We would now like to look at two-dimensional collisions.

And what we'd like to look at is in the laboratory frame-- so I'll call that the lab frame-- in which we have a target particle, which I'm going to call 2, and an incoming particle 1, which is coming in with some initial velocity.

After the collision, let's imagine that the target particle is going out at a certain direction.

So we'll call that 2.

And the target particle has a velocity v2 final.

And the initial particle that's going in this direction-- we'll call that 1.

And that is it's outcoming velocity.

Now in this collision, we want to ask ourselves first, what quantities are constants of the motion.

Well, let's assume no external forces, therefore momentum is constant.

And we can write our momentum equation as m1 v1 initial equals m1 v1 final plus m2 v2 final.

Now recall that momentum is a vector.

And so what we have here are two-- the unknowns here are our two outcoming vectors, v1 final and v2 final.

And a vector in two dimensions has two quantities.

We can discuss-- we can write that as components.

Or we can write it in terms of magnitudes and directions.

Now experimentally, we often will measure the directions of the outcoming particles, which I will now indicate by theta 2 final and theta 1 final.

And so, when we write our two momentum equations, we can either write it as components, or we can write it in terms of magnitudes and directions and do vector decomposition.

Now because we measure the outcoming directions, we're going to choose to do magnitudes and directions.

So let's indicate a little notation.

We'll say that the magnitude of v1 initial is v1 i.

And the magnitude of v2 final is v2 final.

And the magnitude of v1 final is v1 final.

And so now, when we look at our two momentum conditions, we can-- we now also have to introduce unit vectors for directions.

So let's call i hat that way and j hat in this direction.

And in our i hat direction, we have only the incoming momentum.

And we can write that as m1 v1 initial.

It's positive because we've chosen the forward direction as our i hat direction.

Now in terms of the outgoing momentum in the i hat direction, we have to do vector decomposition of both of these vectors.

And they both have positive components.

So we have m1 v1 final-- that's the magnitude-- cosine theta 1 final plus-- positive sign, because they're both in the positive direction-- v2 final magnitude times-- we need that little-- m2 v2 final cosine theta 2 final.

And that is our i hat direction.

Now the j hat direction-- remember, we have to be careful, because we're taking positive j hat up.

So our particle 2 has a positive component in the j direction.

And our particle 1 has a negative component in the j direction.

The incoming momentum-- there's no momentum in the j direction.

So we have a 0.

And that's equal to positive m2 v2 final.

And that's a sine theta 2 final.

Now, here's where you have to be careful, because this one is negative.

Component is in the negative j hat direction.

And we have m1 v1 final sine theta 1 final.

And these two represent our momentum equations.

Now we also have to think-- let's think about energy.

Again, we have to know something about this collision.

And our assumption will be that this particular collision is elastic.

And that means the initial kinetic energy is equal to the final kinetic energy.

Energy is a scalar.

We've been describing our incoming velocity vectors in terms of magnitudes.

So we can write our elastic energy condition as the incoming kinetic energy squared-- that's the kinetic energy incoming-- is equal to 1/2 m1 v1 final squared plus 1/2 m2 v2 final squared.

And that is our kinetic energy condition.

Let's label this equation 1 and equation 2.

Now, it's very important to realize which quantities are given and which we need to solve for.

So in this problem, because the two outcoming velocities-- unknowns-- we have four unknowns.

Those unknowns can be written in terms of the two velocity final and the other one, v2 final.

Those are our vector quantities.

But recall, in terms of the scalar magnitudes, we have that v1 final.

And I'll just write the other ones down-- v2 final, and the two outgoing directions-- theta 1 final and theta 2 final.

So these are our four unknown quantities.

But you can see we only have three equations.

And therefore, if we want to determine the outcomes, we need to measure one additional quantity.

Now that's very useful when doing problem solving, because when you start to read a problem, and you look at what's being measured, you can right away determine which of the four quantities has been given.

You may be given an outgoing magnitude of the velocity, or you may be given one of these scattering angles.

And so that's how we approach two-dimensional elastic collision.

Of course there's algebra now to solve for any particular quantity that you're interested in, provided you have this extra additional information.