to do is to define phi 1 minus phi 2 to be delta, which I will call phase difference.

Then I would like to plot the averaging intensity, I, as a function of delta and see what's going to happen. So this is actually the result. So if I have the x-axis to be delta, which is actually phi 1 minus phi 2, and the y-axis is intensity. Of course, I would like to take out the constant, which is c times n times epsilon 0.

So I am plotting the y-axis' average intensity divided by c times n times epsilon 0. What I'm going to get is something which is actually oscillating up and down, like this.

The maxima value happens when delta is equal to 0. When delta is equal to 0, what is going to happen? This means that cosine delta is equal to what? , Therefore what you are going to get is A1 squared over 2 plus A2 squared over 2 plus A1 A2. This is actually when there delta is equal to 0.

And the intensity, as you reach maxima, and the maxima values is actually 1/2 A1 plus A2 squared, based on these calculations. On the other hand, you can expect that that intensity will reach a minima when delta is equal to which value? Anybody can help me.

AUDIENCE: Pi.

YEN-JIE LEE: Pi, yes. When delta is pi, what is going to happen? Cosine pi is minus 1. So therefore, what you are getting is 1/2 A1 squared plus A2 squared minus 2 A1 A2. And that will give you 1/2 A1 minus A2 squared.

You can see that, when the filter is equal to 0 or when the filter is equal to 2 pi, for example, if you increase the phase difference large enough, or the filter is actually 4 pi, all of those number will keep you maxima constructive interference.

So what does that mean? That means you are adding these two electric fields in the most efficient way. On the other hand, when the value of the filter is equal to pi or equal to 3 pi or equal to 5 pi, et cetera, the intensity, the average intensity reaches a minima.

That means, instead of adding them, you are actually canceling them. You are canceling the electric field of the first and the second electromagnetic wave. And now I give you a maxima intensity, which is 1/2 A1 minus A2 squared. Just a reminder, this A1 and A2 is actually the amplitude of the first and second electric field.

So what will happen if I set A1 equal to A2? If I set A1 equal to A2, that means the minima would be equal to what?

AUDIENCE: 0.

YEN-JIE LEE: 0, yeah, very good. How about the maxima?

AUDIENCE: [INAUDIBLE]

YEN-JIE LEE: It will be A1 plus A2, right? So basically, you are going to get four times larger value compared to the intensity before you add them together. So individual intensity is I. And after adding them together, with delta equal to 0, you are going to get four times larger intensity if the amplitude of the first and second electric field is the same.

So very good. So that's actually the result of the calculation. And you can see that the amount of intensity we can get out of this highly depends on the filter, which is actually the phase difference between the first and the second electric field.

Can we actually get some more feeling about this addition? So what I am going to do is to, again, write everything down in terms of imaginary number or, say, a complex number. So if I rewrite the electric field, E1, as a real part of A1 exponential i phi 1, exponential i omega t minus kz. In the x direction. And I can also rewrite the expression for the second electric field to be the real part of A2 exponential i phi 2, exponential omega t minus kz, again, in the x direction

So if I add these two fields together, what I am doing is like in the complex plane I have an imaginary number contribution in the y direction. And the real part is actually in the x direction. Suppose omega t minus kz is 0. At some instant of time omega t minus kz is equal to 0.

So what I'm doing is I have the first vector, which is actually presenting the contribution of the first electric field. And this electric field is going to be pointing to a direction phi 1 away from the real axis with amplitude equal to A1. This is actually what we learned from the first lecture.

And if I add the second electric field, what I'm going to get is another vector, which is actually A2 in length. And the angle is phi 2, here. So the resulting amplitude is actually when I take the real part of the first and second expression, adding them together.

Basically, I am taking a projection to the real axis. And that is actually the resulting amplitude of the electric field, which is actually the superposition of the first and second field.

So you can see that, when phi 1 is equal to phi 2, what is going to happen is the following. So basically, what you're actually going to get is that you are increasing the length of the routing vector, which is the addition of the two vectors. You are actually getting a maxima out of this addition.

Because phi 1 is equal to phi 2, therefore, these two vectors form a straight line, therefore, you can actually add and get maximum amount of the amplitude out of this.

On the other hand, when phi 1 minus phi 2 is pi, which I define as delta, when this happens, what we are doing is like addition of two vectors, in a complex plane, but they are pointing to the opposite direction. So the first one will be like this. And the second one will be looking like that. And they are actually trying to cancel each other.

So that's actually how you can understand what is happening with different delta values. In this case, delta is equal to 0. The phase difference is equal to 0. And in the second case, phase difference is equal to pi. And then what happened in between is like this.

You are adding them sort of together but not in the most efficient way or the most destructive way. And you are actually evaluating what will be the resulting amplitude by looking at the vector sum of the first and second field.

So I hope that this will give you a some more intuition about what we have been doing. Any questions so far?

So now, we are actually in a very good position once we understand this superposition of the two electric fields and the interference. Basically, the size of the resulting intensity will be highly dependent on the phase difference between the two fields.

Then we are in a very good position to discuss the phenomenon which we just see in the demo. So before I actually perform the calculation and give you the explanation, I would like to take a vote, as of usual.

So the question we are asking is, in addition to what we see in the demo-- we see a colorful bubble-- how thick is the soap film such that you can see color from the reflected light? The first option is maybe it's like 1 millimeter, which is possible. And that is about the size of the head of a pin.

Or it can be 100 micron, so that's actually about the size, the thickness is about the size of the human hair. Or 100 nanometer, which is the size of the virus. How many of you think the thickness is roughly 1 millimeter? Raise your hand.

Nobody thinks so. Really? Actually nobody think that's the case.

How about 100 micron? How many of you think so? How about 100 nanometer? Me How many of you? So that is actually the vote. And we are going to know the result very soon. And how about the rest? Cool.

So now we are going to solve the puzzle. So just a quick reminder about what we have learned from the last lecture. So there is a reason why we have the lecture first on the reflection of an electromagnetic wave before we discuss the color of the bubble.

So from the last lecture, suppose I have two materials, which form an interface between material number 1, with refractive index n1, and the second material has a refractive index n2. If I have an incident wave, incident electromagnetic plane wave, and the incident angle is actually, in this case, 0, that means this incident plane wave is actually propagating in a direction which is actually hitting the surface directly.

So if the initial amplitude is A, what we have learned from last time is that there will be a reflective wave, which is actually R times A. R is actually reflective coefficient. And finally, you have also the transmitted wave, which I call T times A, where is the transmission coefficient.

From the exercise, which we actually already done last time, R is equal to n1 minus n2 divided by n1 plus n2. And the transmission coefficient is actually T equal to 2n1 divided by n1 plus n2. So basically, what I am actually talking about here is a conclusion from the exercise we have done in the last lecture, just a quick reminder.

So I would like to discuss with you various situation related to R value. So the n1 and the n2 are related to the property of the first medium and the second medium. So it could be that n1 is actually greater than n2. So if n1 is greater than n2 in the experimental setup, that means that R will be greater than 0. Because R is actually n1 minus n2 divided by the sum of n1 and n2.

Therefore, what I'm going to get is something like this. So basically, I'm going to have an incident wave like this, where, say, I use the notation pointing upwards. Once they got reflected, it is actually still like this, pointing upward, because the R is actually greater than 0. There's no changing sign in the amplitude. Therefore, there's an no flip in amplitude if n1 is actually greater than n2.

On the other hand, the transmitted wave, if you look at the functional form of the transmitted wave, and transmission coefficient, T is actually equal to 2n1 divided by n1 plus n2. It's always positive. Therefore, will there be any possibility to flip the sign? No. You are absolutely right.

So therefore, what is going to happen is that I will use this little arrow to keep track of the sign change. Basically, you'll see that after it pass through the boundary, there will be no change in sign in amplitude no matter what happens.

On the other hand, if I have the situation n1 smaller than n2, what is going to happen? If you calculate the R value, it will be negative, right? In this case, R will be smaller than 0. So what is going to happen is that, initially, the incident wave has positive amplitude. And I keep track of the sign of this amplitude by this arrow pointing up.

Because the R is actually smaller than 0, therefore, there is flip in sign in the amplitude. So what is going to happen? So the reflective wave will look like this. And I use this arrow to keep track of the flip in amplitude. And finally, as I mentioned before, the transmitted wave, the T, is always positive. Therefore, there will be no change in sign in amplitude.

Finally, the third example is, if I have somehow two different materials, but they have the same refractive index, what is going to happen is that there will be no reflection, and everything goes through.

Even if you have two different kinds of material, but if they have the same refractive index, then what is going to happen is that everything will pass through. And what you are going to get is that you will have no reflected light. Meaning R is actually equal to 0. Any questions so far?

I would like to make sure that everybody understands the consequence of this calculation. So if I introduce no flip in amplitude, this means that this contribution will introduce a filter equal to 0. So basically, there will be no change in the phase, because there's no flip in amplitude.

On the other hand, if there's a changing sign in amplitude, what would be the resulting filter value? Can somebody actually tell me?

AUDIENCE: Pi.

YEN-JIE LEE: It would be pi. Very good. So that means you are getting hit by a phase difference of pi. Therefore, the amplitude changes by a factor of cosine pi, which is minus 1. So that is actually something pretty important when we have the discussion of the soap bubble reflection.

So let me give you a quick example about why is actually the amount of the reflected light and also what is the amount of transmitted light. Let me give you a concrete example.

For example, if I have n1 equal to 1, which is actually the refractive index of the air, and n2 equal to 1.5 If that happens what is the resulting intensity? Just a quick reminder, average I, the average intensity will be equal to c times n times epsilon 0 A squared divided by 2, where A is the amplitude of the electric field. Just a quick reminder. And this 1/2 is coming from the time average, just a reminder.

So now I can go ahead and use these two formula, R and T, to calculate the reflection coefficient and this transmission coefficient. So R will equal to 1 minus 1.5 divided by 1 plus 1.5. So basically, you get minus 0.5 divided by 2.5. And that is actually going to give you minus 0.2.

Of course, I can also calculate what will be the T, which would be a 2 divided 2.5. So basically what you are getting is 0.8. So I can now calculate what will be the resulting intensity of the reflected light. Everybody's following?

So what would be the intensity of the reflected light? This will be equal to minus 0.2 squared, right? Because the average intensity is proportional to A squared. A is actually the amplitude of the electric field. R should tell you what is actually the relative amplitude between the reflected light and the incident light.

Therefore, you are getting hit by 0.2 squared multiplied by the initial intensity. Basically what you are going to get is 0.04 I, initial intensity. So basically 4% of the light is reflected. That may surprise you a bit, right?

Because when you see, for example, the soap bubble, you see that it is still pretty bright, right? But in reality, only 4% of the light or 4% of the intensity got reflected. That is because your eyes is actually having nonlinear. Your eye responds to the-- or, say, receiving or interpreting the intensity is really highly nonlinear.

So basically, you get 4% reflected. And the rest actually goes through. And just to convince you that the total intensity is 100%, we can calculate what would be the intensity of the transmitted light. This will be equal to 1.5-- this is actually related to n2, because the intensity is proportional to c n epsilon 0 A squared over 2-- times T squared. So basically you have a 0.8 squared and the I initial intensity.

And if I calculate this value, basically you are going to get 96% of the initial intensity. So 96% of the initial intensity actually passes through the boundary and continues and propagates in the second medium, which, actually, in this case, is the soap.

So the picture is the following. When 100% of light intensity going towards the boundary, what is going to happen is that 4% of the light got reflected. 4% of intensity got reflected.

And also, because n1 is smaller than n2, therefore, there is a flip in sign in the amplitude. And the rest continues, 96% of them. And there is no flipping sign in the amplitude. Any questions so far? We're really pretty close.

So now we are in a position to discuss what is actually really happening to this soap bubble. So I'm going to keep this result here. And I will now discuss a situation in which you have two interfaces.

So suppose I zoom in, zoom, and zoom in this soap bubble and put it on the board. So this is actually the soap film. And I have now an incident wave, which is actually going into this bubble. So now I have 100%, which is actually going toward this film.

So after this light, this plane wave hits the film, what is going to happen? The first thing which happens is that there will be 4% of the light got reflected. n2 is equal to 1.5. It's the same setup, just a reminder, just to make sure everybody is on the same page.

So 4% of the light got reflected. Of course, the sign changed. 96% of the intensity actually continue. And what is actually happening is that there will be no change in amplitude in sign.

And this is actually not the end of the story, right? Because the light will continue and continue to propagate. What is going to happen is that it will reach another boundary, where the incident light is you're traveling, from n2 refractive index material, to n1, which is actually the air.

Now I have a situation where the light is actually going through the boundary and going out of the air. That means the light is actually going into the bubble. So this is actually inside the bubble.

So what is going to happen is the following. Basically, the calculation is the same, except that now the R is actually 0.2 instead of minus 0.2, right? Because now n2 minus n2 is actually 0.5. 0.5 divided by 2.5 is positive 0.2.

So basically, what you are going to get is a reflected light, which actually doesn't change. It doesn't change the sign of the amplitude.

And what is actually the intensity? The intensity will be 96% times 4%, because only 4% of the light got reflected. And of course, a large fraction of the light actually pass through the bubble, 96% times 96%. And this would be, again, pointing upward, because T is always positive.

Any questions so far. You can see that this actually really interesting, because most of the light actually pass through the bubble. So that's actually already one thing we've learned from this exercise. Now, what is going to happen to this light if I continue and increase the time?

What is going to happen is that this reflected light, from the second surface or second boundary, will go backward and pass through the first boundary again. What is going to happen is the following.

So basically, we're going to get, again, transmitted light and the reflected light. What will be the sign of the reflected light? Will the arrow be pointing up or down?

AUDIENCE: Up.

YEN-JIE LEE: Up, yeah, very good. Right now, if you are bored, then that means I am very successful. So that means I'm getting 4% times 96% times 4%. What would be the sign for the transmitted light? Pointing up or down?

AUDIENCE: Up.

YEN-JIE LEE: Up. Very good, so everybody gets it. And 96% pass through, 96% times 96% times 4% will pass. And of course, I can now continue and continue. What is going to happen is that now you have learned 8.03. You will see that this is a crazy phenomenon.

What is going to happen is that there will be a tiny fraction of the light which is trapped forever between the two surfaces. They are going to be bouncing back and forth, boo, boo, boo, boo, boo, boo, boo, boo, forever.

Of course, the fraction of the intensity is really, really small. Because every time you've got the reflection actually happening, you take a hit of 4%. But since we are talking about theoretical physics, so, theoretically, that would continue forever. That's actually pretty interesting.

And going back to practical situation, basically, I can safely ignore any further reflection, because they are hitting so hard, because every time I get a 4% hit, right? Therefore, I can ignore all the other contribution.

And what we are actually seeing is what? Our eye is here. We see the contribution of the first pass, which is actually reflected from the first surface. The second pass, OK, it pass through the first surface, got reflected from the second boundary, and pass through the first boundary, in the second round, and then, also, reaching your eye.

So what are we looking at ? We are looking at the superposition of two electromagnetic waves coming from one, which is like this, and two, which is actually like this.

The question now is what is the thickness of the film? Now I can define the thickness or, say, the width of this film to be d. Now the question we are actually asking is, what would be the thickness, d, which is needed such that I can have constructive interference? Now the question becomes really clear.

And we can actually calculate that by evaluating the phase difference between the path number one and the path number two. So now, in order to have constructive interference, I need a specific phase difference. But before that, I need to calculate the phase difference first between path number one and path number two.

What will be the phase difference? The phase difference delta will be equal to, of course, pi. This pi contribution is coming from the flip of the amplitude. That will actually give you an pi phase difference.

The second phase difference is coming from the difference in the optical path length. You can see that the first path, it doesn't go into the film. It got reflected, directly. And the second path, which is path number two, it takes more effort or more time for the light to go back and reach your eye.

How big is the path length difference? The size of the path length difference is 2 times d, right? Of course, I need to actually translate that back to the phase. So first, I need to actually calculate how many period. So the length divided by lambda will be the period. So lambda is actually the wavelength of the incident light.

But I am missing a factor here. And can somebody help me? Because this lambda is actually inside the material, right? So which factor, I'm missing?

AUDIENCE: n2.

YEN-JIE LEE: n2, right? Yeah, thank you very much. So basically, inside the material, since the speed of light is 1.5 times smaller than the speed of light in vacuum, therefore, the wavelength is actually lambda divided by n2. And this is actually the number of period.

And now, I need to translate that to phase difference. Therefore, I multiply this by 2pi. So you can see that now I have successfully evaluated or quantified the phase difference between path number one and two. That is there are two contributions.

The first one is pi. It's related to the flip in amplitude. The second contribution, the blue one, is actually coming from the optical path length difference. And of course, we can evaluate that really precisely.

Therefore, we can now quickly conclude that, in order to have constructive interference, I need to have filter equal to 2N pi, where N is an integer. And in order to have destructive interference, I need to have filter equal to 2N plus 1, pi, which is actually the result of the calculation which we have done, I think, before. Yeah, there.

So this is actually based on the calculation we have done in the beginning. So we are really close. So now we have this result, delta is equal to pi plus 2d, times 2 pi divided by lambda divided by n2, right, so this complicated formula?

Now we are in the position to evaluate what would be the phase difference. So the first thing which I would like to discuss is that, when d goes to 0, what does is actually the limit? The limit is when the width of the film is really, really small, it goes to 0.

What is going to happen? You are going to have destructive interference. Why is that? That is because, even when you have d equal to 0, the filter is pi because of the flip in sign in path number one.

The second thing is that now I can calculate what would be the constructive interference width. So this will happen when d is equal to 2N minus 1 lambda, divided by 4 n2. So basically, you can use that formula there and solve d. Then basically that's the formula we are going to get. And I will not go into detail with this.

Any questions so far? So now, the third conclusion is that, if I fix d and the change in lambda, that is actually the more practical situation. Because I have the soap bubble. And it have a well-defined width, which is d.

And what is happening is that I am trying to shine this soap bubble with light with different wavelengths, right? So that is actually the third situation.

If I fix the width of the film, and the change the wavelength, lambda, what I am going to get is that the lambda max, which is the wavelength needed to have constructive interference, will be equal to 4d n2 divided by 2N minus 1. So basically, I can solve the lambda if I am given a d value.

So actually, we already get the answer we are asking in the beginning. The first question is, why do we see color? The second question is, when I see color, what is actually the width of the soap film? We are going to know the result in a moment.

So now, I have this formula in hand. If I have d roughly equal to 100 nanometer, which is the third option we were discussing, that is going to give you lambda maxima equal to 4 times 100 nanometer times 1.5-- n2 is 1.5-- divided by 2N minus 1.

So that is actually 600 nanometer divided by 2N minus 1. Suppose I have N equal to 1, basically I am getting 600 nanometer. Suppose I have N equal to 2, 2N minus 1 is actually 4 minus 1 is 3. Therefore, you get 200 nanometer and 120 nanometer, et cetera, et cetera, which are the required wavelengths in order to have constructive interference between path number one and path number two.

Everybody is following? If I plot the spectra of this lambda max, assuming d is 100 nanometer, what I am getting is like this. So this is a situation of very thin film. So this is the lambda.

What I am getting is that there will be a maxima here, which is actually 600 nanometer. Red color is actually roughly 650 nanometer. This is red light. And this is actually roughly the range of the visible light, which is actually between lambda equal to lambda violet-- violet is equal to 400 nanometer.

So you can see that the first maxima, lambda maxima, where you have constructive interference is at 600 nanometer. So that means you are going to see what kind of color in your soap bubble?

AUDIENCE: Red.

YEN-JIE LEE: You are going to see red, right? And then the next wavelength which you can have constructive interference is 200 nanometer. That is actually shorter than the wavelength of the violet light. It's out of the range of the visible light.

What is going to happen? Your eye will not see it. So the next one would be here, whatever, blah, blah, blah, blah, which I don't care, because they are so short in wavelength. And you cannot see them.

So you can see that, if I have a width which is roughly 100 nanometer, very same situation, what is going to happen? What is going to happen is that you are going to get only one maxima in the visible light range. And therefore, you can see color.

Any questions so far? Now, what I'm going to do is take the same formula here, but now I would like to change this d. So now I would like to change the d to consider a situation where you have a very thick layer.

So now I would like to change the situation to a very thick layer, so maybe I need to erase this part of the board to make some space. So now if I have d equal to 100 micron, what is going to happen?

So I can now still use this formula to calculate what would be the lambda maxima. So lambda maxima will be equal to 600 micron, which is when you have N equal to 1. But this wavelength is way, way larger, much, much larger than the wavelength of the visible light. So it's not going to work.

Therefore, you have to be patient. You have to increase the N value until N is equal to 500. So I am calculation 1, 2, 3, 4, 5, 6, until 500. Ahh, we are in the visible light range, right?

Now, 5000 will give you 600.6 nanometer. Phew. Suddenly, your eye can see it. Very good. That's very nice, right? So I can now put it in my diagram. This is actually the wavelength, again. And, ah, I get one line here.

How about the next one, N equal to 501? I'm going to get 599.4 nanometer. It's pretty close to this one. And the next one would be 598.2 nanometer if N is equal to 502. And what you are getting is that you can see that, no, things are not going very well. They are full spectra, all very, very narrow. Very, very large number of wavelengths can give you constructive interference.

So what is going to happen? What is going to happen is that your eye will see reflection with all kinds of different wavelengths. And what color is that?

AUDIENCE: White light.

YEN-JIE LEE: White! You are going to see something which is white. So that is actually the answer to our question. So what would be the required thickness? The required thickness is something like 100 nanometer. So we can see how thin is the water bubble.

That may surprise some of you, right? Most of you actually didn't think that's actually that thin. Secondly, if d equal to 0, you are going to have destructive interference. That means there will be no reflected light. Everything is going to pass through. And the bubble is like transparent.

Finally, when the bubble is really thick, you are going to see white. So let me finish this lecture with a demonstration here. First, before I turn off the light, I would like to turn this on.

So what I have here is a very complicated machine. It's not that complicated, actually. So basically, I have a light source here, which emits light with all kinds of different wavelengths. And I have this little device here. There's soap solution inside. And I can actually rotate from outside. You see how sweet is this setup.

And I can actually create a soap film out of this. You can see that I am rotating and trying to actually project the result on the wall. You can see that, initially, there's nothing really striking in the beginning. You can see that the light is which color?

AUDIENCE: White.

YEN-JIE LEE: It's white, right? Remember, because of the optical setup, this image is actually upside down. So the upper edge of that image is actually the lower edge of my setup, which is the lower edge of my soap film.

We have gravity, right, so that I can walk around. Due to gravity, you can see that it will form a thicker and thicker layer in the bottom of my experimental setup or in the upper edge of the image. On the other hand, due to gravity, the upper edge or the lower edge of the image will become thinner and thinner as a function of time.

At some point, the color will start to show up. As you can see now, since we wait long enough, the soap film becomes thinner and thinner. And you can see that there are colors popping up. It's like a rainbow. Why is that? Because I am varying the thickness of the film as a function of the vertical distance.

So therefore, you can see that this is actually showing you that different d value will give you very different colors. And if we wait long enough, basically what we are going to get is that the whole film will become more and more colorful. And I am sure that after this class, you can walk out of the classroom and explain to your friend why the soap bubble is colorful.

Thank you very much. And if you have any questions, I will be around. And of course, if you want to make your own soap bubble, you can actually go ahead and play the demo here.

So hello, everybody. So we are going to show you a demonstration, which we can see colorful interference pattern from a soap film. So basically, the experimental setup up is like this. Basically, we have light, which is trying to shine this thin layer of soap film.

And the result is actually projected on the screen. And you can see, at first, you don't really see a lot of colorful pattern, because the thickness of the film is still rather large, rather thick. Therefore, you don't really see a lot of pattern.

But as a function of time, you can see that this pattern is actually changing. Because of gravitational force, you will be able to see that the lower part of the film becomes thicker and thicker. And the upper part of the film, which you see that upside down on the screen, is actually becoming thinner and thinner.

As we actually discussed during the class, when soap film is thin enough, there will be only one or only a few maximas in the interference pattern as a function of wavelengths, which happened to be inside the visible light range.

And you can see it now, already, this colorful pattern really develops. It's really beautiful. And you can see that, in the lower part of the experiment, really have authentic color, because there are multiple maximas in the visible light range.

On the other hand, in the upper part of the film, you typically have very little number of maximas or only have one maxima, in the visible light range, as a function of wavelength. Therefore, you see really, really dramatic and very, very colorful pattern develop from this experiment.