Topics: Electric Charge; Electric Fields; Dipoles; Continuous Charge Distributions
Related Reading:
        Course Notes (Liao et al.):   Section 1.6; Chapter 2

Topic Introduction

Today we review the concept of electric charge, and describe both how charges create electric fields and how those electric fields can in turn exert forces on other charges. Again, the electric field is completely analogous to the gravitational field, where mass is replaced by electric charge, with the small exceptions that (1) charges can be either positive or negative while mass is always positive, and (2) while masses always attract, charges of the same sign repel (opposites attract). We will also introduce the concepts of understanding and calculating the electric field generated by a continuous distribution of charge.

Electric Charge

All objects consist of negatively charged electrons and positively charged protons, and hence, depending on the balance of the two, can themselves be either positively or negatively charged. Although charge cannot be created or destroyed, it can be transferred between objects in contact, which is particularly apparent when friction is applied between certain objects (hence shocks when you shuffle across the carpet in winter and static cling in the dryer).

Electric Fields

Just as masses interact through a gravitational field, charges interact through an electric field. Every charge creates around it an electric field, proportional to the size of the charge and decreasing as the inverse square of the distance from the charge $\left(\vec{\mathbf{E}} = k_e \dfrac{Q}{r^2} \hat{\mathbf{r}} \right)$. If another charge enters this electric field, it will feel a force $\left(\vec{\mathbf{F}}_E = q\vec{\mathbf{E}} \right)$. If the electric field becomes strong enough it can actually rip the electrons off of atoms in the air, allowing charge to flow through the air and making a spark, or, on a larger scale, lightening.

Charge Distributions

Electric fields "superimpose," or add, just as gravitational fields do. Thus the field generated by a collection of charges is just the sum of the electric fields generated by each of the individual charges. If the charges are discrete, then the sum is just vector addition. If the charge distribution is continuous then the total electric field can be calculated by integrating the electric fields $d\vec{\mathbf{E}}$ generated by each small chunk of charge dq in the distribution.

Charge Density
When describing the amount of charge in a continuous charge distribution we often speak of the *charge density*. This function tells how much charge occupies a small region of space at any point in space. Depending on how the charge is distributed, we will either consider the volume charge density $\rho = dq/dV$, the surface charge density $\sigma = dq/dA$, or the linear charge density $\lambda = dq/d\ell$, where V, A and ℓ stand for volume, area and length respectively.

Electric Dipoles
The electric dipole is a very common charge distribution consisting of a positive and negative charge of equal magnitude q, placed some small distance d apart. We describe the dipole by

its dipole moment p, which has magnitude $p = qd$ and points *from* the negative *to* the positive charge. Like individual charges, dipoles both create electric fields and respond to them. The field created by a dipole is shown at left (its moment is shown as the purple vector). When placed in an external field, a dipole will attempt to rotate in order to align with the field, and, if the field is non-uniform in strength, will feel a force as well.
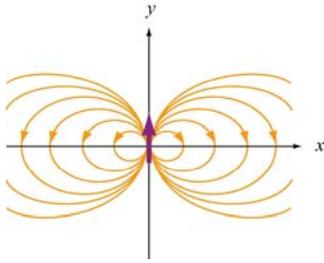
Important Equations

Electric force between two charges:
$$\left|\vec{\mathbf{F}}_E\right| = k_e \frac{qQ}{r^2},$$

       Repulsive (attractive) if charges have the same (opposite) signs

Strength of electric field created by a charge Q:
$$\vec{\mathbf{E}} = k_e \frac{Q}{r^2}\hat{\mathbf{r}} = k_e \frac{Q}{r^3}\vec{\mathbf{r}},$$

       $\hat{\mathbf{r}}$ points from charge to observer who is measuring the field

Force on charge q sitting in electric field E:
$$\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$$

Electric dipole moment:
$$\left|\vec{\mathbf{p}}\right| = qd$$

       Points from negative charge $-q$ to positive charge $+q$.

Torque on a dipole in an external field:
$$\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$$

Electric field from a discrete charge distribution:
$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{\left|r_i\right|^2}\hat{\mathbf{r}}_i = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{\left|r_i\right|^3}\vec{\mathbf{r}}_i$$

Electric field from continuous charge distribution:
$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\int_V \frac{dq}{r^2}\hat{\mathbf{r}}$$

Charge Densities:
$$dq = \begin{cases} \rho dV & \text{for a volume distribution} \\ \sigma dA & \text{for a surface (area) distribution} \\ \lambda d\ell & \text{for a linear distribution} \end{cases}$$

Important Nomenclature:

A hat (e.g. $\hat{\mathbf{A}}$) over a vector means that that vector is a unit vector ($\left|\hat{\mathbf{A}}\right| = 1$)

The unit vector $\hat{\mathbf{r}}$ points *from* the charge creating *to* the observer measuring the field.