

We want to look at this pulley system.

We want to find out what this force here is, for example, with which this block is being pulled.

Now we have two massless pulleys here and two moving parts.

And one key component of this problem is to derive the acceleration constraint.

How are we going to do that?

Well, we have to look at this string here.

First of all, it's a fixed string length.

And that will help us.

Ultimately, this is just a distance that connects all of these objects.

And if we eventually differentiate that twice, we get to an acceleration.

So the first thing we're going to do is to figure out what the string length is to then derive the acceleration condition.

Let's begin by first identifying the fixed length in this problem.

So we have a little distance here.

We call it  $s_b$ .

And we have a distance here that's fixed,  $s_a$ .

And we furthermore know that this block here is fixed to the ground.

And we can choose a coordinate system origin.

Let's say we do that here.

We know that we have the distance  $d$  here to this little block.

The moving parts, block 1 and block 2-- so we have to assign position functions.

And actually this one here is  $x_1$ .

It goes up to here because we want to measure the string length, ultimately, and we know this portion here.

So we can subtract it.

And then of course we need also  $x_1$ , so that goes here--  $x_2$ .

So now we can tally up the length.

Let's start with this portion here.

That is  $d$  minus  $s_a$  minus  $x_2$ .

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And then we have a half circle here,  $\pi r$ .

And we know actually that there is another half circle that's going to come there, so we can just write  $2\pi r$  immediately.

And then we have this part here.

That one is  $x_1$  minus  $s_b$ , and then minus  $s_a$  minus  $x_2$ .

And then finally we have this portion, and that is  $x_1$  minus  $s_b$  minus  $x_2$ .

OK, so the next step is that we need to simplify this a little bit.

So we have one  $x_1$  and another one here,  $2x_1$  plus  $x_2$ ,  $x_2$ ,  $x_2$ -- actually  $3$ -- minus  $3x_2$ .

And then we have all sorts of consonants.

We have  $d$ , we have  $s_a$ ,  $s_b$ , the  $2\pi r$ .

But, as you will see, if we differentiate this out here, that will actually all fall away.

So we're going to make our life easy and just add a constant here.

And so we want to now do the second derivative here of our string length with respect to time, because these all position functions.

So we can differentiate those.

What's important of course here is this string length is not changing with time.

So actually we know that that derivative will be 0.

And we can just write this up here because  $x$  differentiated twice is  $a$ .

So  $2a_1$  minus  $3a_2$ .

And we immediately see from that that  $a_1$  equals  $3/2 a_2$ .

So this is our constraint condition that we will need later.

For now, we need to continue with setting up free body diagrams of all four objects.

Let's start that with object 1.

What's acting on object 1?

Well, we have  $F$  here.

Why don't we just write it as magnitude.

We have  $F$ , and then we have here a tension that goes to the pulley B. That one is different from the tension in the string.

So we're going to call this  $T_B$ .

And then we have object 2.

Oh, and of course,  $i$  hat goes in this direction because it follows the motion of the object.

We have here kind of the reverse.

Now we have a tension of this string here that's attached to pulley A, but it's different from this string tension-- that's a specific one--  $T_A$ .

And we also have a  $T$  from the string here.

So we'll add a  $T$ . And then if we look at pulley A, we have two string tensions,  $T$  and  $T$ .

And here we again have a  $T_A$ .

And pulley B,  $T_B$ , and two  $T$  over here.

So that means we can write down our equations of motions using Newton's Second Law,  $F$  equals  $ma$ .

And since this is just going in the  $\hat{i}$  direction, we can just write down the four equations following the four free body diagrams here.

So we have  $F - T_B = m_1 a_1$ .

We have  $T + T_A = m_2 a_2$ .

Then we have  $2T - T_A = 0$ .

That is 0 because we're dealing with two massless pulleys there.

That means  $m$  is 0 and so our acceleration term is 0.

So the  $m$  is 0 here.

And finally, we have  $T_B - 2T$ .

No, not  $2T$ .

And that one is also 0 because it is a massless pulley.

So here we have our four equations of motion that fully govern this pulley system.

And we can use it then, for example, to find this pulling force here.

We know what  $T_B$  is from the equation down here-- so  $2T$ .

We know what  $T_A$  is.

It's also  $2T$ .

And we can then solve this for  $F$ .

The only sticky part is that we have this  $a_1$  and  $a_2$  in here.

But for that, we derived this constraint condition here.

So with that one, we can fully solve this.

Otherwise, we would have one too many unknowns.

Alternatively, if one were to be interested in one of these accelerations, then this equation system can also be solved for  $a_1$  or  $a_2$ .