I would now like to calculate the potential function for two other conservative forces that we encounter all the time, spring forces and gravitational forces.

Will begin with spring forces.

So first off, let's begin with some type of coordinate system.

Suppose we have a spring.

We have a block.

This is my point x equilibrium, unstretched length of the spring.

And now whether we stretch it or compress it here, I'll stretch it.

I'll introduce a coordinate function x.

Union vector i, and my spring force, f, is -kxi, where x can be positive for a stretch spring, force is in the negative i-hat direction.

When x is negative, negative times negative, means the spring force is in the positive i-hat direction when it's compressed.

So this is a force that's a restoring force, always back to equilibrium.

This is an example of a conservative force.

And now let's calculate the change in potential energy.

And then introduce a zero reference point and get a potential function for this force.

So the first calculation is straightforward.

So, if we took our displacement to be dx i-hat.

And we take the dot product of fds, we get dx i-hat dot i-hat is 1, times dx.

And that's just the x component of the force displacement.

And minus sign because the x component of the force is -kx.

And so now, if we were to start our system, so, again, for our initial state, let's say that the block from the unstretched position, is stretched xi.

And our final state, whether it's stretched or compressed, it won't matter, but we'll just stretch it out a little bit more to x-final.

So now let's calculate the change in potential energy between our initial and our final states.

Now, recall there's a minus sign, because it's -w conservative.

That's minus the integral x-initial to x-final of -kxdx.

We have 2 minus signs.

That's our integration variable, if you want to see all the detail.

And when you simply integrate x-prime dx-prime, you get x-prime squared over 2.

And so the change in potential energy between these two states is 1/2 kx-final squared, minus 1/2 kx-initial squared.

And that's physically meaningful if I take my system initially.

And the system ends up in the final state.

Then what I'm calculating is minus the work done by the spring force on the block as it goes from the initial state to the final state.

I'm not talking about the work that an external agent does in stretching or compressing it.

This is explicitly the work done by the spring force on the block as it goes from the initial state to the final state.

I introduced the minus sign in our definition of potential energy.

And so this quantity represents the negative of the work done by the spring force as a system goes from the initial to the final states.

Now, you may have already thought about it, but the reference point that we're going to use is x equals zero.

The unstretched length of the spring.

And our reference potential at that point will be zero.

So our reference point is actually the zero-point for the potential.

And then our arbitrary state, we can call this the referent state, if you like.

That's probably better than the reference point.

Our arbitrary state will be at some arbitrary stretch or compress.

And our potential energy function, at that reference, minus the potential energy at the arbitrary state, minus the reference state.

Well, if we set xi equal to 0 and x-final equal to x, as we have in this expression, we simply get 1/2 kx squared.

So the potential energy function equals our reference potential, plus 1/2 kx squared.

And we have defined that our reference potential to be zero, it could have been anything.

But we're making it 0 so that we get a nice function, 1/2 kx square.

And again, it's always worthwhile to plot that function.

It's a nice parabola.

U of x, x.

And you can see down here, that's our reference point.