8.022 - FALL 2002 - QUIZ#2 NOV 14, 2002

1. SHORT ANSWERS

A: INDUCTOR

B: $v=c$, NO FRAME!

C: SHORT A TO B, MEASURE $I_{TH} \Rightarrow R_{TH} = \frac{V_{TH}}{I_{TH}}$

D: (b)

E: (a)

F: (a).

2. INDUCTOR W/ CONSTANT CURRENT SOURCE

A: $I_2(t=0) = I_{02} = 0$ (CONTINUITY FOR L).

$I_1(t=0) = I_{01} = I$ (CONSERVATION)

$I_2(t=\infty) = I_1(t=\infty) = \frac{I}{2}$

B: KIRCHHOFF #1 $I = I_1(t) + I_2(t)$, #2: $-L\frac{dI_2}{dt} - I_2R + I_1R = 0$

C: $I_1(t) = \alpha + \beta e^{-\lambda_1 t}$ $I_2(t) = \gamma + \delta e^{-\lambda_2 t}$

$$\left.\begin{array}{ll} t=0 & I_{01} = \alpha+\beta = I \quad I_{02} = \gamma+\delta = 0 \\ t=\infty & I_{\infty 1} = \alpha = \frac{I}{2} \quad I_{\infty 2} = \gamma = \frac{I}{2} \end{array}\right\} \Rightarrow \begin{array}{l} \alpha = \beta = \frac{I}{2} \\ \gamma = -\delta = \frac{I}{2} \end{array}$$

$I = I_1 + I_2 \Rightarrow I_1 = \frac{I}{2}\left(-1 + e^{-\lambda_2 t}\right) + I \Rightarrow I_1 = \frac{I}{2}\left(1 + e^{-\lambda_2 t}\right)$

BUT $I_1 = \frac{I}{2}\left(1 + e^{-\lambda_1 t}\right) \Rightarrow$ (TAKE LN'S) $\boxed{\lambda_1 = \lambda_2}$

$-L\frac{I}{2}\lambda_2 e^{-\lambda_2 t} - \frac{IR}{2}\left(1 - e^{-\lambda_2 t}\right) + \frac{IR}{2}\left(1 + e^{-\lambda_1 t}\right) = 0 \overset{t=0}{\Rightarrow} \boxed{\lambda_2 = \lambda_1 = \frac{2R}{L}}$

D: $V_{AB}(t) = V_B - V_A = -I_1 R = -\frac{IR}{2}\left(1 + e^{-\frac{2R}{L}t}\right)$

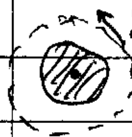
E: $U_L = \frac{1}{2}L\left(\frac{I}{2}\right)^2 = \frac{LI^2}{8}$

3. A DIFFERENT KIND OF COAXIAL LINE

A: $I = \int_0^{R_a} J_0 e^{-\frac{\rho^2}{R_a^2}} \hat{k} \cdot 2\pi\rho\, d\rho\, \hat{k} = J_0 \pi R_a^2 \int_0^{R_a} e^{-\frac{\rho^2}{R_a^2}} d\left(\frac{\rho^2}{R_a^2}\right)$

$= J_0 \pi R_a^2 \left[-e^{-\rho^2/R_a^2}\right]_0^{R_a} = J_0 \pi R_a^2 \left(1 - \frac{1}{e}\right)$

B: CURRENTS AXIALLY SYMMETRIC $\Rightarrow \vec{B} = B\hat{\phi}$ EVERYWHERE



$\odot\hat{k}$ $\quad \rho < R_a$: $2\pi\rho B_1 = \frac{4\pi}{c} \int_0^{\rho} J_0 e^{-\rho^2/R_a^2} 2\pi\rho\, d\rho$

$\Rightarrow B_1 = \frac{2\pi J_0 R_a^2}{c\rho} \left[1 - e^{-\frac{\rho^2}{R_a^2}}\right] \Rightarrow \vec{B} = B_1 \hat{\phi}$

$R_a < \rho < R_b$: $2\pi\rho B_2 = \frac{4\pi}{c} I \Rightarrow B_2 = \frac{2I}{c\rho} \Rightarrow B_2 = \frac{2\pi J_0 R_a^2}{c\rho}\left(1 - \frac{1}{e}\right)$

$\Rightarrow \vec{B} = B_2 \hat{\Phi}$

$\rho > R_b$: $2\pi\rho B_3 = \frac{4\pi}{c} \underbrace{(I - I)}_{0} \Rightarrow B_3 = 0.$

C: $u_B = \frac{B^2}{8\pi} = \begin{cases} \frac{B_1^2}{8\pi} & \rho < R_a \\ \frac{B_2^2}{8\pi} & \cancel{R_a}\rho > R_a \\ = 0 & \rho > R_b \end{cases}$

$U_B = \underbrace{\int_1 u_B\, dv}_{\rho < R_a} + \underbrace{\int_2 u_B\, dv}_{R_a < \rho < R_b} + \underbrace{\int_3 u_B\, dv}_{\rho > R_b \to 0}$

$L = \frac{2U}{I^2}$ OR $L = \frac{\Phi}{cI}$

$\Phi = \int \vec{B} \cdot d\vec{a}$

4. INDUCED ELECTRIC FIELD

A: $\oint \vec{E} \cdot d\vec{l} = A(y_1 - y_2)(x_2 - x_1)$ B: $\vec{B} = B(t)\hat{k} \Rightarrow \Phi = B(t)(y_2 - y_1)(x_2 - x_1)$

C: $\oint \vec{E} \cdot d\vec{l} = -\frac{1}{c}\frac{d\Phi}{dt} \Rightarrow A(y_1 - y_2)(x_2 - x_1) = -\frac{1}{c}\frac{dB}{dt}(y_2 - y_1)(x_2 - x_1) \Rightarrow \boxed{B(t) = cAt + C}$

D: $\vec{\nabla} \times \vec{E} = \hat{k}\left(-\frac{\partial E_x}{\partial y}\right) = -A\hat{k}$, $-\frac{1}{c}\frac{dB}{dt} = -A\hat{k} \Rightarrow \vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{dB}{dt}$

E: YES SINCE $\vec{\nabla} \cdot \vec{E} = 0 \Rightarrow \boxed{\rho = 0}$