

I'd now like to talk about the velocity of the center of mass for a system of particles.

So let's take a system, which I'll just outline by this.

And in that system, we have a bunch of particles, particle 1, particle 2-- let's refer to this as the j -th particle and some point x_{cm} .

And if I want to talk about the position of the center of mass, I can choose a point s .

And if I want to define that vector R_{cm} , then what I have to do is draw a vector to each object R_{sj} .

And we saw that this velocity, the position of the center of mass with respect to this origin s is the sum and $m_j r_{sj}$.

And that's divided by the total mass and m total.

And j goes from 1 to n , where n is the number of particles in the system.

Now if I want to find the velocity of the center of mass, then I can just differentiate this.

And I'm dropping the point s for the moment, but let's just differentiate 1 to n .

And you'll see why.

And when I differentiate the position vector of the object, that's the velocity of the object divided by j goes from 1 to n , the total mass.

Now why did I drop the position?

Because if you have any two fixed points-- so if I chose another fixed point, say, over here p , then this distance R_{sp} -- we'll call it vector from s to p -- this is a constant.

And if I draw position vector with respect to p -- now the point here is that this is a constant distance, because this is a fixed-- these are fixed points.

Then if you were to draw your vector triangle, which is the position of the object with respect to s -- that's this vector-- is equal to that fixed position vector from s to p , plus the vector from p to j , and I differentiate this, dr_{sj}/dt .

Well, this derivative of a constant vector, this is 0 plus dr_{pj}/dt .

And so we see that the velocity j is independent of the choice of point s .

You choose any other fixed point and you get that velocities $\frac{dr_s}{dt}$ equals $\frac{dr_p}{dt}$ for all fixed points p .

And that's why in this expression, when we differentiate the velocity, even though we had an index s , we dropped that.

And so our conclusion is that we can treat that we have the velocity of the center of mass of this system is equal to the sum $m_j v_j$.

$\sum_{j=1}^n m_j v_j$ divided by the total mass.

Now what's interesting here is, why is this an important quantity?

Let's just add that if we want to talk about the acceleration of the center of mass, I do exactly the same type of calculation.

I just differentiate.

And I get the mass of the j -th particle times the acceleration of the j -th particle divided by the total mass.

And our next step is to understand why this is an important quantity for a system of particles.