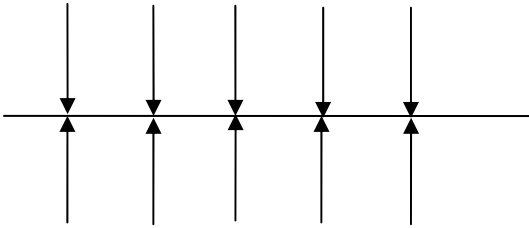


Solution to Problem Set #2

Problem 1 (5 Points) Consider an infinite plane (non-conducting) with a uniform negative surface charge density σ

(a) Draw a sketch of the electric field (using field lines) close to the plane.

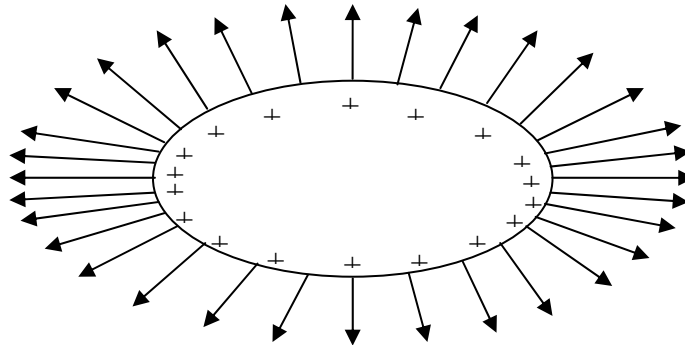


(b) Explain in words why the field has to look the way you have drawn (using symmetry arguments). (Keep your explanation to 2-3 sentences)

Field can only have component perpendicular to the plane: At each point, there is an equal amount of charge to the right and to the left. The horizontal components always cancel.

Problem 2 (5 Points) Consider an ellipsoidal conducting object carrying a positive charge Q (shown below).

(a) Draw a sketch of the electric field (using field lines) very close to the surface of the object. Your sketch should show where the magnitude of the electric field will be biggest.



(b) Explain in words how the fact that we are looking at a conducting object determines the direction of the field relative to the surface at very small distances to the surface.

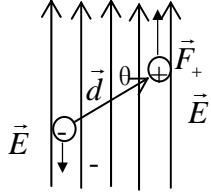
(Keep your explanation to 2-3 sentences)

In electrostatics, \mathbf{E} is perpendicular to the surface of the conductor. If not then, there would be a tangential component results in movement and re-distribution of charges.

Problem 3 (5 points):

Consider an electric dipole consisting of point charges $+q$ and $-q$, separated by a fixed distance \mathbf{d} .

(a) Determine the net force and net torque on the dipole in a uniform electric field \mathbf{E} as a function of the angle between the axis of the dipole and the direction of the field.



The force acting on $+q$ is $\vec{F}_+ = q\vec{E}$.

The force acting on $-q$ is $\vec{F}_- = -q\vec{E}$.

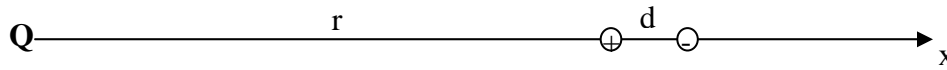
The net torque is $\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 = q(\vec{r}_1 - \vec{r}_2) \times \vec{E} = q\vec{d} \times \vec{E}$

Therefore the magnitude of the net torque is

$$\tau = qdE \sin \theta$$

and the direction of the net torque is perpendicular to this paper (outward), which is determined by the right hand rule of cross product.

(b) Now the dipole is brought into the field of a fixed point charge Q , which is at distance r from $+q$ and distance $(r+d)$ from $-q$. We observe that the dipole accelerates towards Q . Is Q positive or negative?



Since $+q$ is closer to Q , force acting on $+q$ is stronger than force acting on $-q$. This indicates that Q is negative.

(c) If the dipole has mass m , what will be the initial acceleration of the dipole in case (b)?

$$m\vec{a} = \vec{F}_+ + \vec{F}_- = -kq|Q|\left(\frac{1}{r^2} - \frac{1}{(r+d)^2}\right)\hat{x}$$

$$\text{Since } r \ll d, \text{ we have } \frac{1}{(r+d)^2} = \frac{1}{r^2}\left(1 + \frac{d}{r}\right)^{-2} \approx \frac{1}{r^2}\left(1 - \frac{2d}{r}\right) \Rightarrow$$

$$\vec{a} = -\frac{kq|Q|}{m}\left(\frac{1}{r^2} - \frac{1}{(r+d)^2}\right)\hat{x} = -\frac{kq|Q|}{mr^2}\left(1 - \left(1 - \frac{2d}{r}\right)\right)\hat{x} = -\frac{2kqd|Q|}{mr^3}\hat{x}$$

Problem 4 (5 Points): Young + Freedman, Challenge Problem 22.65

$$\text{a) } \int \rho dv = Q \Rightarrow \int_0^R 4\pi r^2 \rho(r) dr = Q$$

$$\Rightarrow \int_0^{R/2} 4\pi r^2 \alpha dr + \int_{R/2}^R 4\pi r^2 2\alpha(1-r/R) dr$$

$$= \frac{4}{3}\pi\alpha r^3 \Big|_0^{R/2} + \frac{4}{3}\pi 2\alpha r^3 \Big|_{R/2}^R - \frac{4\pi}{4R} 2\alpha r^4 \Big|_{R/2}^R = \frac{5}{8}\pi\alpha R^3 = Q \Rightarrow \alpha = \frac{8Q}{5\pi R^3}$$

b) Gauss's law: $\oint \mathbf{E} \cdot d\mathbf{s} = \int \rho dv / \epsilon_0$

$$\text{For } r \leq R/2: 4\pi r^2 E = \frac{4}{3} \pi r^3 \alpha / \epsilon_0 \Rightarrow E = \frac{\alpha r}{3\epsilon_0} = \frac{8Q}{15\pi R^3} \frac{r}{\epsilon_0}$$

$$\text{When } r = R/2, E = \frac{4Q}{15\pi\epsilon_0 R^2}$$

For $\frac{R}{2} \leq r \leq R$:

$$\begin{aligned} 4\pi r^2 E &= \int_0^{R/2} 4\pi r^2 \alpha dr + \int_{R/2}^r 4\pi r^2 2\alpha(1-r/R) dr \\ &= \frac{4}{3\epsilon_0} \pi \left(\frac{R}{2}\right)^3 \alpha + \frac{4\pi 2\alpha}{\epsilon_0} \left(\frac{1}{3} r^3 \Big|_{R/2}^r - \frac{1}{4R} r^4 \Big|_{R/2}^r \right) = \frac{4}{3\epsilon_0} \pi R^3 \alpha \left(2\left(\frac{r}{R}\right)^3 - \frac{3}{2}\left(\frac{r}{R}\right)^4 - \frac{1}{32} \right) \\ &= \frac{4}{3\epsilon_0} \pi R^3 \frac{8Q}{5\pi R^3} \left(2\left(\frac{r}{R}\right)^3 - \frac{3}{2}\left(\frac{r}{R}\right)^4 - \frac{1}{32} \right) = \frac{4}{3\epsilon_0} \frac{8Q}{5} \left(2\left(\frac{r}{R}\right)^3 - \frac{3}{2}\left(\frac{r}{R}\right)^4 - \frac{1}{32} \right) \Rightarrow \\ E &= \frac{Q}{60\pi\epsilon_0 r^2} \left(64\left(\frac{r}{R}\right)^3 - 48\left(\frac{r}{R}\right)^4 - 1 \right) \end{aligned}$$

$$\text{When } r = R/2, E = \frac{4Q}{15\pi\epsilon_0 R^2}$$

$$\text{When } r = R, E = \frac{Q}{4\pi\epsilon_0 R^2}$$

$$\text{For } r \geq R: 4\pi r^2 E = Q / \epsilon_0 \Rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$\text{When } r = R, E = \frac{Q}{4\pi\epsilon_0 R^2}.$$

$$\text{c) } \frac{\frac{4}{3} \pi \left(\frac{R}{2}\right)^3 \alpha}{Q} = \frac{\frac{4}{3} \pi \left(\frac{R}{2}\right)^3}{Q} \frac{8Q}{5\pi R^3} = \frac{4}{15}$$

$$\text{d) } E = \frac{\alpha r}{3\epsilon_0} = \frac{8Q}{15\pi R^3} \frac{r}{\epsilon_0}$$

$$m\ddot{r} + \frac{8Qe}{15\pi R^3} \frac{r}{\epsilon_0} = 0 \Rightarrow \omega = \sqrt{\frac{8Qe}{15\pi m\epsilon_0 R^3}} \text{ and } T = 2\pi \sqrt{\frac{15\pi m\epsilon_0 R^3}{8Qe}}$$

e) No, because the force acting on the electron is no longer proportional to the distance the charge deviates from its original point.

Problem 5 (5 Points): Young + Freedman, Exercise 23.39

$$\text{a) } E = \frac{q}{4\pi\epsilon_0 r^2} \text{ when } r \geq R \text{ for both spherical shell and solid conductor.}$$

$$\text{Then we have } U(R) = \int_R^\infty E(r) dr = \frac{q}{4\pi\epsilon_0 R}.$$

$$\text{b) } -1200 = k \frac{q}{15\text{cm}} \Rightarrow q = k^{-1} \times 1.2 \times 10^3 \times 0.15 = -20\text{nC}$$

c) No, because the amount of net charge on the balloon is very small.

Problem 6 (5 Points): Young + Freedman, Problem 23.61

a) Using the result from 23.57, we have

$$E(r) = \frac{V_{ab}}{\ln(b/a)} \frac{1}{r} = \frac{50 \times 10^3}{\ln\left(\frac{14 \times 10^{-2}}{90 \times 10^{-4}}\right)} \times \frac{1}{7 \times 10^2} = 9.71 \times 10^4 \text{ V/m}$$

$$\text{b) } 9.71 \times 10^4 \text{ V/m} \times q = 10 \times 30 \times 10^{-6} \times 9.8 \times 10^{-3} \Rightarrow q = 3 \times 10^{-11} \text{ C}$$