Let's look at some examples of one dimensional elastic collisions with no external forces between two particles.

So suppose I have particle 1 and particle 2, and we have them moving on a frictionless surface.

And let's choose a reference frame in which we'll call-- the laboratory frame-- in which the initial velocity of particle 2 0.

So our frame is called the laboratory frame.

And in this frame we're thinking of particle 2 as our target.

And the target is at rest.

And particle 1 is moving in with some initial velocity.

And so here is our initial picture.

And then in our final state, both the target and the particles can be moving.

So we'll just indicate v1 final.

And we'll have target was also moving with v2 final.

And we have i hat.

So let's now look at some examples where we're analyzing this type of collision in which both energy and momentum are constant.

And for a particular example that we want to look at, we'll have m2 is twice m1.

Now, for our two equations that we're going to write about, we'll choose components, and we'll write our energy equation as 1/2 m1 v1 x initial squared plus 1/2 m2-- so now we're going to write 2 m1.

But object 2 is at rest, so we don't have to worry about that one yet.

And so, our initial kinetic energy is just equal to the final kinetic energy-- 1/2 m1 v1 x final squared.

And now I'm going to substitute in 2m1 for the mass of m2 v2 x final squared.

And that's our energy condition.

And then our momentum condition is that the incoming momentum, 1/2 x initial is equal to the outgoing momentum, m1 v1 x final.

And again, I'm going to substitute for m2.

That's 2m1 v2 x final.

And these two equations represent a system of two equations with two unknowns.

We'll treat this as givens along with the initial velocity of particle 1.

And our target here is to solve for the 1x final and v2 x final.

So we have two equations and two unknowns.

Now it's a quadratic equation.

So we have to identify-- from a problem solving strategy, we have to identify which quantity we're going to solve first.

And so, let's solve for v1 x final, which means we need to eliminate v2 x final.

And the way I'll eliminate v2 x final is I'll use the momentum equation.

And notice that the m's will cancel in the momentum equation.

And so, if I divide through by 2m in the momentum equation and bring this term over to the other side, equation 2a becomes v2 x final equals-- I'm dividing through by 2-- 1/2 v1 x initial minus 1/2 v1 x final.

So this gives me a target-- v2 x final-- I can substitute into there.

This equation, again, can be cleaned up.

Let's clean it up before.

We can get rid of the halves.

We can get rid of the m1's.

And so the quadratic equation that we're going to be working with is v1 x initial squared equals v1 x final squared.

And I'm dividing through by 2.

So I have the factor 2 plus 2 v2 x final squared.

Now that's our energy equation.

And here's our momentum equation.

So if we substitute this in and square it, we can now substitute and we get v1 x initial squared equals v1 x final squared plus 2.

Now we make our substitution.

So that's 1/2 v1 x initial minus 1/2 v1 x final, quantity squared.

So let's now expand this.

And we have to be careful not to make any mistakes.

v1 x initial squared equals v1 x final squared.

Now if I pull the 2 out, I got a quarter in front, and divide that by 1/2.

So I get 1/2 v1 x initial squared plus another 1 plus a 1/2 v1 x final squared.

Those represent the-- squaring out those two terms.

Now the cross-term will have a factor of 1/2 in the front but a factor of 2.

So we have a simple cross-term of v1 x initial v1 x final.

Now, when we-- let's collect terms.

Let's bring everything over to this side so we have a 0 equals 3/2 v1 x final squared minus 1/2 v1 x initial squared and minus v1 x initial v1 x final.

Now I always like to just write this up in a simple way to use the quadratic formula.

So I'm going to divide through by 2/3.

And I get minus 1/3 v1 x initial squared minus 2/3 v1 x initial v1 x final.

And this is now a simple application of the quadratic formula.

Negative b is plus 2/3 v1 x initial plus or minus.

And we're going to interpret those two roots in a moment.

We have to factor 2/3 v1 x initial squared minus 4ac.

So that's another minus sign with a plus.

So that's plus 4/3 v1 x initial squared, all to the square root, and everything divided by 2.

Now here, let's just look at this factor.

This is 4/9 plus 4/3.

4/3 is 12/9, so that's 16/9, which is very convenient, because when you take the square root that's 4/3.

So we get equal to 2/3 v1 x initial plus or minus 4/3 v1 x initial divided by 2.

Now we see that there's two different roots.

So when you add them, you get 2 v1 x initial divided by 2.

So there's one solution.

And when you subtract them, you're getting 2/3 minus 4/3.

That's negative 2/3 divided by 2.

So that is another solution-- v1 x final equals negative 1/3 v1 x initial.

Now let's think about the meaning of these two possible solutions.

This solution has v1 x final equals v1 x initial.

So that's the initial conditions.

Just repeat it.

And that will always be the case.

One solution will describe the initial state, and the other solution will define the final state.

You can check for yourself that if you just put vx1 final into this-- equal to vx1 in the initial-- into this momentum equation, then v2 x final is 0, which just repeats the initial conditions.

So this solution is the initial state.

And this solution, here, is the final state.

Now just to complete the picture, v2 x final-- well, that's equal to 1/2 v1 x initial minus 1/2 times v1 x final.

But v1 x final is negative 1/3.

So we have a 1/2 minus 1/2 half times minus 1/3.

So that's 1/2 plus 1/6, which is 4/6 or 2/3 v x initial.

So that's 2/3 the v1 x initial.

And that represents the solution to this particular problem.

Now, of course what you want to do is, you want to check-- we know the momentum condition is already satisfied.

But just as a check, you would like to put it into the energy condition just to make sure that when you square these things out you get the right terms.

However, we're very confident of our result, because we've already reproduced the initial conditions.

And that wouldn't happen if we made some algebraic mistake.

So that, by itself, is a sufficient check that this is a correct solution.

One thing we should sit back and think about is that when we use the energy and momentum that we're inevitably dealing with quadratics.

And so we expect to use-- to solve a quadratic equation at one point.

And here it was right here.

Now we're going to find another way to do this by linearizing the system, which will be a lot easier.