

Well, we've been calculating angular momentum about a point.

Recall our definition of angular momentum.

We've been looking at cases where the particle is moving linearly.

Now let's look at a case where the particles are undergoing circular motion.

So suppose we have a particle that's undergoing circular motion m .

And let's choose some axes.

And our particle, we denote that it's rotating about the vertical axis.

I'm going to call that axis \hat{k} .

And I'll make that ω^2 , ω_z positive.

So it's rotating about the k -axis, the z -axis.

Now, when we calculate the angular momentum, we know because it's rotating about the z -axis, this is our z -axis, that the particle has a velocity tangential to the circle.

So its momentum is tangential to the circle.

And we draw our vector \mathbf{r}_s to where the object is.

Now, you could solve this in Cartesian coordinates but then you might have to do some vector decomposition.

But because there's a central point to this motion, whenever there's a central point we like to choose cylindrical coordinates.

And the way we'll do that is we'll define some angle θ .

If this were my plus x and my plus y -axis, then that's consistent with \hat{k} being up in our definition of ω .

And I'll define an \hat{r} unit vector, which is pointing radially outward from the center of the circle.

And a $\hat{\theta}$ vector, which is tangent to the circle.

And now I can calculate-- I can write down, say the radius of this circle is r .

Then \hat{l} of \hat{s} , the vector \hat{r} s has radius r pointing outward.

And the momentum vector is pointing tangential.

And that's a very easy cross-product to make \hat{r} hat cross $\hat{\theta}$ hat.

That's what we're defining to be \hat{k} hat, maintaining the cyclic order.

And so, we get $r p \hat{k}$ hat.

Now for circular motion the momentum magnitude is $m v$, the magnitude of the $\hat{\theta}$ component.

We've made that positive, which is $m r \omega$.

And so \hat{l} is-- There's an r here and another r there.

So we get $m r^2 \omega \hat{k}$ hat, This is our vector ω .

Now this turns out to be the moment of inertia of a point particle located at the center.

And so we conclude that the angular momentum is proportional to the angular velocity.

Now, that's no surprise in this particular case.

Why?

Because the vectors \hat{r} s and \hat{p} are in the plane of motion.

And whenever you take a cross-product, \hat{l} is perpendicular to both of those vectors so it's perpendicular to the plane of motion.

And therefore, \hat{l} has to point in the z direction.