

Now we'd like to analyze in more depth our result that for a system of particles-- so let's indicate our system.

We had particle 1.

We have our j th particle.

And we have a particle N . So here's our system of particles where the total force caused the momentum of the system of particles to change.

Now, I'd like to examine that concept of the total force.

Before we said that our total force on the j th particle-- we just wrote it like this.

And I'm going to put a little t up here for the moment.

Because when we examine what force we mean here-- and I also want to put a little boundary around our system.

And let's now consider another particle internal to the system.

And let's try to identify the types of forces on the j th particle.

We can really have two types of forces here.

Our first force can be an interaction between these two particles.

So what I'll write is the force on the j th particle due to the interaction between the k and the j th particle.

And I'm going to put a little sign up here.

I'm going to write this internal.

What do I mean by internal?

This is a force strictly between the internal particles in the system.

Now, of course, we know that there must be a force on the k th particle due to the interaction between the j th and the k th particle.

And we can call that internal.

So what we have here is that we can divide-- there can still be other forces acting on the j th particle.

And we'll do a decomposition like this.

We'll say that the total force on the j th particle can come from some external forces.

There could be an object outside our system.

If these were interacting gravitationally, there could be a planet outside here.

And this could be a moon.

And our system is just the moons.

That would be an external gravitational force, plus the total internal forces.

So I'm going to keep this same color.

Internal on the j th particle.

Now how do we write this total internal force?

Well, we're interested in the force on the j th particle.

But the internal forces can come from all of the other particles in the system.

So what we're looking at here is for a sum over all of the possible interactions where the other particles, k , go from 1 to N . And we have to be very careful here that in this sum k cannot be equal to j .

Now this sum-- again, because it's a little bit tricky to understand-- is the internal force on the j th particle.

Here's the k th one.

But this could be a sum.

I'll just draw one here.

This is the internal force on the j th particle due to particle number 1.

And so we're adding as k goes from 1 to N all of these internal forces.

But we're excluding the case when k equals j , because that would be a force of an object on itself.

And this quantity here, we can write as the total internal force on the j th particle.

So in summary, we see that the total force on the j th particle is equal to the total external forces.

I didn't say total there.

I'm assuming there could be many different types of internal forces plus the total internal force.

A little bit later on, we can drop the T's for simplicity of notation.

But this is our big idea, that a force on the j th particle, external plus internal.

And now when we look at this sum, and we want to now apply our main idea, we have that the force, which we're writing as-- let's explore this.

Our total force is the sum of the forces on the j th particle.

And we've now done this decomposition.

I'm going to drop total.

So it's the sum of the external forces on the j th particle.

j goes from 1 to N .

And here, we have a sum of the internal forces.

So we have our sum j .

It goes from 1 to N of the internal forces on the j th particle.

Now we want to apply Newton's second law.

And the concept is very straightforward.

But the mathematical expression can be a little bit messy.

We know by Newton's second law that the sum of a pair of internal forces is zero-- third law, by Newton's third law.

So what we're saying here is, as an example, for Newton's third law-- let's just focus on this particular pair-- that $F_{\text{internal } kj}$ plus $F_{\text{internal } jk}$ is zero.

So this is the statement that internal forces cancel in pairs.

And so when I look at this total internal force, which is the sum of all of these pairs of internal forces, I can see that

the total internal force has to be zero.

So internal force cancel in pairs.

Now here, we can see it another way if we want to look at this notation.

We took the sum.

j goes from 1 to N of $F_{\text{internal } j}$.

Now we use our definition for F_{internal} .

This where things get a little bit messy.

k goes from 1 to N . k not equal to j .

j goes from 1 to N . $F_{\text{internal } kj}$.

This looks terribly messy.

But what we're saying is this sum is just a sum of pairs.

And every single pair in this adds to 0.

So what we have for our statement now is that the total force is the sum of the external forces, plus the sum of the internal forces which we've now said that cancels in pairs.

So let's rewrite that as the total force-- now instead of writing this sum, let's write it as the sum of the external forces.

And the internal forces cancel in pairs.

And so this is now our force on our system.

It's only the external force.

And now we can recast our Newton's second law for a system of particles with the following statement that the external force causes the momentum of the system to change.

And this becomes our expression for Newton's second law when we apply it to a system of particles where the beauty of this idea is that no matter how complicated the interaction is inside the system, all of those interactive pairs sum to zero.

And so only thing that matters is the external force in terms of changing the momentum of the system.

And now we'll look at some applications of that.