We would now like to look at some applications of the rocket equation, in which we have an external force plus dmr dt, where m and r is the mass of the rocket at time t times the relative velocity of the exhaust velocity was equal to m of r dv rocket, dt.

And now we want to consider some special cases.

So our first example will be a case when there is no external force.

Now, this might be characteristic of a rocket that is far away from any type of gravitational interaction and is either speeding up or slowing down.

This is a special case that will enable us to understand our problem.

Now, we'll draw a picture at time t.

Just to remind ourselves, we have mass of the rocket.

It's moving with a velocity v of r of t, and we're going to call this our plus i hat direction.

And then at time t plus delta t, we have our fuel delta, delta m fuel, which was equal to minus delta m rocket moving with the relative velocity u plus v of r of t plus delta t.

And our rocket itself is moving with v of r of t plus delta t.

So in our coordinate system, we have to remember that the exhaust speed is relative to the rocket, and we define that to be u.

Since the rocket is moving forward and we've chosen plus i hat in that direction, it's minus u i hat, and we can write our vectors vr as v of r i hat.

So our rocket equation in this special case becomes dmr dt, minus u i hat, equals mr dvr dt i hat.

Now, in this equation, remember that it's important to recall that m of r is a function of time.

Unless we have an explicit model for how the mass is ejected, in principle we can't solve for the time behavior, but we can look at this differential equation just by eliminating the time completely by multiplying through by dt, in which case we have minus dmr u equals m of r dvr.

Now, this is separable and I can integrate it, so I'll bring the terms minus dmr over mr on this side, times u, and that's equal to dvr on that side.

Now, when we integrate, we're going to be integrating mass of the rocket, say at some initial time, t initial, to mass of the rocket at some final time.

And over here, we're going to be integrating the velocity from some initial time to some final time.

When both these integrals are straightforward, on the left-hand side we have a natural log, so we have minus u, natural log of mrt final over mrt initial.

And this side, we just have the velocity difference, v of r at t final minus vr at t initial.

Now, again, we always check our minus signs, but recall the mass of the rocket is decreasing.

So the log of a fraction is negative, so we have a positive quantity on this side.

And that's indicating that the velocity has increased on that side, and so we conclude that the velocity of the rocket at its final time is equal to the velocity of the rocket at its initial time.

Now, here, I'll keep this minus sign in here-- minus natural log of mrt final over mrt initial.

Now, in our answer, we need to know what the mass of the rocket is at the final time, so in general this equation is still not going to give us an explicit expression for v of t final, unless we have one special case in which our final time is when all the fuel has been burned.

And our initial time is the rocket with all the fuel there, and then this ratio is a known ratio.

And you can look at a variety of different examples for that.

So there is an example of the solving the motion of a rocket with no external forces.