

One of the most common motions we see in our everyday lives is the path of an object moving through space.

Now, this type of motion has a very famous name called projectile motion.

And when we look at it, let's introduce a coordinate system.

\hat{i} and \hat{j} .

Here's our y plus y -axis and our x -axis.

Then, in order to understand the kinematics of this motion, we'd like to apply Newton's second law.

So separately we'll draw our object.

It has gravitational force acting downward.

Remember our unit vectors in the \hat{j} direction.

And so when we write our equations of motion, f equals ma .

We have two different directions.

In the \hat{j} direction, we have the gravitational force downward minus mg .

And remember here that g is our positive quantity, 9.8 meters per second, and we have $m a_y$.

Newton's second law equates these two different things.

And that's-- we'll write it like that.

So those two quantities are equal.

And so our conclusion is that the acceleration is equal to minus g in the y direction.

Now, likewise, keep in mind that we have our horizontal direction as well.

And notice that we're now assuming that there's no horizontal forces.

In the real world, there can be all types of air resistances.

But here, for the simplicity of this model, we have no horizontal forces.

So we have $0 = a_x t$.

And so we have our separate equation, $a_x t = 0$.

Now, both of these equations are equations that we've already been working with in kinematics.

Here we have a constant acceleration, and here we have zero acceleration.

So using the results that we had earlier from integration, we can write down the equations of motion in the y and the x directions.

First, we'll write the velocity.

v_y as a function of time is equal to some initial value.

And because the acceleration is negative, minus gt .

You can test your integration technique to see that.

And the position as a function of time, remember, is just some constant value.

This is our constant y_0 .

And we have plus $v_{y0} t - \frac{1}{2} g t^2$ again doing integration.

Now, the horizontal equation of motion, the $x(t)$, because there is no acceleration in the horizontal direction, this is a constant value given by the initial value of the component of the velocity in the x direction and the position $x(t)$ is then equal to some initial position plus $v_{x0} t$.

And so those are our functions of time for the components of the velocity and the components of position.

Now, for our particular example, it's much easier.

Here we have $x_0 = 0$.

Notice that our object is starting at the origin.

And so that tells us that x as a function of time is just $v_{x0} t$.

I'm dropping the parentheses t .

Remember, that's not a product.

That's just a function of time.

And given this fact, that tells us that t is x divided by V_x nought, and now I can take this value of t and put it into our vertical equation and I get V_y is some initial value plus V_y nought x over V_x nought, substituting for t , minus $1/2 g x^2$ over V_x nought squared.

When we described projectile motion, we had an equation describing y as a function of t , an equation describing x as a function of t -- the horizontal motion and the vertical motion-- and here we have a separate equation, which is describing the y as a function of x .

Notice there is no time involved in this equation.

Now, let's try to look at graphically what we're seeing here.

So, for instance, when we look at a plot of the motion of y versus x , we can see the vertical component going up and down and we can see the horizontal component moving to the right while the object is following this trajectory.

And this is a parabolic trajectory of y as a function of x .

Now, if we wanted to actually plot out y as a function of t , then here we can look at the simulation where we're just looking at the y component and you can see that that motion too is a parabola.

However, what's crucial to look at is that it's y as a function of t and not y as a function of x .

Finally, we can look at the horizontal motion and again recall that when we looked at the trajectory as y as a function of x , you can see the horizontal motion component.

Let's plot that separately, x as a function of t .

And when we make that plot, you can see that this is just a linear equation in time where there is some initial condition, x nought, and so that's the plot of x as a function of t .

And so we see three separate representations of this motion.

y as a function of x , y as a function of t , and x as a function of t .