Class 07: Outline

Hour 1:

Conductors & Insulators

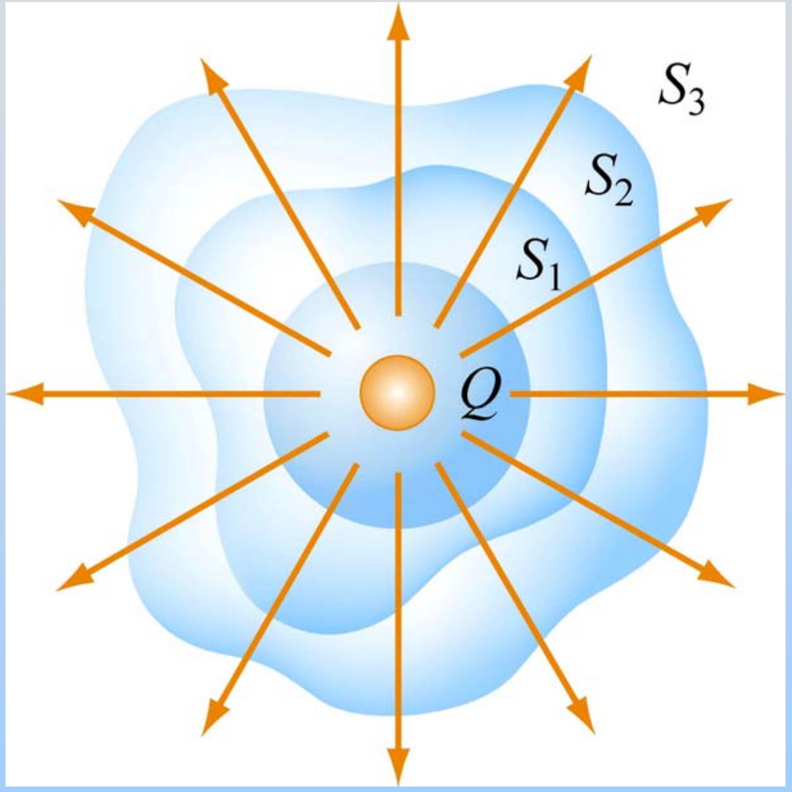
Expt. 2: Electrostatic Force

Hour 2:

Capacitors

Last Time:
Gauss's Law

Gauss's Law



$$\Phi_E = \underset{\substack{\text{closed} \\ \text{surface S}}}{\oint\!\!\!\oint} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{enc}}{\varepsilon_0}$$

In practice, use symmetry:

- Spherical (*r*)
- Cylindrical (*r*, ℓ)
- Planar (Pillbox, *A*)

Conductors

Conductors and Insulators

A conductor contains charges that are free to move (electrons are weakly bound to atoms)

Example: metals

An insulator contains charges that are NOT free to move (electrons are strongly bound to atoms)

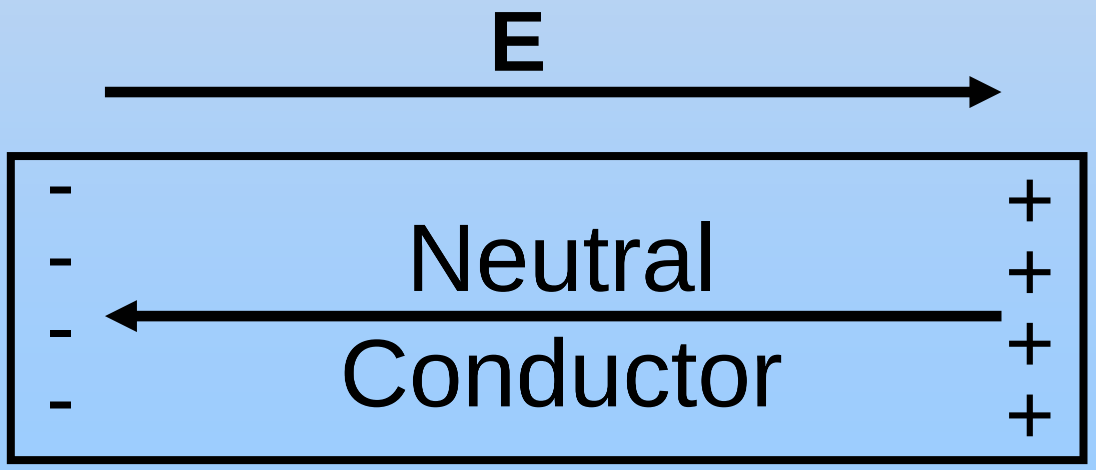
Examples: plastic, paper, wood

Conductors

Conductors have free charges

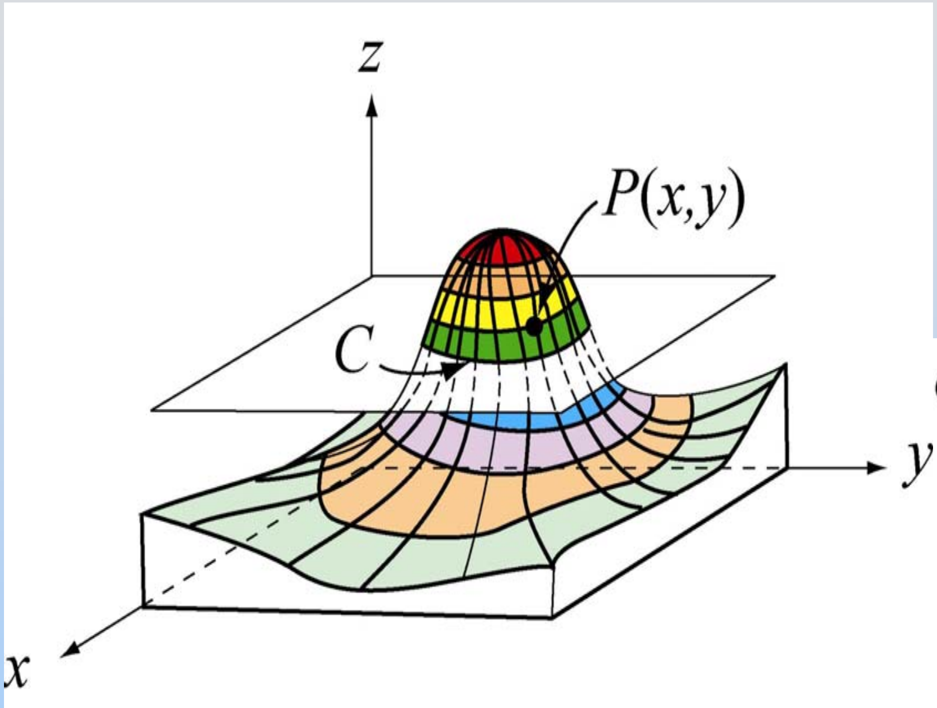
→ E must be zero inside the conductor

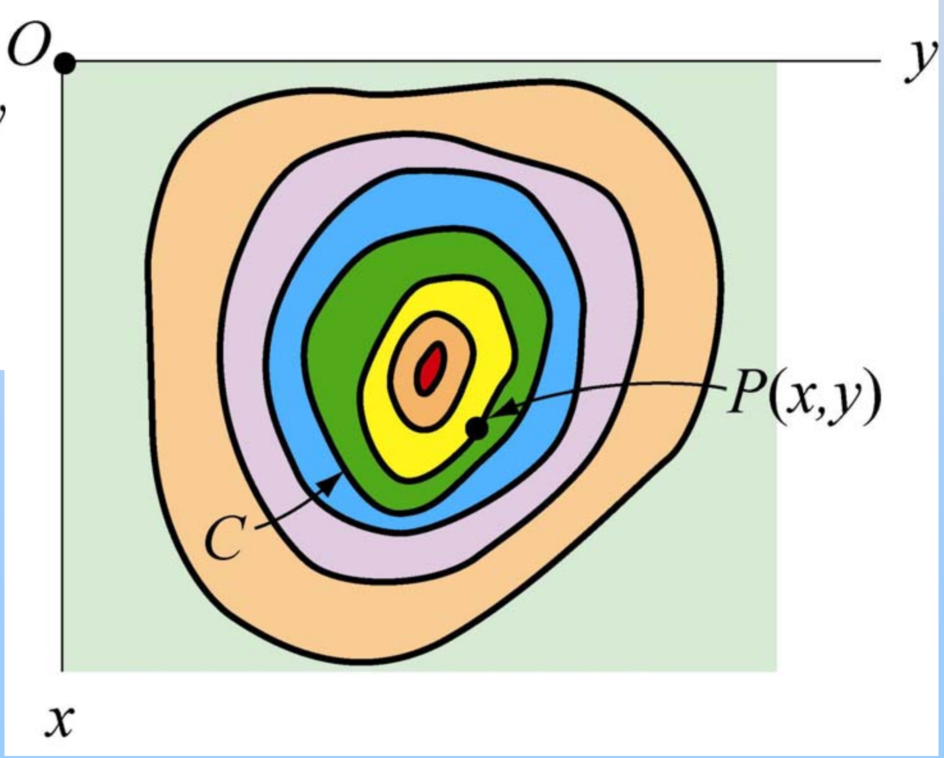
→ Conductors are equipotential objects



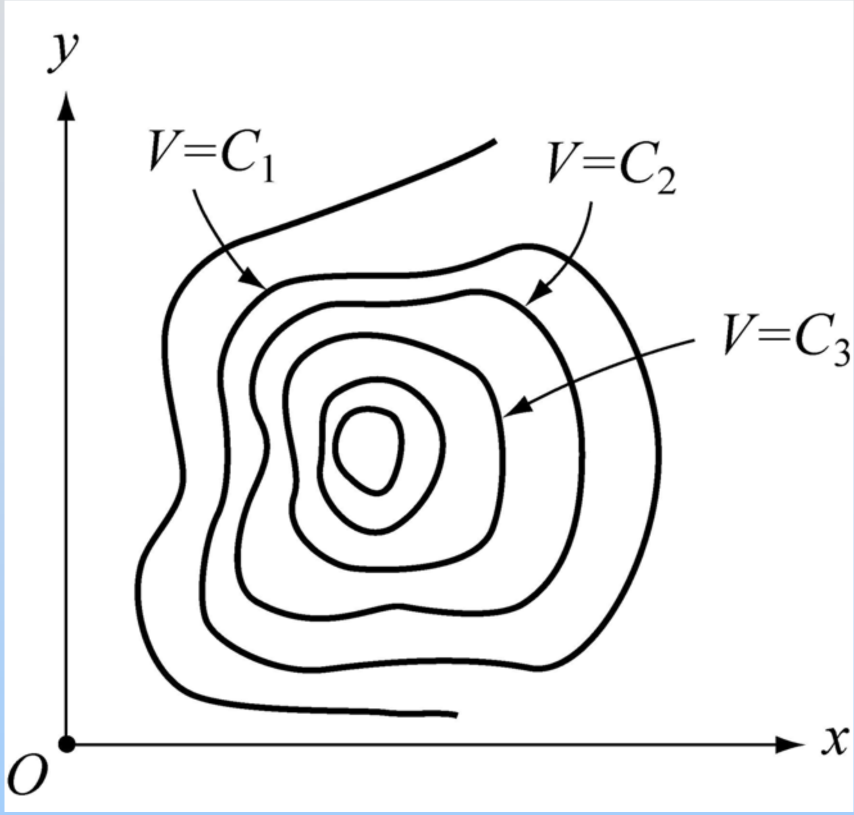
Equipotentials

Topographic Maps





Equipotential Curves

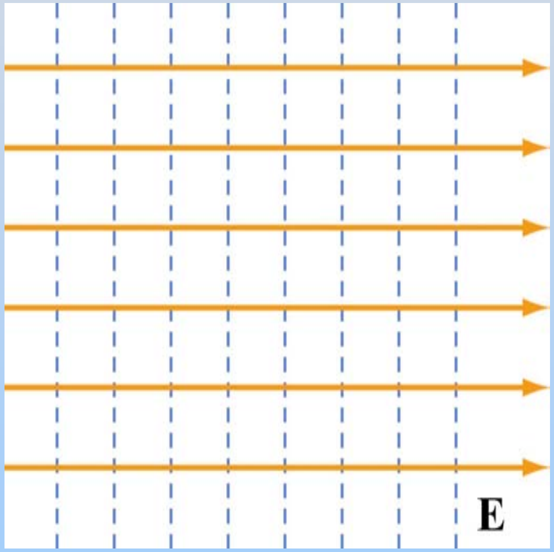


All points on equipotential curve are at same potential. Each curve represented by V(x,y) = constant

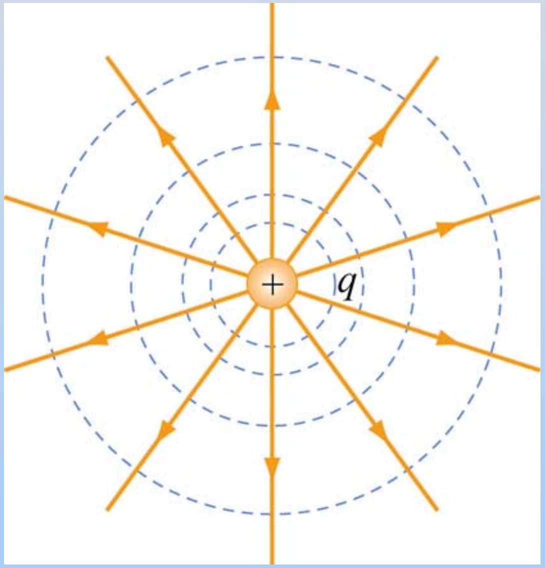
PRS Question:
Walking down a mountain

Direction of Electric Field E

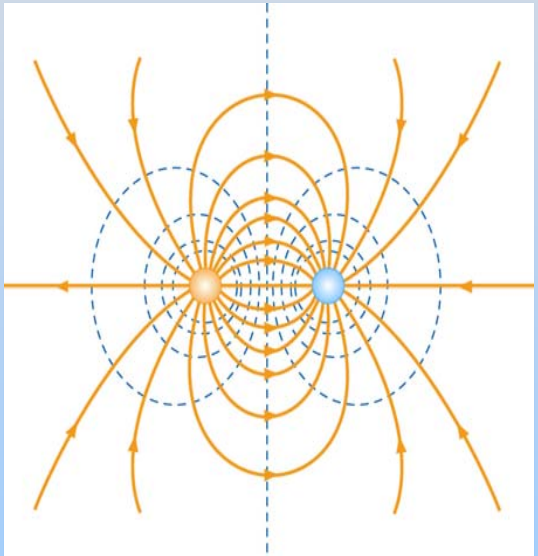
E is perpendicular to all equipotentials



Constant E field



Point Charge



Electric dipole

Properties of Equipotentials

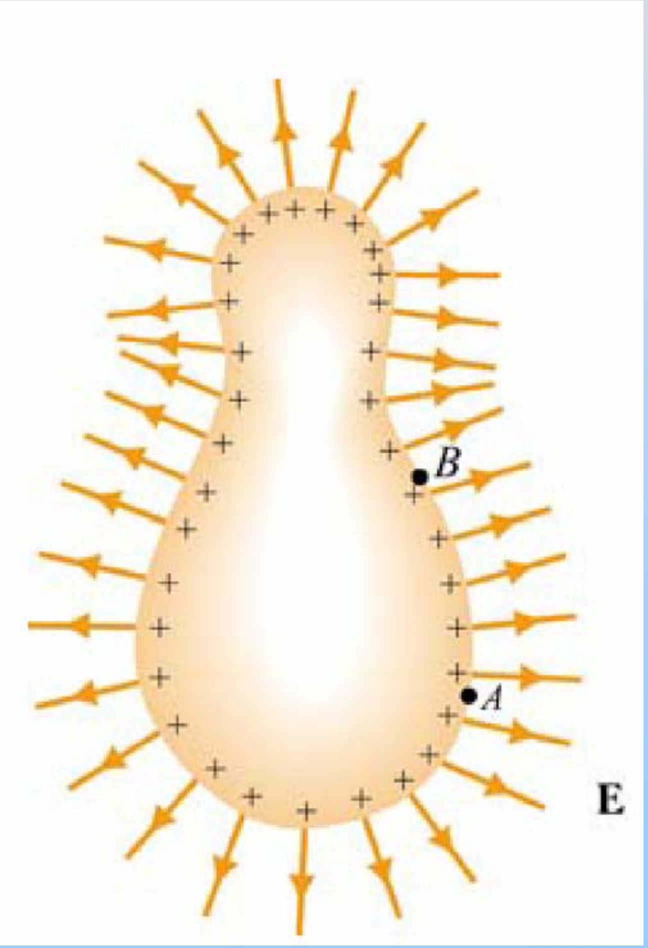
- E field lines point from high to low potential
- E field lines perpendicular to equipotentials
  - Have no component along equipotential
  - No work to move along equipotential

Conductors in Equilibrium

Conductors are equipotential objects:

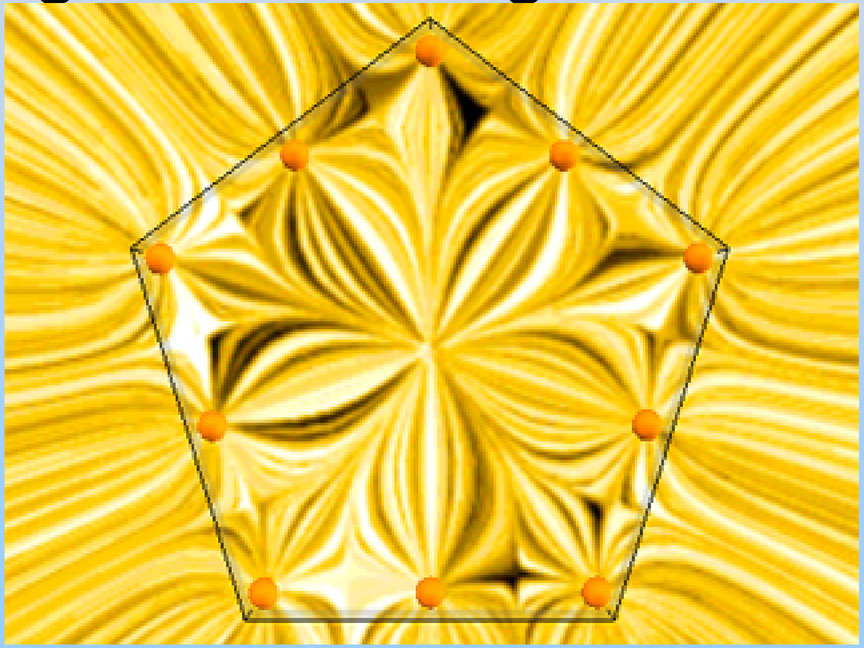
1) E = 0 inside
2) Net charge inside is 0
3) E perpendicular to surface
4) Excess charge on surface

$$E = \sigma / \varepsilon_0$$



Conductors in Equilibrium

Put a net positive charge anywhere inside a conductor, and it will move to the surface to get as far away as possible from the other charges of like sign.



http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/electrostatics/34-pentagon/34-pentagon320.html

Expt. 2: Electrostatic Force

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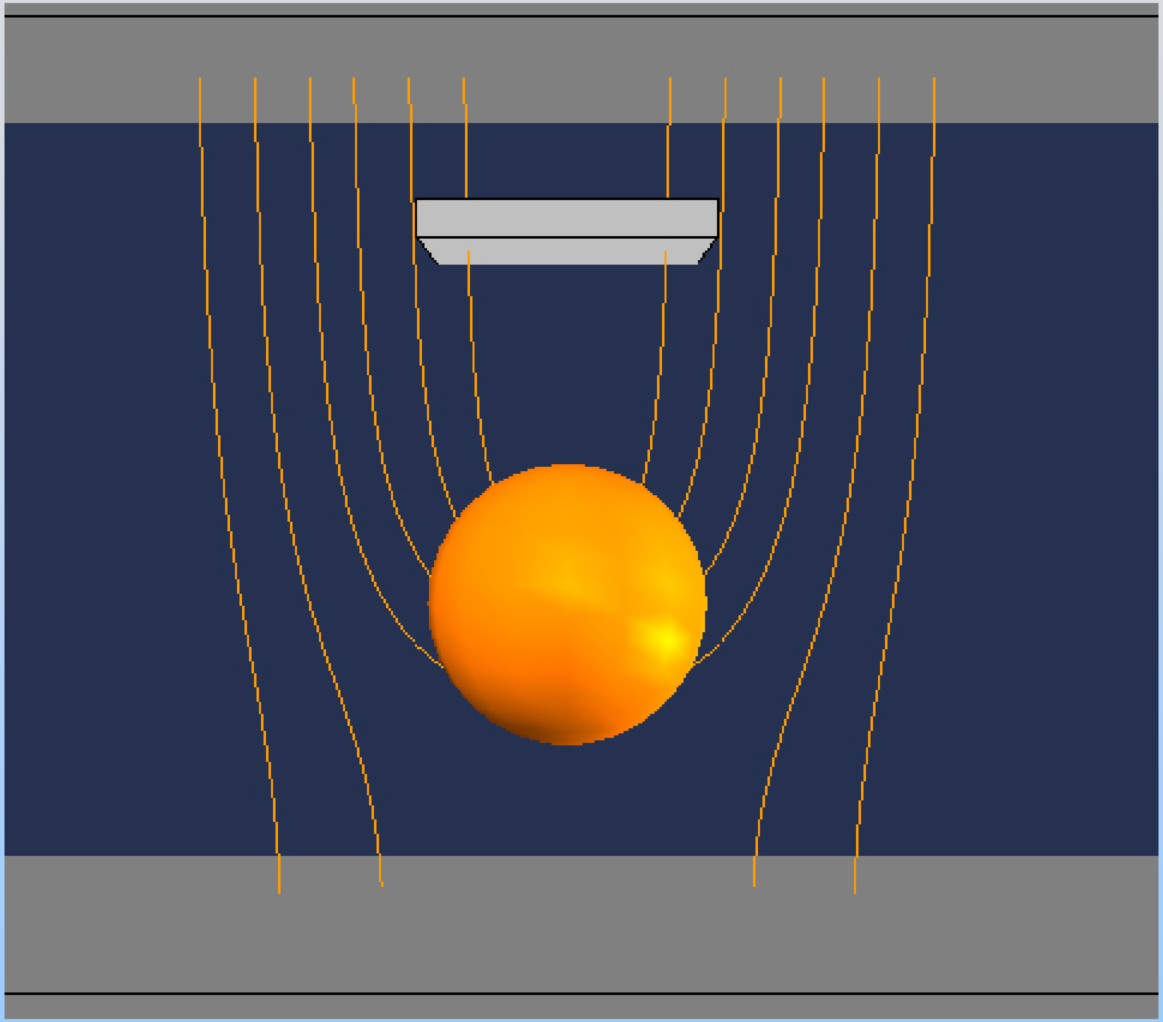

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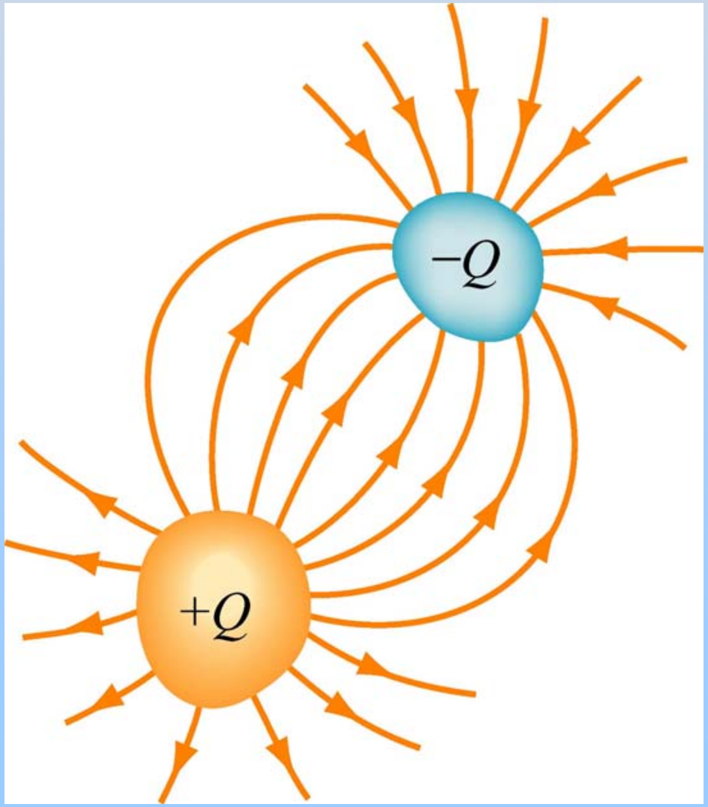
Experiment 2: Electrostatic Force

Capacitors and Capacitance

Capacitors: Store Electric Energy

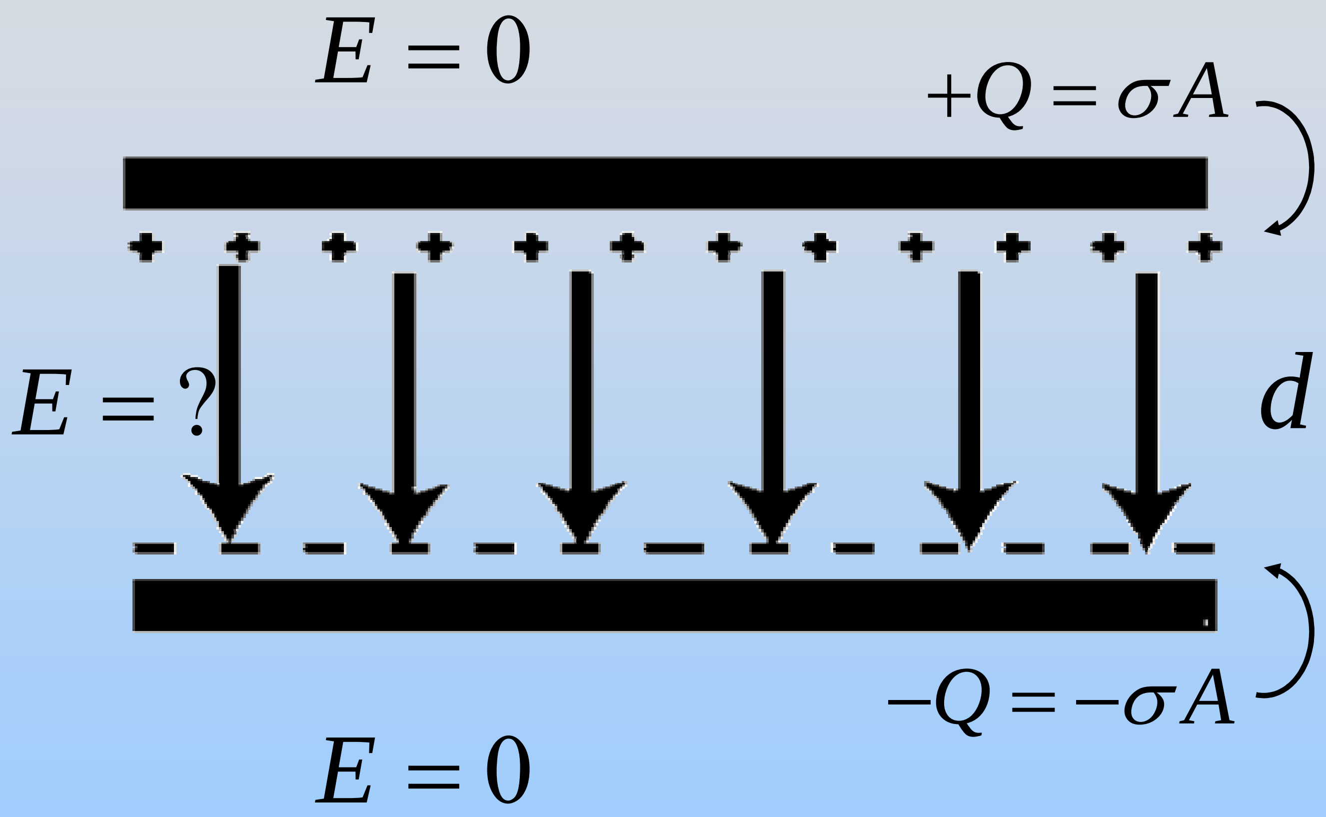
Capacitor: two isolated conductors with equal and opposite charges Q and potential difference ΔV between them.



$$C = \frac{Q}{|\Delta V|}$$

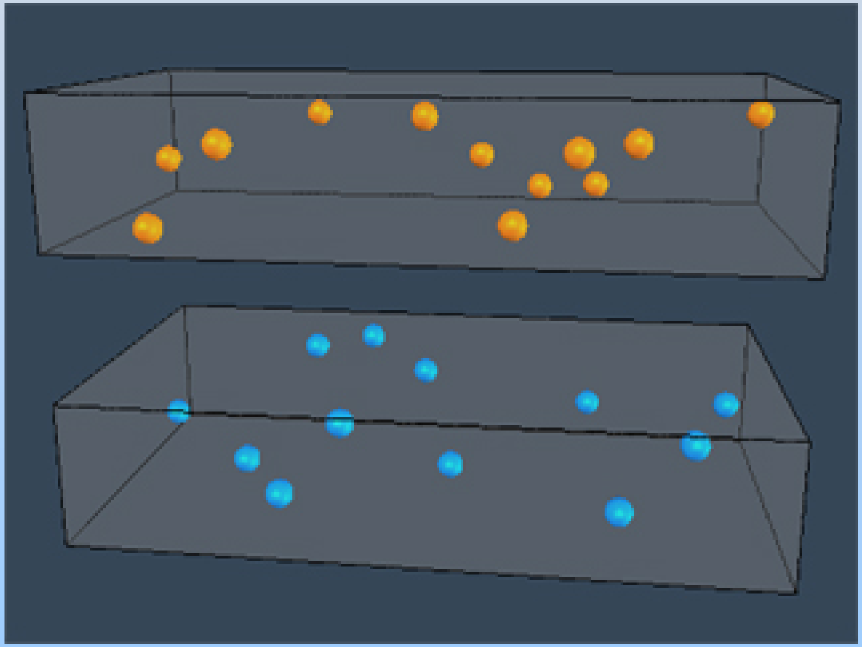
Units: Coulombs/Volt or Farads

Parallel Plate Capacitor



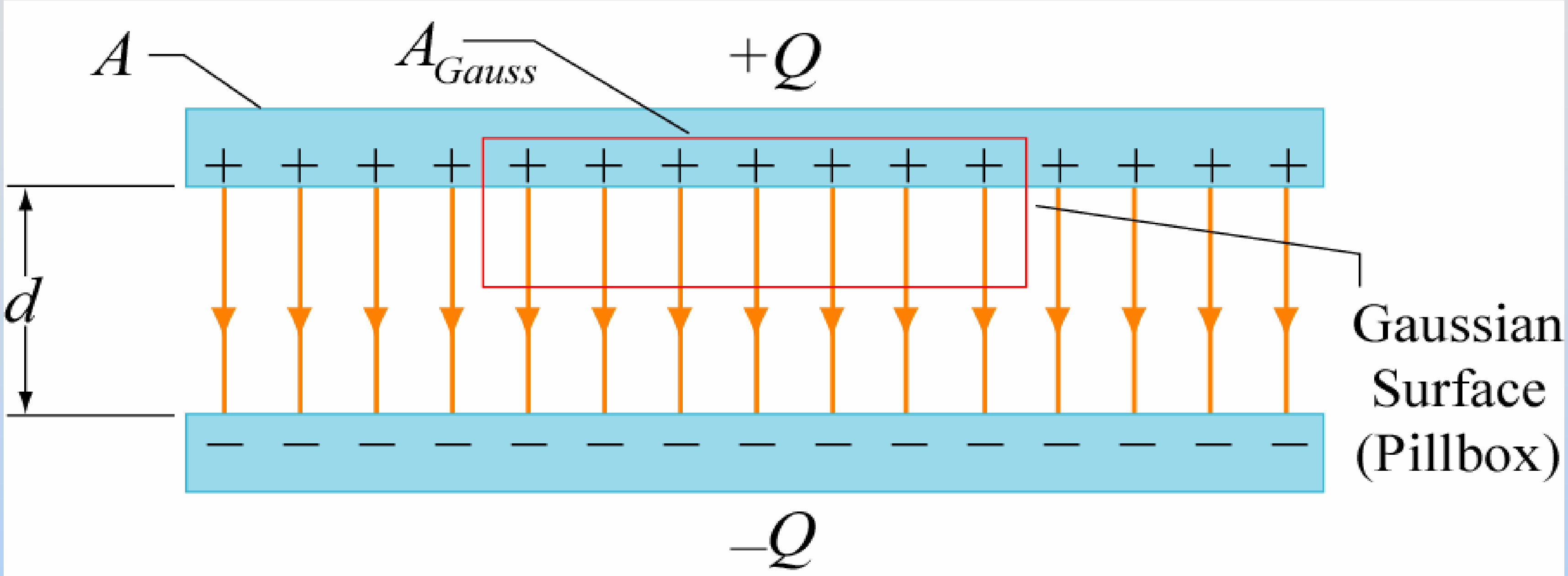
Parallel Plate Capacitor

When you put opposite charges on plates, charges move to the inner surfaces of the plates to get as close as possible to charges of the opposite sign



http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/electrostatics/35-capacitor/35-capacitor320.html

Calculating E (Gauss's Law)



$$\oint\!\!\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0} \qquad E\left(A_{Gauss}\right) = \frac{\sigma A_{Gauss}}{\varepsilon_0} \qquad \boxed{E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}}$$

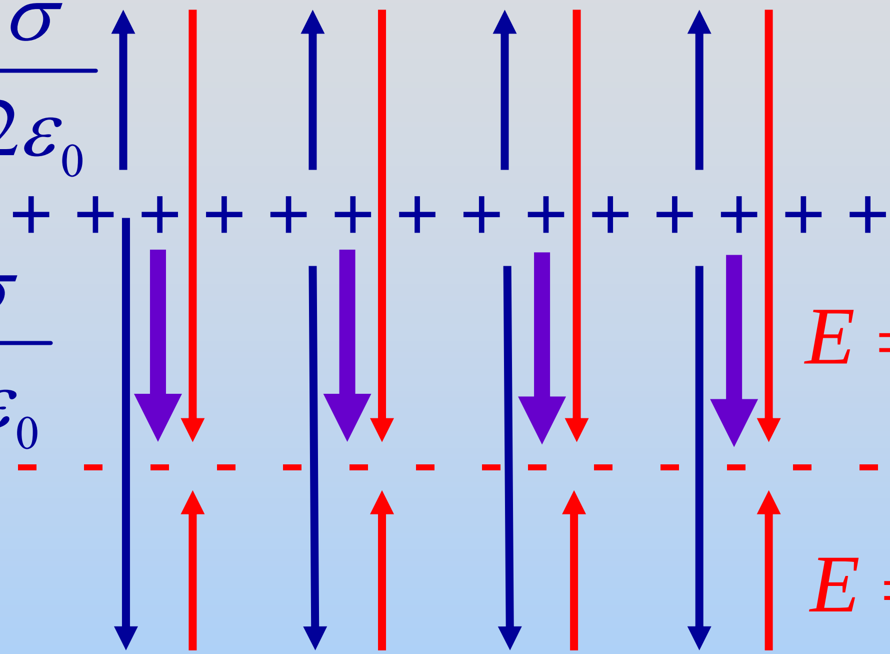
Note: We only "consider" a single sheet! Doesn't the other sheet matter?

Alternate Calculation Method

$$E = -\frac{\sigma}{2\varepsilon_0}$$

Top Sheet:

$$E = \frac{\sigma}{2\varepsilon_0}$$



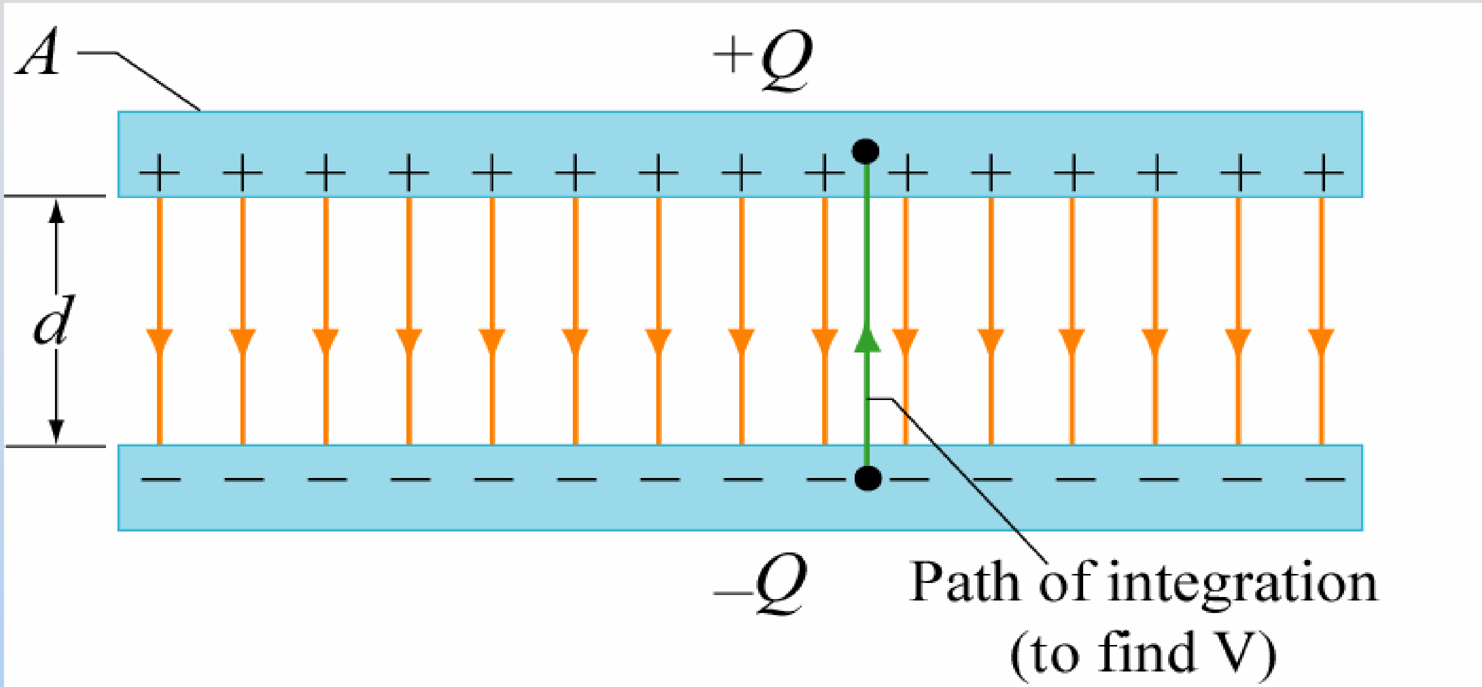
$$E = \frac{\sigma}{2\varepsilon_0}$$

Bottom Sheet:

$$E = -\frac{\sigma}{2\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

Parallel Plate Capacitor



$$\Delta V = -\int_{bottom}^{top} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = Ed = \frac{Q}{A\varepsilon_0} d$$

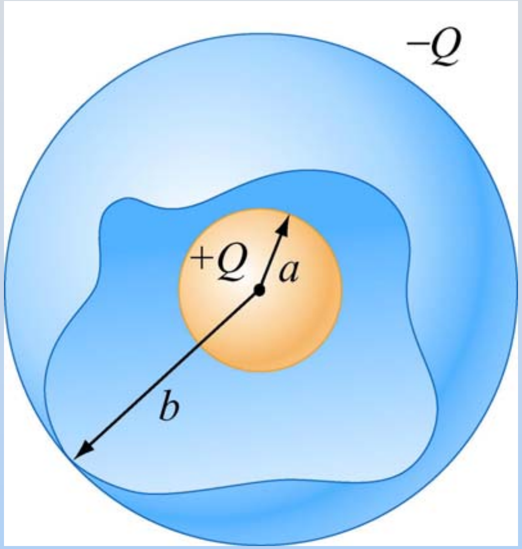
$$C = \frac{Q}{|\Delta V|} = \frac{\varepsilon_0 A}{d}$$

C depends only on geometric factors A and d

Demonstration:
Big Capacitor

Spherical Capacitor

Two concentric spherical shells of radii *a* and *b*



What is E?

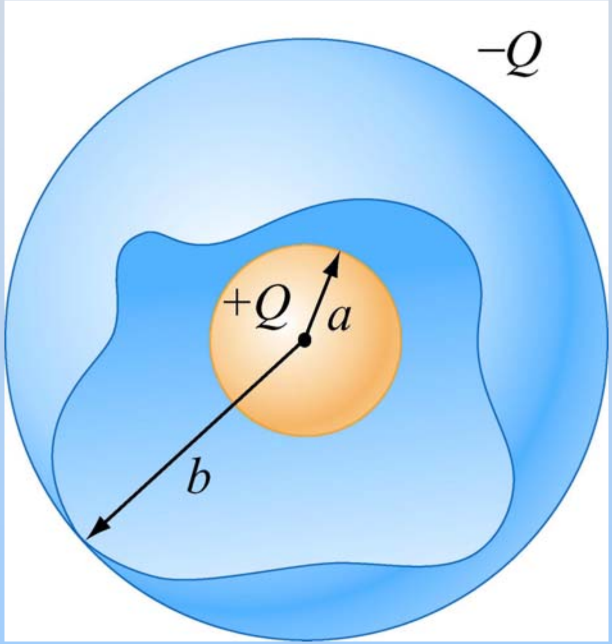
Gauss's Law → E ≠ 0 only for $a < r < b$, where it looks like a point charge:

$$\vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$$

Spherical Capacitor

$$\Delta V = -\int_{inside}^{outside} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = -\int_a^b \frac{Q\hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2} \cdot dr\,\hat{\mathbf{r}} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$$

Is this positive or negative? Why?



$$\boxed{C = \frac{Q}{|\Delta V|} = \frac{4\pi\varepsilon_0}{\left(a^{-1} - b^{-1}\right)}}$$

For an isolated spherical conductor of radius *a:*

$$C = 4\pi\varepsilon_0 a$$

Capacitance of Earth

For an isolated spherical conductor of radius *a:*

$$C = 4\pi\varepsilon_0 a$$

$$\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{F/m} \qquad a = 6.4\times10^{6}\,\mathrm{m}$$

$$C = 7\times10^{-4}\,\mathrm{F} = 0.7\mathrm{mF}$$

A Farad is REALLY BIG! We usually use pF (10^{-12}) or nF (10^{-9})

1 Farad Capacitor

How much charge?

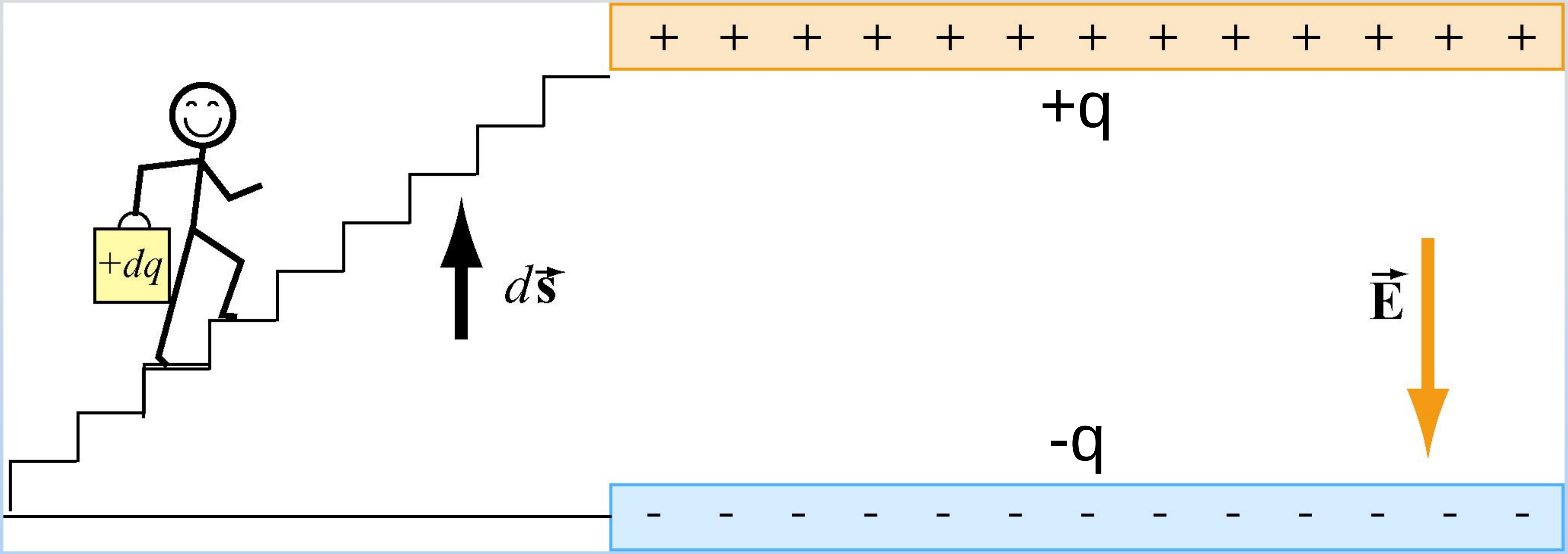
$$
\begin{aligned}
Q &= C\left|\Delta V\right| \\
&= (1\,\mathrm{F})(12\,\mathrm{V}) \\
&= 12\,\mathrm{C}
\end{aligned}
$$

PRS Question:
Changing C Dimensions

Demonstration:
Changing C Dimensions

Energy Stored in Capacitor

Energy To Charge Capacitor



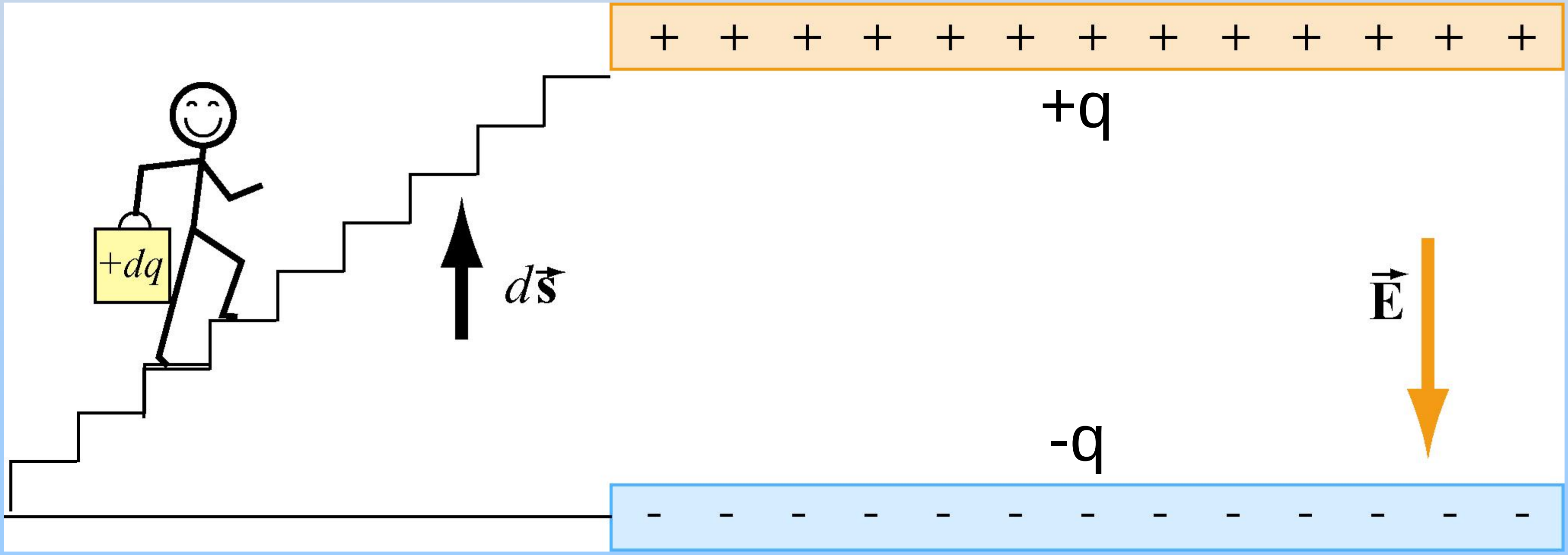
1. **Capacitor starts uncharged.**
2. **Carry +*dq* from bottom to top.**
   Now top has charge q = +dq, bottom -dq
3. **Repeat**
4. **Finish when top has charge q = +Q, bottom -Q**

Work Done Charging Capacitor

At some point top plate has +q, bottom has –q

Potential difference is $\Delta V = q / C$

Work done lifting another dq is $dW = dq\ \Delta V$



Work Done Charging Capacitor

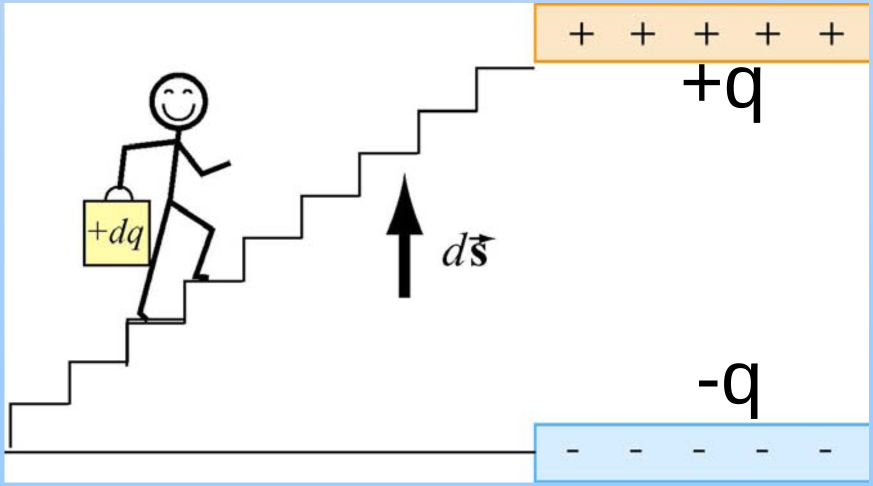
So work done to move *dq* is:

$$dW = dq\,\Delta V = dq\frac{q}{C} = \frac{1}{C}q\,dq$$

Total energy to charge to *q* = *Q*:

$$W = \int dW = \frac{1}{C}\int_0^Q q\,dq$$

$$= \frac{1}{C}\frac{Q^2}{2}$$



Energy Stored in Capacitor

Since $C = \dfrac{Q}{|\Delta V|}$

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q|\Delta V| = \frac{1}{2}C|\Delta V|^2$$

Where is the energy stored???

Energy Stored in Capacitor

Energy stored in the E field!

Parallel-plate capacitor: $C = \dfrac{\varepsilon_o A}{d}$ and $V = Ed$

$$U = \frac{1}{2}CV^2 \quad = \frac{1}{2}\frac{\varepsilon_o A}{d}\left(Ed\right)^2 = \frac{\varepsilon_o E^2}{2} \times (Ad) = u_E \times (volume)$$

$$u_E = E \text{ field energy density} = \frac{\varepsilon_o E^2}{2}$$

1 Farad Capacitor - Energy

How much energy?

$$U = \frac{1}{2} C \left| \Delta V \right|^2 = \frac{1}{2} (1\,\mathrm{F})(12\,\mathrm{V})^2 = 72\,\mathrm{J}$$

Compare to capacitor charged to 3kV:

$$U = \frac{1}{2} C \left| \Delta V \right|^2 = \frac{1}{2} (100\,\mu\mathrm{F})(3\,\mathrm{kV})^2 = \frac{1}{2} (1\times 10^{-4}\,\mathrm{F})(3\times 10^{3}\,\mathrm{V})^2 = 450\,\mathrm{J}$$

PRS Question:
Changing C Dimensions
Energy Stored

Demonstration:
Dissectible Capacitor