YEN-JIE LEE: Welcome back, everybody to 8.03. So today we are going to continue discussions on the examples which we started the last time, sound waves, and, this time EM waves, which can be described by wave equations. So far, what we have learned is that there are three different kinds of systems we've discussed in a lecture or in a textbook. And the first one is actually a string, a very long string, system with constant tension and mass on the string. And the behavior of the string obey wave equation, and can be described by a wave equation.

We also can produce a density wave with a spring. And basically the density wave or spring can also be described by wave equations. So that's as you described in the textbook. Finally, last time we actually discussed sound waves. We have an open pipe, and then we can have air inside the pipe. And the behavior of the air, or the molecules inside the open pipe, can be described by wave equations. Crashes So what we are going to do today is to discuss with you a special kind of wave, which is electromagnetic waves. And that's actually slightly different from what we have learned in the last few lectures. And we see what this is different today in the lecture.

All right. So this essentially is a reminder of Maxwell's equations. So basically what is written here is the differential form of Maxwell's equations. So the first law is Gauss law. It says should the divergence of e, the electric field, is equal to rho, which is the charge density as a specific point, divided by epsilon zero, which is actually a constant. We'll call it permittivity of this constant. OK? Which should relay the divergence of e and the density of the charge at this specific point.

And the second law is actually Gauss law for magnetism. This is actually the divergence of b equal to 0. So divergence b is always equal to zero because we haven't yet discovered the magnetic monopole yet. Right? So maybe you have discovered it one time, at some time, in your experiment. Please tell me now. I want to be the first with who knows how to do that. [LAUGHS] All right? So promise me.

The third one is Faraday's law. It's curve of e equal to minus partial e partial t and the b, as a reminder, is a magnetic field vector. And in the last law is actually Ampere's law. It's actually the curve of b equal to mu 0. Mu 0 is actually a constant, permeability. Which would lay the current and displacement current. Epsilon 0, partial e, partial t, to the curve of b. OK?

And I would like to draw your attention to these term. This very important term is actually Maxwell's addition. OK? Without Maxwell's addition, there would be no electromagnetic wave. Then you could not see me. OK? [LAUGHS] All right.

So, what we are going to discuss today is a simpler case at the beginning. So what will happen if we go to a vacuum? Going to vacuum means there will be no material charges floating around, and that means rho will be equal to 0. Therefore, the divergence of e will be equal to 0. And also in the last question Ampere's law, say which is that the current density will be equal to 0. Therefore, the function of curl of b equal to mu 0, epsilon 0, partial e, partial t. OK?

So before we go into the discussion of Maxwell's equation's implication, I would like to remind you about some mathematics which will be used in this lecture. I hope you have seen this in other courses or 8.02. So as you can see, we use del here, which is a vector. This vector is defined as partial x, partial y, and partial z, in the x, y, and z direction. OK? So this is actually some kind of operator. You see that again. A lot more operators in 8.04.

And we make this definition because I'm lazy. Because I don't want to write so many partial, partial x, partial, partial y, partial, partial z again and again. Therefore we define del, which is like this. Looks really crazy, but it really makes our lives much easier. OK? So that's the whole reason. As a physicist.

And as we discussed before, we have divergence, which is defined here: del times a, both of them are vectors. Y is the operator vector, the other y is actually really a vector. You basically get partial ax, partial x, plus partial ay, partial y, plus partial az, partial z. So basically you just multiply them like a normal operation. And you can actually get this question. OK. Then finally there's curl. Curl is actually del cross a. So basically, maybe in the past you see this complicated formula. You know, it had maybe no meaning to you.

And one easy way to remember this curl is to not care about this. Don't look at the right hand side part. But just remember that you can actually construct this curl by determining a matrix. In the matrix, I can fill the first row by x, y, and z unit vector. And the second row, I filled it with the counting of del. Finally I feel the content of the matrix with a vector. Then you will be able to calculate the determine of this back matrix, then you naturally would get this very long formula. So you don't really need to remember the formula, but you will be able to know how to calculate it really easily. OK? OK.

So we talk about divergence. We talk about curl. What does that mean? Divergence, curl, what does that mean? So divergence is actually some kind of measure which measures how much the vector v spreads out, or diverges, from a point of interest. So in this example, this vector field-- vector field means at any point in the space which I am discussing, there is a vector associated with that. I call it vector field. We know scalar field very much. For example, the temperature as a function of position is a scalar field, right?

So, every point you have scalar corresponding to that point. And in the case vector field, every point you have a vector connected to that point. And if I arrange the vector field like that, each arrow is actually a straight dimensional the vector. Then if I evaluate the divergence and you see the heart, it looks like something is really spreading out from the center of that graph. And that will give you positive divergence. OK?

So that's the physical meaning of this formula. And the second formula which we discussed is the curl. So curl is actually del cross a. It's a measurement of how things are curling around a point of interest. OK? So you can see that if I arrange my vectors in a space like that, then you will see that something is really rotating around that specific point. Therefore, if we evaluate the curl, you would get the nonzero value. So that's actually the physics intuition which we can or the mathematics intuition which we can actually get before the discussion of Maxwell's equations.

So if you accept those ideas, let's take a look at what we have here, especially in the vacuum case. OK, so in a vacuum case, you have a curl of e equal to minus partial p, partial t. What does that mean? That means, if you change the magnitude of the magnetic field, now we introduce a curling around thing in the e field. So if you change the size of the b field, then the e field will start to curl around, doing this. And on the other hand, if you change the electric field, that will do something, which is curling around in the b field. All right, so do you have any questions?

I hope everybody's familiar with this notation. So from here actually Maxwell, see the light. [LAUGHS] Can you see it? Maybe not yet. Maybe we are slightly slower than Maxwell, but we will see that together in this lecture. For that I would need as usual help from the math department.

So we are going to use this identity. This identity is curl of curl of a would be equal to del, divergence of a, minus del dot del, a. So this is an identity which we learned from the math department. And of course, if you are patient enough, you can actually expand all those terms and compare the left hand side of the formula and right hand side of the formula. And you will see that really this works. So I'm not going to do that here in front of you.

So if you accept this is an identity, and then usually when we have del times del, we call it Laplace. And usually we write it as del squared. With this formula, I can now put my electric field into this formula. assuming that I am working in a situation of a vacuum and I plug in my electric field into that formula. Then this is actually what I am going to get. Curl of e. And this will be equal to del, divergence of e minus del squared e. OK?

And based on the four Maxwell's equations, we can immediately recognize that divergence of e is equal to 0, because I don't have charges around. Therefore you, cannot not introduce a gradient or divergence. You can introduce positive divergence in the electric field. Therefore, when you evaluate the divergence of the electric field, that is equal to 0. According to that formula, Gauss law. And you can also take a look here. We have curl of e, according to this formula. Basically you can conclude that this will be equal to minus partial b, partial t according to Faraday's law.

So if I look at the left hand side, that would be equal to the curl of minus partial b, partial t. And this will be equal to basically, I can take the minus sign out and take the partial partial t out. And basically you have curl of b. And according to Ampere's law, this would be equal to minus mu 0, epsilon 0, partial square e, partial t. OK, everybody is following? So basically what I have been doing is copy the left-hand side and make use of the Ampere's law. And basically you get minus mu, epsilon zero, partial square e, partial t squared

And this thing, the left hand side, is equal to the right hand side. On the right hand side, what is left? This is equal to 0. So this is gone. This is equal to minus del squared, e. I can cancel the minus sign. Then basically what I am going to get is del squared e. And this will be equal to mu zero epsilon 0, partial square e, partial t squared. Wow, this is what? This looks like, what? Wave equation again. Oh my god.

[LAUGHTER]

But there is some difference. This is different from what we've seen before, right? Before, the wave equation only has partial squared partial x squared. This time, you have this del square. Very strange, right?

So what is this? Del square is actually partial square, partial x squared cross partial square partial y square, plus partial square, partial z square is the operator, which will have three components. And basically, if you do this calculation, you are going to have how many times? If you do this del square e, how many times you have? You have how many? Any anybody help me?

Yeah, you have three times in x direction, you have three times in y direction, you have three times in z direction. Therefore how many times? You have nine times, because each operator is acting out the vector. OK. So it's very important because this is a common mistake. So you have nine times. and it looks really like the wave equations it tastes like wave equation, it looks like equation, it feels like equation - that wave equation - and therefore is really the wave equation, right?

[LAUGHTER]

OK so this is a three-dimensional wave equation. Very cool. So we are increasing the dimension. So I can write it down more explicitly. So basically what I'm getting is partial square e partial x squared, partial square e, partial y square, plus partial square e, partial z square. And this is equal to mu zero, epsilon zero, partial square e, partial t squared. OK?

So Maxwell sees this when he adds this additional term here. As you can see, if I don't have this additional term, the displacement of current from Maxwell, what is going to happen? This curve of b will be equal to zero. So what is going to happen to this identity? This left hand side part will be equal to zero. There will be no electromagnetic waves. OK?

So that's really thanks to Maxwell's work. And this is actually really an equation which changed the world, because that actually gave us a lot of insights about how we can send energy, how we can actually understand the phenomenon related to light. So what is the velocity of this wave equation? The velocity, Vp, would be equal to what we usually call c. Because you have been using this constant for a long time. And that will be equal to 1 over square root of mu zero epsilon zero. And to measure the speed of light, it takes a long time to achieve that. Let's take a look at the history.

So the first attempt was done by Galileo so 1638. He was doing an experiment, and that he was trying to track the speed of light. But he was not super successful. So his conclusion was that if the speed of light is not instantaneous, then it is super fast. He says it's at least 10 times faster than the speed of sound. He said OK, this is super awesome, very fast.

OK. So that's what he found. And later Romer actually made use of the orbit of Jupiter. Basically he use Jupiter and Jupiter's satellite to measure the speed of light. So when the earth is closer to Jupiter, then somehow the satellite of Jupiter appears faster than when the earth is actually away from Jupiter. Because that light have to travel through additional time. Two times the radius of the orbit the Earth. Basically that's the math that he was using.

He is actually making the first computative measurement of the speed of light. And what number he found is 2 times 10 to the 9 meters per second. Then finally, again using the star the observation as a tool to actually calculate the speed of light. James actually nailed it. He found a value which is really close to the current understanding of the speed of light, which is 3 times 10 to 9 meters per second.

Therefore, if you calculate that using all those constants here, you will be able to see that, indeed, from Maxwell's equation, you get-- oh, it should be 3 times 10 to the 8, not to the 9. I was saying 10 to 9. It should be 3 times 10 to the 8 meters per second. So indeed, this equation is actually predicting the speed of light to be 3 times 10 to the 8 is matching the experimental result.

So that is pretty nice. And you may ask a question-- so wait a second. You said this is actually an electromagnetic wave, right? So that's actually what I was talking about. But this equation only talks about electric fields. What is happening to the magnetic field? What happened? Can we actually choose arbitrary magnetic fields? Is a magnetic field also described by the three dimensional wave equation, right?

The answer is that indeed you can actually do the same exercise. You can now instead of plugging in electric field, you can plug in a magnetic field. And you will extract exactly the same conclusion.

You will conclude the del square B, will B equal to Mu 0, epsilon 0, partial square, B partial to square.

OK, so it is actually very important to see that the magnetic field also obey this wave equation.

OK?

And also from Maxwell's equation, you can see that the changing electric field will produce a curling around a magnetic field. The same thing also happens here. A changing magnetic field also produce a curling around electric field.

So what does that mean? That means E, electric field create magnetic field. Magnetic field create electric field. And this happens all the time. Therefore, one cannot live without the other. They are living together. They are all together, forever.

All right, so what is actually oscillating is actually both electric field and the magnetic field, right? So you may ask, OK, we are talking about vacuum.

Vacuum means there is no material, no charge, no whatsoever in vacuum. So what is actually oscillating? Who is oscillating? Is the electric field and the magnetic field. This so-called field, all those vectors-- which are actually oscillating-- it's not the material, but all those vectors associated with the space, which is actually oscillating up and down.

All right, so originally I would like to show you a pulse of light in front of you. And show that it's moving, but it's too fast. So I couldn't do that.

[LAUGHTER]

Fortunately, we have photos. Photos are actually collected the recorded photons. Emitted from the object of the interest. So this is actually how we make applesauce at MIT. We shoot-- bullet through the apple then we have the sauce.

[LAUGHTER]

But not sure if that's tasty enough or not. But that's how we do it in MIT-- MIT style.

And the good thing is that this kind of technique is improved dramatically in these days. I would like to show you a short video, which is actually recording a video of-- it's recording experiment, which you shoot some beam of light through some plastic container.

And the speed of this recording corresponds to one trillion frame per second. So this is super fast recording. And they can actually reconstruct the propagation of light through this bottle. The credit is actually to the Media Lab Camera Culture group.

And let's take a look at the video. Just one second. OK, so this is actually recording at one trillion frame per second. So you can see that there's a light pulse-- a very short pulse created. And is really pass through the bottle. And it can be recorded with the technique created by Media Lab.

So you can see that the pulse is really propagating through it. And the reason why we can see the pulse is because there are air, there are material which will actually change the direction of the light. And therefore, those are recorded by the camera. And they take trillions of frames of this thing, and put them together. Then basically-- and they take many, many frames, and they put them together to reconstruct this movie.

So as you can see that indeed you can see the propagation of the light through this kind of video. So I hope that we enjoyed this video.

And let's actually take a look at some concrete example which make use of the wave equation, which we did right here. So let's consider a plane wave solution. Things we are entering a three dimensional world. So that's actually consider so-called a plane wave.

So in this example, I am considering the electric field that's actually equal to the real part of E0 exponential i, kz minus omega t. And this electric field I actually consider here is in the x direction.

And if I write all the terms from this expression expressively, that's actually what I'm getting is-- x component will be E0, cosine kz minus omega t, 0 and 0.

So what does this mean? What is actually a plane wave? The plane wave basically is actually fielding the whole space. What I mean by plane wave is I feel the whole space with electric field. This electric field only have one-- only one direction have non-zero value, which is x direction in this example.

And then the other direction, there's no-- the magnitude is actually equal to 0. So that's actually what I mean by plane wave. And also the electric field is filling a whole space in the discussion-- in the example which I discussed here. And if I define my coordinate system like this, x is in the horizontal direction. Then that means everything is actually-- all the electric field is actually pointing toward the x direction in this coordinate system.

So we have discussed progressing wave in the past few lectures. Can somebody actually tell me the direction of propagation of this plane wave? So the hint is that this is actually equal to

E0, that the magnitude of the x component is equal to E0, cosine kz minus omega t.

What is actually the direction of propagation of this electric field?

AUDIENCE: z.

YEN-JIE LEE: It's in the z direction. Yeah, very good. Because we know that this is actually going in the positive z direction. Because this is actually kz minus-- there's a minus sign-- omega t. So therefore it's going toward the positive z direction.

Not x direction. x direction is where the electric field is pointing to. And the direction of propagation is toward the z direction. So there's a difference.

So first thing which I would like to do is to check if this so-called plane wave solution actually satisfy the equation-- the wave equation which we derive here. Del square E equal to Mu 0 epsilon 0, partial square E, partial t square.

So I can now plug that in to that equation. I can now plug in to this equation. If I plug in the wave-- the plane wave solution, which I have here to that equation-- basically, I can get the left-hand side. The left-hand side of the equation, you will get minus E0.

Only one term which contribute is the partial square E partial z square term which contribute. Right? Because the magnitude of the electric field only depends on z and t. Therefore, you get minus E0, k square, cosine, kz minus omega t in the left-hand side of the wave equation.

How about the right-hand side? Right-hand side actually you are taking partial derivative, which is fed to t. Basically, you get minus Mu 0-- Mu 0, epsilon 0-- I copied from that formula there. And you basically get omega square out of it because of the partial square, partial t square operator.

And then you basically get cosine kz minus omega t. And of course I missed the E0 term. E0 should be copied from-- on there.

So now I can show that-- OK, this cancel. Basically, this is the same cosine kz minus omega t. And E0 also cancel. And I can cancel the minus sign.

What I'm going to get is k square is equal to Mu 0, epsilon 0, omega square. So that means there should be a fixed relation between k and omega, which is actually omega over k will be equal to 1 over square to the Mu 0, epsilon 0. And this is equal to c.

If this is satisfied, then the plane wave is a solution to the wave equation-- only when this is actually satisfied. Otherwise we can write arbitrary plane wave equation, but they are not the solution of that equation from Maxwell's equation.

So now, I have derived the electric field and also know the relation between omega, the angle frequency, and the k, the wave number. And now, what about magnetic field?

So I just mentioned before, magnetic field cannot live without electric field. And electric field cannot live without magnetic field.

So what is actually responding magnetic field? We can actually evaluate that. So now, the question is what is actually the magnetic field? And how is that vary as a function of time and as a function of position in the space?

So we are facing a choice. So there are two equations, which relate electric field and the magnetic field. It is actually very important you make the right choice when you start your calculation. So we can use Faraday's law. We can also use Ampere's law. But there's only one, which is actually much easier to derive a solution, which is the choice of Faraday's law.

If you choose to use Ampere's law to evaluate B, then you are going to get a really super complicated problem to solve. But on the other hand, if you choose to use Faraday's law to solve this problem, then you can see that the unknown is the magnetic field-- the field which I would like to evaluate.

And the expression for the B is actually rather simple. It's actually just a partial derivative, partial B, partial t. So it's pretty simple and you can actually evaluate the known part. This curl looks pretty complicated. So you can actually evaluate that because you know what is the electric field.

On the other hand, if you will use Ampere's Law then you will be in trouble because you don't know what is a B, xBy and Bz. And you have to evaluate curl. And you get a lot of terms, and that is actually equal to something from-- the information from the electric field. And that would be very difficult to evaluate.

So therefore, what we are going to do is to use Faraday's law, curl of E will be equal to minus partial B, partial t. So basically, as I mentioned in the beginning, we can make use of the equations the determinant of matrix to evaluate the curl.

So therefore, I am going to use that. And then what I'm going to get is x, y, z unit vector for fill the first row. And the partial partial x, partial partial y, partial partial z, which fill the second row of the matrix. Then I get Ex, 0, 0 because the electric field is only in the x direction.

And this will be equal to-- only two terms survive because of these two 0's. So all other terms are killed, and only two terms are now 0. The first term is actually partial Ex, partial z in the y direction. And the second term is actually minus partial Ex, partial y in the z direction.

Any questions? Am I going too fast? All right, so you can see that the electric field only depends on the position z. It's independent of y. Therefore, partial Ex, partial y, is actually equal to 0.

Wow, this become much, much easier because there's only one term which is surviving. This is a operator. Then basically what we're going to get is I can now calculate partial Ex, partial z based on that equation, E0 cosine kz minus omega t. Then basically, what I can get is minus-- I get a K out of it, E0 sine kz, and it's omega t.

So this is actually the result of the left-hand side. The right-hand side of that equation of the Faraday's law is minus partial B, partial t. So this will give you equal to minus partial B, partial t.

So very important-- I don't want to drop the y direction. So this is just y direction. And this is actually a vector and this is also a vector.

So what I could do is to do a integration over t. And those will cancel the minus sign. So if I integrate over t, then basically what I'm going to get is K over omega is 0, cosine kz minus omega t in the y direction only. So I'm doing a integration of t, cancel the minus sign, then this is what you want to get.

And of course, k/omega is actually 1/c. So therefore, you have E0-- you can actually simplify this fraction-- and this is actually equal to E0 over c, cosine, kz minus omega t in the y direction.

OK, look at what we have learned from here. What we have learned from here is that-- I got started with a plane wave solution of the electric field. And I can show that only when omega over k is equal to the speed of light this is actually a solution to my wave equation.

And also because the electric field and the magnetic field have to satisfy the Maxwell's equation all the time-- because that's the fundamental law-- therefore, I can use those equations to evaluate and to find what is actually the corresponding magnetic field.

And using Faraday's law and plugging in and the solving the question, I will be able to figure out that B is also what kind of wave? B is also what kind of wave I was talking about-- also?

AUDIENCE: Plane wave.

YEN-JIE LEE: Plane wave, right? It's also plane wave. You see? So if I got started with a plane wave in the electric field side, and I also get the plane wave in the magnetic field side. They are proportional to each other.

Originally, the magnitude of the electric field is E0. The corresponding magnetic field-- the magnitude is proportional to E0. But there's a factor of 1/c difference between the magnetic field amplitude and the electric field amplitude.

The third thing which we learned from here is that electric field is actually in the x direction. B field is actually not in the x direction, it's in the y direction.

What we learn from here is that the direction of the B field can be determined by a simple calculation. So basically, the B is proportional-- the magnitude of B is proportional to the electric field. But you have to multiply the magnitude by 1/c.

And also this is actually not correct because the B is actually in the y direction. So the original direction of the electric field is in the x direction.

Also we know the direction of a propagation is in the z direction. Therefore, if I take unit vector K-- K is actually the wave number, but now I make it a vector and I take the unit vector is equal to z-- so direction of propagation.

If I make this definition then I can now rewrite this relation. Basically, I can express the magnitude of B by K hat, which is the direction of propagation cross the E field.

And we can check this. And then basically what you are going to get is z cross E-- then actually really z cross x, you are going to get y direction.

And that is actually telling you that B and the E have a rather simple relation. And also you don't really need to go through all those calculation again because now you can see that if you know the direction of propagation and you know the direction of the electric field, then you can already evaluate what will be in the B field.

So we will take a five minute break. We'll come back in 29, and we will continue the discussion of this solution. Let me know if you have any questions about the content we discussed.

Welcome back, everybody. So we will continue the discussion of what we have learned from the wave equation. So basically we start with plane wave in the electric field. And this electric field is in the x direction. And we evaluated the corresponding B field which is in the y direction.

And what we found is that actually we can find a pretty simple relation between electric field and the magnetic field, which is actually magnetic field vector is equal to 1/c, K hat cross E. And the K hat now which is-- you find here-- is actually the direction of propagation.

So basically, in this case in the discussion we had before, the direction of propagation is in the positive z direction. So if I go ahead and visualize the whole-- solution-- plot the magnetic field and the electric field is a function of z, x and the y-- it's a function of z actually here. And I only evaluate the value at x equal to 0, and the y equal to 0.

And basically, this is actually what you have. So basically, you have two sine wave. One is actually pointing to the x direction. And the other one is actually pointing to the y direction, which is the B field. And those lines doesn't mean a lot because those lines are just connecting the end point of all those vectors.

So you can see that they are cosine wave structure when you connect all those vectors. And keep in mind that those are evaluated at x and y equal to 0. Therefore, what we actually get is actually a lot of vectors.

So those individual arrows are vectors. And this whole thing-- this whole electromagnetic wave is propagating to the positive z direction. And those electric field and the magnetic field are propagating at the speed of light, which you see.

And also you can see that the magnitude-- also I plotted here-- the magnitude, there's no phase difference between electric field and the magnetic field.

This is actually not always the case. In which we will show a example probably later in the lecture. So in general, what we can actually do is to write down a general expression for the plane wave.

So for example, I can have a plane wave, which is actually propagating in some direction. Which is actually given by this K vector. K vector is actually giving you information about the wave number. And also the direction of propagation.

And in this case, what I am trying to construct is a solution, which is actually propagating along in the direction of the K vector. And the electric field is actually going to be pointing to a direction perpendicular to the direction of the K vector.

So basically, what I can do is I can write this plane wave in this functional form. E0 is actually a vector, which is actually telling you the direction of the electric field-- E0 vector-- is actually have this function of form.

And the K vector is actually placed in the exponential function-- inside the exponential function. Exponential i, k dot r minus omega t.

And what is actually r? r is actually x x hat, plus y y hat, plus z z hat. And omega is actually the angular frequency which we are familiar with and that's actually equal to c times the magnitude of K, which is actually the wave number.

And you can actually show that-- OK, indeed this expression can satisfy the wave equation, which we did right for the electric field. And of course there are some requirements, which is actually that the direction of the electric field have to be perpendicular to the direction of propagation. Which you can actually derive that.

And finally, this expression B field equal to 1/c. K, which is the direction of propagation cross E field is still valid because basically we have shown that it works for the plane wave pointing to the x direction propagating to the z direction.

We can always redefine the coordinate system because we can actually rotate this coordinate system and the physics should not change. Therefore, you must see that this expression must be still valid. And also that the direction of the electric field, which is actually proportional to E0, must be perpendicular to the direction of propagation.

So that is actually what we can actually learn a general description of electric field pointing to some random direction.

So we have talked about the progressing wave solution and also the plane wave and also the corresponding magnetic field. I hope that you can actually apply this-- the technique which we learned here-- if you are given a magnetic field, you must know that there must be a corresponding electric field because they cannot be separated from each other.

And you can actually obtain the corresponding electric field if you are given magnetic field by using Maxwell's equations.

So what is going to happen is that now if I emit this photon-- or say this electromagnetic wave from the light source, for example, that one-- the one of which is pointing at my face.

Basically, my face is going to bounce some of the electromagnetic field around. And some that actually go out of the window. And then when they go out the window, maybe they are lucky they are not hitting any building in the MIT. Then what is going to happen is that they're going to propagate forever toward the end of the universe.

Really, they are going straight forever as you can see from this solution. It's like some kind of wave propagating forever at the speed of light. If they don't encounter anything before the end of life of the electromagnetic wave, it's going to be propagating forever toward that direction-- escaping from that window.

So that is actually fascinating and-- but we would like to introduce some more excitement to see what is going to happen. So what I'm going to do is now instead of only discussing about the plane wave-- what I'm going to do is that I would like to add a perfect conductor into the game and see what is going to happen.

So what do I mean by a perfect conductor? A perfect conductor can be seen in a musical, like in a concert.

[LAUGHTER]

But the one which I am talking about is not that one, which is also fascinating, but this is a different system. The interesting thing is that both the conductors in the concert and this one is very busy. It's a very busy system.

What do I mean by perfect conductor? That means all the little charges inside the conductor can move freely. So if they move they don't actually cause any energy. They can move around-- all the electrons inside the conductor can be moved freely without costing anything, without any of this energy dissipation.

So that's actually what I mean by perfect conductor. What do I mean by a very busy system? That means whenever there are any distortion on the electric field-- any electric field approaching to this conductor-- what is going to happen is that this conductor will, oh, this is electric field, so I have to move from some of my electrons. Then it's going to cancel all the electric field inside the conductor because it cost nothing.

So you have fast-- really fast the react to this change in the electric field and they really carefully arrange all the electrons. And so that the electric field is canceled. Otherwise, all those electrons will continue to move around until this happens-- this cancellation happens. So that's actually what I mean by a busy world and what I mean by a perfect conductor.

If I put this conductor into game, what is going to happen? What is going to happen is that if I consider a situation-- if I have my x's defined here pointing up to be the x-axis, pointing to the right to the z-axis, pointing to the-- pointing toward you is actually the y-axis.

So I can now again take the plane wave which I started with. There will be a plane wave like this. And it's going toward a piece of perfect conductor. What is going to happen is that as I actually mentioned before there are many charges all over the place. They are going to quickly rearrange-- all those charges to cancel the electric field.

So if you have a plane wave going toward the perfect conductor at the surface of the perfect conductor-- the electric field will become 0.

But if you have only one plane wave it cannot be-- the magnitude cannot be equal to 0 because I know the functional form. I know that the functional form of that electric field is E0 cosine kz minus omega t. If I place this perfect conductor at Z equal to 0, then I can evaluate the electric field is not equal to 0 because it is actually equal to E0 cosine minus omega t.

So what can I do to cancel the electric field? This is actually very similar to the situation when you have a progressing wave on this string hitting a wall. Because the magnitude of the string which is fed to the equilibrium position is actually equal to 0.

That's actually what we have learned in the last few lectures. And this is actually exactly the same situation, right? You have a progressing wave. And there is some kind of boundary, which is actually when this progressing plane wave encounter this perfect conductor.

There-- the electric field-- the boundary condition-- has to be E is actually-- E, x, y, 0, which is actually the position of the z of the perfect conductor. As a function of time will be equal to 0.

The whole plane will have 0 electric field.

So that means there must be what kind of wave? There must be a reflective wave because of the presence of the perfect conductor. It's actually similar to the situation which we discussed there's a progressing wave hitting the wall. And this string wall system-- there will be a reflecting wave coming out of it.

So therefore, what we are expecting is some kind of-- refracting wave which actually cancel the magnitude of the electric field at Z equal to 0. And then this progressing wave is going to the left-hand side direction.

So now, I can actually write down the incident wave-- expression. The incident wave-- I call it Ei, this is Ei-- is expressed as E0 over 2, cosine kz minus omega t. This is actually what I putting to the system. The magnitude is E0 over 2, and it's actually propagating toward the z direction, as you can see from here.

And that the direction of the electric field is in the x direction. And of course the E field will have a corresponding magnetic field, which is actually-- you can actually write it down directly using this formula-- B equal to 1 over c, K cross E. K here is z, therefore you can quickly evaluate and then conclude that the magnetic field must be in the y direction.

And that the magnitude of the magnetic field would be E0 divided by 2 c. Cosine Kz and this omega t. So that is actually the incident wave. And of course I also need, as I discussed, there must be a reflective wave, Er, which you actually cancel the electric field at z equal to 0. If that cancels the incident wave, that means the magnitude must be in the opposite direction of the incident wave.

Therefore, I can quickly write down what would be the resulting reflective wave that would be equal to minus E0 over 2, cosine minus Kz, minus omega t in the x direction. And then the corresponding B field, I can also write it down using exactly the same formula.

And basically what I conclude is that this will be equal to E0 over 2 c, cosine minus kz, minus omega t in the y direction. So you can actually check this expression after the direction.

So now, I would like to check what would be the magnitude of the electric field at z equal to 0. So basically, at z equal to 0, you have something which is proportional to cosine minus omega t for the incident wave. And then the magnitude is E0 over 2.

And if you evaluate z equal to 0, basically you get minus E0 over 2 cosine minus omega t. Therefore, they really cancel and give you the desired boundary condition, which is actually E equal to 0 and the surface of the perfect conductor.

So that's very nice. And this is actually the physics of which we already learned from this string wall system.

So what I can do now is to calculate the total electric field if I add them together. Basically, I would get E-- total electric field, which is actually overlapping the incident and the reflective wave. What I am going to get is Ei plus Er.

And basically, what I get is E0 over 2 because the incident wave and the reflective wave of the electric field is always in the x direction. Therefore, I only need to take care of the x direction. So basically, I have cosine kz minus omega t, minus-- right, because there's a minus sign here-- minus cosine minus kz, minus omega t in the x direction. And there should be--

And of course this is a cosine minus cosine. So we have all the formulas-- one from, for example, Wikipedia, or from your textbook. So you can actually calculate this-- rewrite this expression to be E0 sine omega t, sine kz. And then this is actually in the x direction.

Everybody's following? I hope it's not too fast. All right, and of course, I can also calculate the corresponding B field. So it's actually again, exactly the same thing-- Bi plus Br. And basically, I will skip the step. Basically, you can add this term and that term. And you will be able to conclude that the B field will be equal to E0 over c cosine omega t, cosine kz in the y direction.

This is actually pretty interesting. If you look at this result, I have a electric field, which is proportional to E0, the magnitude, sine omega t, and sine kz. What does that I mean? This is a special kind of wave which we learned before. What kind of wave is this?

AUDIENCE: Standing.

YEN-JIE LEE: It's a standing wave because the shape is actually fixed, the sine kz. And the magnitude is actually changing up and down at the angle frequency omega t. It's a standing wave.

Another thing which is really interesting is that if we look at the expression of a electric field and the magnetic field-- if we compare that-- one is actually sine, sine. The other one is cosine, cosine.

That's kind of interesting because this is actually different from what we actually usually learn from the progressing wave solution, or traveling wave solution.

Where the electric field and the magnetic field are in phase. There's no phase difference. In the case of the superposition of the incident and the reflective wave-- the solution of a standing wave-- actually you can see that the phase of the B field and the E field are different.

Finally, very important-- you will see that-- look at this expression-- B equal to 1/c, K cross E-- that means this only work for traveling wave.

Clearly, this doesn't work for standing waves. So very important. So don't blindly apply this expression. This is only useful for the traveling wave solution. And you can see a very concrete example here. This doesn't work for standing waves.

That's kind of interesting. And if you look at this result, you will see that if I don't have magnetic field-- if I only have the electric field-- there will be a instant of time, for example, t equal to 0. When t is equal to 0, sine is equal to 0.

What is going to happen? You will have no electric field. That means electric field completely disappear because we are operating this system in vacuum. There's nowhere to hide. Where is the energy? The energy, fortunately-- electric field have a very good partner, which is actually B field.

All the energy's actually stored in the form of magnetic field. You can see that now magnetic field is reaching the maximum.

So of course I can now calculate the Poynting vector. Poynting vector is E cross B divided by mu zero. And these will be equal to one over Mu 0, Ex, By, and the z direction. There's only one term which survive.

So Poynting vector is not pointing vector. It's not pointing around. There's a gentleman who is called Poynting and he has a vector. And this vector is a directional energy flux. It's a directional energy flux, or the rate of energy transfer per unit area. So that is actually the meaning of Poynting vector.

And then each magnitude is proportional to E cross B divided by Mu 0. So I can calculate that. Basically, I have the E and the B-- Ex and By, then I can calculate. That would be equal to E0 square, over Mu 0, sine omega t, cosine omega t, cosine Kz, sine Kz in the z direction because I have x cross y. And I'm going to get the z direction.

And I can simplify this. I have the sine, cosine. And also all have cosine, and sine. Basically, you can simplify this expression and get E0 squared divided by 4, Mu 0c sine 2 omega t, sine 2kz in the z direction.

So you can see that the directional energy flux is in the z direction. It has a vector-- it has a wave number 2 times of the original wave number. And it's actually going up and down 2 times of the speed of the oscillation of the original electromagnetic wave.

And this energy is actually vibrating up and down. And the shape of this energy transfer Poynting vector is actually a sine wave. So that this is actually how the microwave actually works.

So basically, what we are doing is to have generate microwave inside your device. And in the oven this microwave is actually bouncing back and forth because you have metal walls, which actually bounce the electromagnetic field back and forth. And it really can cook the food by vibrating the molecules inside the food back and forth.

So as you can see the magnitude of the Poynting vector is actually isolating up and down. That actually cause additional vibration and that heat up the food.

So after this lecture, you will be able to say proudly that you understand the physics of microwave oven.

[LAUGHTER]

Thank you very much. I hope you enjoyed the lecture today. And you have any questions, I will be here.