I'd now like to consider a very unusual problem, and we saw a demo of this.

Suppose you have a yoyo, and the yoyo an inner radius b and an outer radius R. And the yoyo is rolling without slipping along the ground.

And what we have here is we're pulling the yoyo with a string that's wrapped around the spool, and I want to find F max such that it rolls without slipping.

If I don't pull it hard enough, the wheel will roll without slipping.

And if I pull it harder, then this F max, the wheel will start to slip.

So now let's analyze this problem, and let's use our dynamics approach.

Let's choose an i hat, a j hat, k hat.

As before, we'll define an angle theta.

And now we'll apply both Newton's second law for linear motion and our torque about the center of mass.

Now, we have to consider our forces.

So I'm going to put the forces on this diagram.

We have the normal force from the ground pointing up.

We have the gravitational force pointing down.

And remember, it's rolling without slipping.

So the wheel is-- the contact point is instantaneously at rest.

But because the center of mass of the wheel is accelerating, once again, in order to keep a equal to R alpha, then we need some type of torque that will produce a non-zero alpha.

And that's going to come from a non-zero static friction.

So once again, we have a case, unlike a wheel which is just rolling along a horizontal plane, because we're pulling this yoyo's string, the static friction is not 0.

Static friction depends on everything else that's happening in the system.

And now we can apply Newton's second law, F equals ma.

And so in our x direction, we have F minus F equals ma.

And if we look at the torque about the center of mass is Icm alpha.

Then this is our x equation.

Now here, both the pulling force and the static friction both exert torques about the center of mass.

The static friction will exert a torque in the direction k hat, and the pulling force will exert a torque in the minus k hat direction.

So our torques-- the normal force does not produce any torque about the center of mass, nor does gravity.

So what we have is fs times the radius of the wheel-- that is the torque due to static friction-- minus b times the pulling force F, and that's equal to Icm alpha.

So we have our two dynamic equations.

And again, let's recopy our rolling without slipping condition, now expressed in terms of acceleration.

So now what I'd like to do is solve for this force F. And the way I'll do it is I'll write down, from this condition, alpha in terms of a/R. And from this equation, I have that-- so alpha is a/R. And from the top equation, I have that a is F minus f static over mR, times R.

So I can replace the alpha in this equation, and I get fsR minus bF is Icm F minus fs over mR.

And now I just need to collect terms.

I'll bring the static friction term to this side and my f term to that side.

And so I get fs times R. And when I bring the static friction term over to that other side, I get Icm over mR.

Let's just check dimensions.

Icm is mR squared.

So these terms have the dimensions of length, so looks like I'm OK.

And now I bring the F to the other side, and I get F times b plus Icm over mR. I can now solve for F, and I get f static R plus Icm m/R-- it's a complicated answer-- divided by b plus Icm divided by mR.

Now, if I want to ask the question, what is the maximum force I can pull in which it just slips, what's physically happening is the harder I pull, the bigger the static friction is.

I pull harder, static friction gets bigger.

I pull harder, static friction gets bigger.

But static friction can only have a maximum value.

That hasn't changed.

The maximum value of static friction is the coefficient of static friction times a normal force, which, in this case, is just normal balance in gravity, mg.

So this is the maximum value that static friction can have.

And so now I get F max-- this is, again, a little complicated-- mu s mg times R plus Icm divided by mR divided by b plus Icm divided by mR.

And that is the maximum force that I can pull this yoyo with that it still rolls without slipping.