Massachusetts Institute of Technology
Physics 8.03
Practice Final Exam 3

Instructions

Please write your solutions in the white booklets. We will not grade anything written on the exam copy. This exam is closed book. No electronic equipment is allowed. All phones, tablets, computers etc. must be switched off.

Formula Sheet Exam 3

Maxwell Equations in vacuum

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\partial B_z}{\partial t} \; ; \; \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t} \; ; \; \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t}$$

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = \mu_0\epsilon_0\frac{\partial E_z}{\partial t} \; ; \; \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} = \mu_0\epsilon_0\frac{\partial E_x}{\partial t} \; ; \; \frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} = \mu_0\epsilon_0\frac{\partial E_y}{\partial t}$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \; ; \; \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

Wave equation for EM fields in vacuum

$$\frac{\partial^2 E_i}{\partial x^2} + \frac{\partial^2 E_i}{\partial y^2} + \frac{\partial^2 E_i}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 E_i}{\partial t^2} \text{ where } \; i = x, y, z$$

$$\frac{\partial^2 B_i}{\partial x^2} + \frac{\partial^2 B_i}{\partial y^2} + \frac{\partial^2 B_i}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 B_i}{\partial t^2} \text{ where } \; i = x, y, z$$

EM energy per unit volume and Poynting vector:

$$U_E = \frac{1}{2}\epsilon_0\vec{E}^2 \quad U_B = \frac{1}{2\mu_0}\vec{B}^2 \quad \vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

Transmission and reflection

$$R = \frac{z_1 - z_2}{z_1 + z_2} \quad T = \frac{2z_1}{z_1 + z_2}$$

Phase velocity and impedance:

$$v = \sqrt{\frac{T}{\mu}}, \quad Z = \sqrt{T\mu} \;\; \text{(string)}$$

$$v = \sqrt{\frac{1}{LC}}, \quad Z = \sqrt{\frac{L}{C}} \;\; \text{(transmission line)}$$

Electric field from an accelerated charge:

$$\vec{E}(\vec{r}, t) = -\frac{q\vec{a_\perp}(t - r/c)}{4\pi\epsilon_0 r c^2}$$

Total power emitted by the accelerated charge:

$$P(t) = \frac{q^2 a^2(t - r/c)}{6\pi\epsilon_0 c^3}$$

Fourier Transform

$$f(t) = \int_{-\infty}^{\infty} d\omega \; C(\omega) e^{-i\omega t} \qquad C(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \; f(t) e^{i\omega t}$$

Interference of two sources with amplitudes A_1 and A_2 with a relative phase difference δ:

$$< I > \propto (A_1^2 + A_2^2 + 2A_1 A_2 \cos\delta)$$

Interference of N fields of equal amplitude with phases $\delta_{m+1} - \delta_m = \delta$:

$$< I > = < I_0 > \left[\frac{\sin(N\delta/2)}{\sin(\delta/2)} \right]^2$$

Single slit diffraction where β is the phase difference between rays coming from edges and the center of the slit:

$$< I > = < I_0 > \left[\frac{\sin(\beta)}{\beta} \right]^2$$

Electric field transmission and reflection ratios, magnitude and sign, for radiation incident normally on an interface between lossless dielectrics with indices of refraction n_1 and n_2.

$$\frac{E_t}{E_i} = \frac{2n_1}{n_1 + n_2} \quad \frac{E_r}{E_i} = \frac{n_1 - n_2}{n_1 + n_2}$$

Optical path length between A and B:

$$\int_A^B n(x) dx$$

Problem 1 (30 pts)

Answer each conceptual questions separately. Each is worth 5 pts.

a. Circularly polarized light of intensity I_0 is incident on a filter that transmits one linear polarization. What is the transmitted intensity?

b. Electronics used at the LHC accelerator uses 1ns square pulses. What is the approximate range of frequencies (bandwidth) required to send such pulses?

c. Deep water waves dispersion relation is $\omega = \sqrt{gk}$. What are the phase and group velocities and what is their relative magnitude at a given k?

d. A green laser ($\lambda = 500$ nm) is set up on the Moon, sending a light beam towards the Earth. What is the approximate size of the light spot on the Earth's surface if the width of the laser beam on the Moon is 1 meter and the distance between the Moon and the Earth is 380,000 km? The beam is diffraction limited.

e. You are sitting upright on the beach near a lake on a sunny day wearing your Polaroid sunglasses. When you lie down on your side, facing the lake, the sunglasses don't work as well as they did while you were sitting upright. Why not?

f. Suppose monochromatic light falls on a diffraction grating. What happens to the pattern of principle maxima if the same light falls on a grating that has more lines per centimeter? What happens to the pattern of principle maxima if a longer-wavelength light falls on the same grating?

Problem 2 (30 pts)

A particle of mass m and charge q is constrained to move without friction along the $\hat{x}$ direction. The particle is illuminated by a traveling electromagnetic plane wave with the associated B-field:

$$\vec{B}_{wave}(\vec{r}, t) = B_0(\hat{y} + u\hat{z})\cos(kz - \omega t)$$

where B_0 is constant and u is an unknown (constant) parameter. The wave is traveling in open space.

a. Calculate the parameter u, justify the calculation.

b. Write down the electric field of the incoming wave $\vec{E}_{wave}(\vec{r}, t)$

c. Calculate the acceleration $\vec{a}(t)$ of the charge q. You may neglect any energy losses in calculating the motion of the charge.

d. Derive the electric field $\vec{E}_{rad}(\vec{r}, t)$ radiated by the charge toward the following three positions. What is the polarization in these three locations?

   (1) $\vec{r} = r\hat{x}$

   (2) $\vec{r} = r\hat{y}$

   (3) $\vec{r} = r\hat{z}$

Problem 3 (40 pts)

A simple grating that has 5 long, narrow slits is glued on top of a block of glass with index of refraction n. A monochromatic source of plane waves illuminates the grating from inside the glass. The wavelength of the light in vacuum is λ. Figure 1 shows a cross section of the grating; the length of the slits is perpendicular to the page. The source is off-axis at an angle θ from the normal to the grating as shown (the sign of θ is important!). The slits are very narrow compared to their separation d and the wavelength of the illuminating light, and the screen is very distant compared to d.

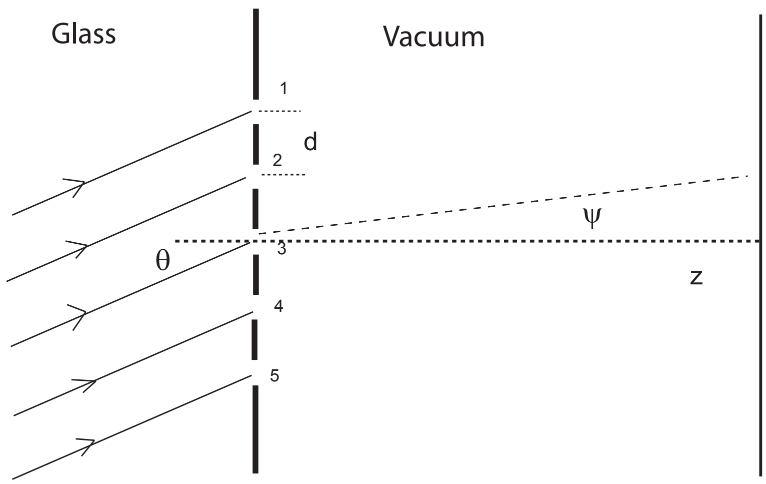

Figure 1: Grating

a. First consider the case where $\theta = 0$. Show, using a simple diagram, that the phase difference δ between rays from adjacent slits which are viewed far away at an angle ψ with respect to the z axis is $\delta = \frac{2\pi d}{\lambda} \sin\psi$.

b. Now consider an arbitrary θ. What is δ?

c. Write an expression in terms of d, θ, λ and ψ for the intensity I that will be viewed on the screen.

d. Sketch the intensity I as a function of $\sin\psi$. Be sure to specify the location of the first interference maxima and minima.

e. Now consider the same grating with the two outside slits blocked (slits 1 and 5). Write an expression for the intensity observed on the screen.

f. Make the sketch of the new intensity versus $\sin\psi$ and compare it to the sketch obtained for the 5 slits. What are the new locations of the first maxima and minima? How has the magnitude of the maxima changed?