Dynamics - Newton's Laws

Kinematics:

- mathematical description of motion
  - definition of position, velocity and acceleration and their relationships and time evolution

Dynamics:

- why do bodies move as they do?
- What causes a body to accelerate?
  - we shall see that forces are responsible for acceleration

The properties of force and the relationships between force and acceleration are given by Newton's three Laws of Motion.

The First Law describes the natural state of motion of a body on which no forces are acting. The other two laws deal with the behavior of a body under the influence of forces.

Galileo (d 1642) began the modern development of the science of mechanics.

- Free fall objects have constant acceleration — independent of weight (mass).
- Leaning Tower of Pisa experiments.

Newton

1687 - Principia Mathematica

- Laws of motion
- Law of Universal Gravitation

- widely regarded as the greatest scientist ever lived.
- Credit for inventing calculus.

Newton (1642-1727) incorporated Galileo's findings into a formulation of dynamics which is called "Newtonian Mechanics".

Newton's laws are not a perfect description of how nature behaves.

It fails:

i) Atoms and nuclei → Quantum Theory

ii) Motion at High Velocities → Theory of Relativity

To study dynamics we must introduce two new concepts not required in kinematics:

- force
- mass

Forces

Force is a central concept in all of physics. It is a vector quantity – must describe the direction in which it acts as well as its magnitude.

In everyday use a force is used to describe a – push or a pull –

- stretched spring
- rubber bands
- taut cables
- ropes

} exert forces on objects

- liquids } forces on walls
- gases } buoyant forces

- sliding surfaces } frictional forces

Above are all exemples of "contact forces" — objects in direct contact with each other.

Some forces are "action-at-a-distence-forces"
- No direct contact required.
  - gravity
  - electromagnetic

<u>Fundamental Forces in Nature</u>

i) Gravitational
- Acts between all bodies having mass
- Always attractive
- Weakest force in nature
- Exchange particle ⟷ graviton?
- Do accelerating masses radiate energy?
  - Active area of research (gravity waves!)

ii) Electromagnetic
- Attraction or repulsion between electric charges
- Except for gravity, all other macroscopic forces have origin in em.
- Exchange particle is the photon
- Accelerating charges give off photons

iii) Strong Force

- A nuclear force acting between nucleons in the atom (between quarks)
- Nuclear glue
- Attractive or repulsive (complicated behavior)
- Exchange particles (mesons / gluons)

iv) Weak Force

- Seen in interactions between elementary particles
- Neutron decay: $n \rightarrow p + e^- + \bar{\nu}$ / β-decay.
- Field particle first observed experimentally three years ago (CERN) / Theoretically predicted many years ago.
- Exchange particles $W^{\pm}, Z^0$ ($Mc^2 \sim 80$ GeV)

v) Fifth Force

- PRLett Jan 6/86
- Efrös' Expt: Tested equality of gravitational and inertial masses for materials
- Author's contribute slight difference (?) in response to 5'th force. Relate 5'th force to Hypercharge → Baryon Number. (Neutrons + Protons) + Strangeness
- Range ~ few hundred meters (repulsive)
- Field Particle → Hyper photon?
- Many precise experiments underway.
- Existence not entirely conclusive at this time

[Baryons / Unit Mass is max in center of periodic table.]

The structure and behavior of the entire universe can be described by the action of four fundamental forces.

	Range	Relative strength	Field Particle
Strong Force (hadronic-nuclei)	10^{-15} m	1	mesons/gluons
Electromagnetic Force (charges)	∞	10^{-2}	photon
Weak Force (leptonic)	10^{-17} m	10^{-13}	$W^{\pm}$, Z^{0}
Gravitational Force (masses)	∞	10^{-38}	graviton ?

Relative strength 2-protons 10^{-15} m apart

Continuous theoretical work ongoing to develop a common unified description in terms of a single model for all forces. Such a model would make (hopefully) predictions which can be checked by experiment.

Gauge Theories have shown us how to combine

Electromagnetic } Electro-weak
Weak }

↳ At high energies the two forces have comparable strength and arise from a common description.

Nobel Prize : Weinberg / Salam / Glashow

- Work on Standard Electro-weak Model.
- Predicted existence of neutral currents
- Subsequently Z^0, $W^{\pm}$ Intermediate Vector Bosons
  - ↳ Discovered 81 GeV (CERN-1983)

GUTS : Grand Unified Theories

- Attempt to link
  - Strong
  - Electromagnetic } Electro-weak
  - Weak. }
- proton decay, etc ?
  - ↳ $\tau_p > 10^{32}$ yrs. [Some difficulty with this result for theory]

Newton's First Law

"Every body continues in its state of rest, or in uniform motion in a right line (straight line) unless it is compelled to change that state by forces impressed upon it."

$$\vec{F} = 0 \implies \vec{V} = \text{constant}$$

- fixed direction
- fixed magnitude

Most bodies we normally see, stop when left alone.

- presence of frictional forces
- air pucks, air-tracks give a hint of the pesistence of motion.

Particles in a vacuum, celestial bodies persist in a state of uniform motion.

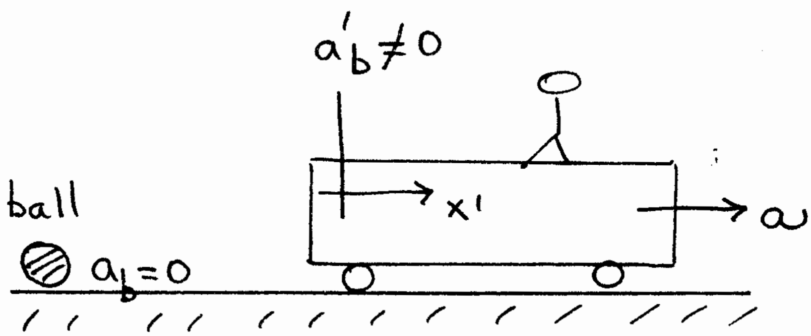
A body with no forces acting on it is called a <u>free body</u>.

Newton's First Law $\implies$ Law of Inertia

- Expresses the tendency of bodies to maintain their original state of motion.

Reference Frames

Law is not valid in all reference frames.
Valid in special frames
↳ Inertial Reference Frames



- Ball fixed in ground frame
- Train accelerates
- Ball appears to be spontaneously accelerated even though it has no forces acting on it.

Test for <u>inertial reference frame</u>:
- Take a free body (no forces acting)
- If it persists in a state of uniform motion
  ⇒ Inertial Reference Frame.

Any other reference frame in uniform translational motion relative to the first is also an inertial frame. A frame in accelerated motion relative to the first is not an inertial frame.

Earth based frame: Not inertial → Effects small

Centrepetal Acceleration (Rotation/Equator) = .034 m/s^2
(Revolution) =

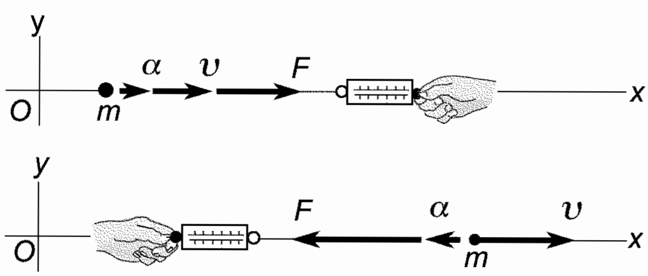
Newton's Second Law

- Established the relationship between force, acceleration, and mass.

"The change in motion (acceleration) is proportional to the motive force impressed; and is made in the direction of the right line (straight line) in which the force is impressed."

$$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$$

↑ more correct statement, will discuss later



- A law of nature
- Precise definition of force
- Valid only in inertial reference frames.
- $\vec{a} \propto \vec{F}$ and in the same direction.

Units:

<u>SI</u>

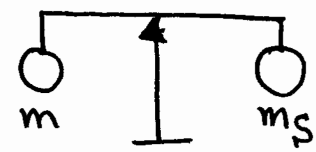
$$F = m\,a$$
$$[kg]\left[\frac{m}{s^2}\right]$$
$$= N \quad (\text{Newtons})$$

$$\left.\begin{array}{l} m = 1\,kg \\ a = 1\,m/s^2 \end{array}\right\} F = 1\,N.$$

<u>British</u> $\left.\begin{array}{l} m = 1\,\text{slug} \\ a = 1\,ft/s^2 \end{array}\right\} F = 1\ \text{pound}.$

Definition of Mass

- Standard of Mass (1kg)
- Compare masses by balancing.
- Not good in outer space/gravity
- ↳ Comparison of weights (see later)



Procedure

- Common force acting on standard and unknown masses.
- Under action of the force the bodies will accelerate relative to each other.
- Do experiment in an inertial frame.

m_s, a_s mass, acceleration of standard

m, a mass, acceleration of unknown.

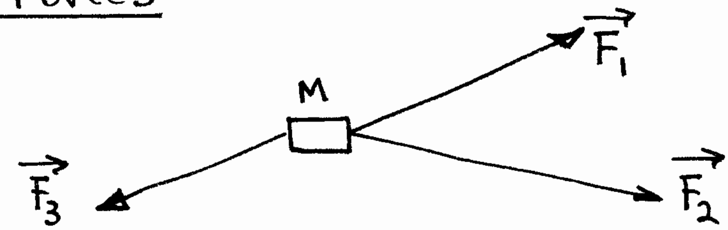
$$F = ma = m_s a_s$$

$$\therefore \boxed{\frac{m}{m_s} = \frac{a_s}{a}}$$

- Mass ⇒ a measure of resistance (inertial quantity) a body offers to changes in its velocity
- Large masses have small accelerations due to given force.
- Mass is an additive property of matter.

$$m = m_1 + m_2$$

Superposition of Forces



- Suppose several forces act on body.
- How does it move?

Principle of Superposition

If several forces $\vec{F}_1, \vec{F}_2, \vec{F}_3$ act simultaneously on a body then the acceleration is the same as that produced by the single force:

$$\vec{F}_{Net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

$\uparrow$ $\vec{F}_{Net}$:
- Net/resultant force.
- Vector sum of individual forces.

Law of Nature: Tested Experimentally.
e.g. Planetary motion.

Newton's Second Law

$$\vec{F}_{Net} = m\vec{a}$$

$$\vec{F_1} = m\vec{a_1}$$
$$\vec{F_2} = m\vec{a_2}$$
$$\vdots \qquad \vdots$$

Forces produce individual accelerations

$$\vec{F_1} + \vec{F_2} + \vec{F_3} + \cdots = m[\vec{a_1} + \vec{a_2} + \vec{a_3} + \cdots]$$

$$\vec{F}_{Net} = m\vec{a}$$

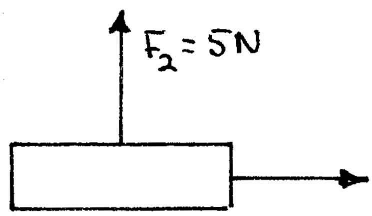
$$\vec{a} = \frac{\vec{F}_{Net}}{m}.$$

In Rectangular (x, y, z) Coordinate System.

$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$
$$\sum F_z = ma_z$$

Equations of Motion

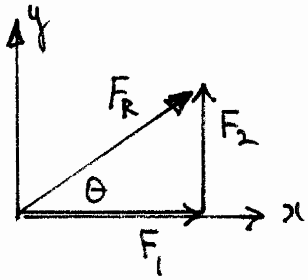
Example



$m = 7/3$ kg

What is acceleration of object?

Find resultant force F_R:
choose a coordinate system.



$$\vec{F_1} = 7\hat{x} + 0\hat{y}$$
$$\vec{F_2} = 0\hat{x} + 5\hat{y}$$

$$\vec{F_R} = \vec{F_1} + \vec{F_2} = 7\hat{x} + 5\hat{y}$$

$$F_R = \sqrt{7^2 + 5^2} = 8.60\,N$$

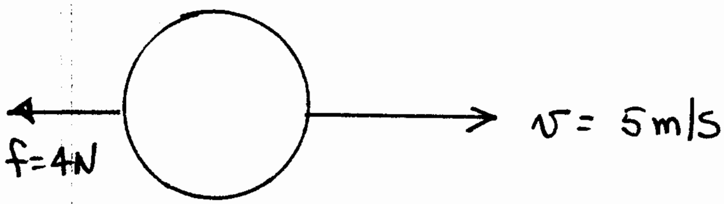
$$\tan\theta = \frac{F_{Ry}}{F_{Rx}} = \frac{5}{7} \qquad \theta = 35.5^\circ$$

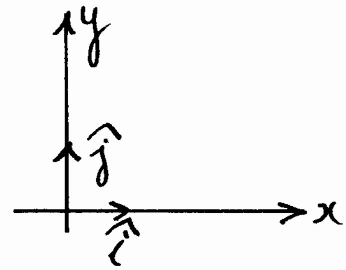
$$\vec{a} = \frac{F_R}{m} = \frac{7\hat{x} + 5\hat{y}}{7/3} = 3\hat{x} + \frac{15}{7}\hat{y}$$

[Same direction as $\vec{F_R}$]

$$a = \frac{8.60}{7/3} = 3.69\ m/s^2$$

Example





A flat disk of mass m = 2kg slides on a frozen lake with an initial speed v = 5m/s. Frictional force has a constant value f = 4N opposite to motion. How far does the disk slide before coming to rest?

Disk has no vertical motion — Ignore!!
Sum of all vertical forces equals zero.

Choose coordinate system :

$$-f\,\hat{i} = m\vec{a}$$

$$\vec{a} = \frac{-f}{m}\,\hat{i} = -\frac{4N}{2kg}\,\hat{i} = -2\hat{i}\ m/s^2$$

1-D Kinematics

$$v_f^2 - v_0^2 = 2as$$

↑ distance travelled.

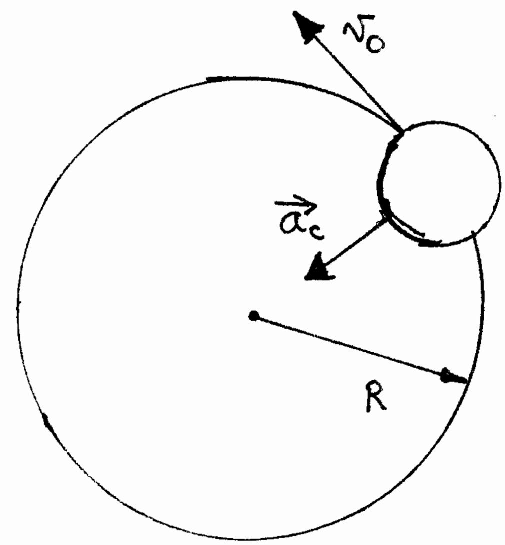
$v_f = 0$ when disk stops.

$$\therefore -v_0^2 = 2as$$

$$s = -v_0^2/2a = -5\times5/(2\times(-2)) = \underline{6.25\,m}$$

Example

A stone is whirled in a circular path at constant speed.



$m = 100g$
$R = 2m$
$v_0 = 3 m/s$

What force must be acting on the stone?

$$a_c = \frac{v^2}{R} = \frac{(3\ m/s)^2}{2} = 4.5 m/s^2 \quad \text{[Towards center]}$$

$$\sim \tfrac{1}{2} \text{ gee}$$

$$F_c = ma_c = (0.1)\, 4.5 = 0.45 N \quad \text{[Towards center]}$$

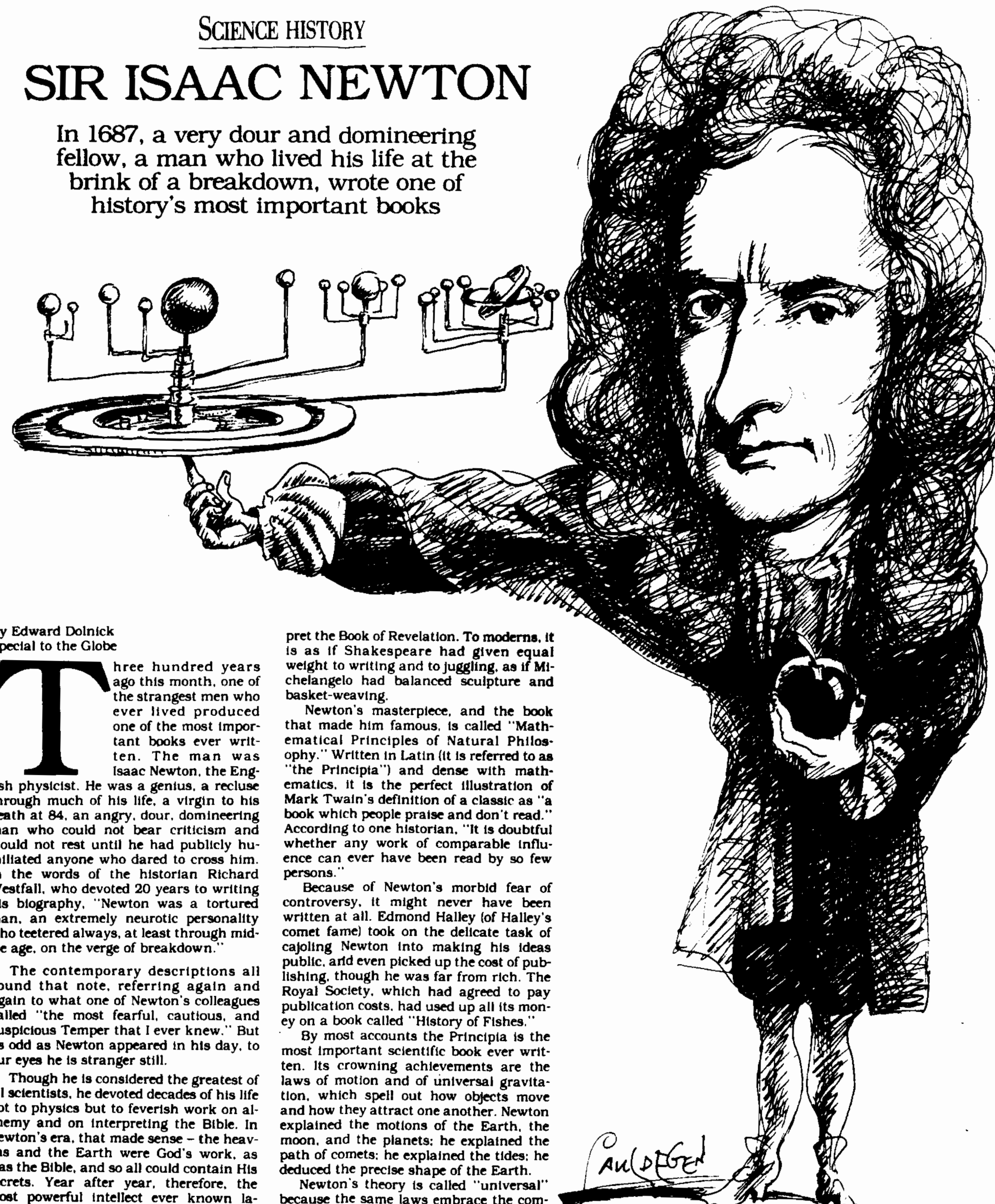
Sci-Tech
SCIENCE · HEALTH · TECHNOLOGY

Deaths 42

SCIENCE HISTORY

SIR ISAAC NEWTON

In 1687, a very dour and domineering fellow, a man who lived his life at the brink of a breakdown, wrote one of history's most important books



By Edward Dolnick
Special to the Globe

Three hundred years ago this month, one of the strangest men who ever lived produced one of the most important books ever written. The man was Isaac Newton, the English physicist. He was a genius, a recluse through much of his life, a virgin to his death at 84, an angry, dour, domineering man who could not bear criticism and would not rest until he had publicly humiliated anyone who dared to cross him. In the words of the historian Richard Westfall, who devoted 20 years to writing his biography, "Newton was a tortured man, an extremely neurotic personality who teetered always, at least through middle age, on the verge of breakdown."

The contemporary descriptions all sound that note, referring again and again to what one of Newton's colleagues called "the most fearful, cautious, and suspicious Temper that I ever knew." But as odd as Newton appeared in his day, to our eyes he is stranger still.

Though he is considered the greatest of all scientists, he devoted decades of his life not to physics but to feverish work on alchemy and on interpreting the Bible. In Newton's era, that made sense – the heavens and the Earth were God's work, as was the Bible, and so all could contain His secrets. Year after year, therefore, the most powerful intellect ever known labored to turn lead into gold and to interpret the Book of Revelation. To moderns, it is as if Shakespeare had given equal weight to writing and to juggling, as if Michelangelo had balanced sculpture and basket-weaving.

Newton's masterpiece, and the book that made him famous, is called "Mathematical Principles of Natural Philosophy." Written in Latin (it is referred to as "the Principia") and dense with mathematics, it is the perfect illustration of Mark Twain's definition of a classic as "a book which people praise and don't read." According to one historian, "It is doubtful whether any work of comparable influence can ever have been read by so few persons."

Because of Newton's morbid fear of controversy, it might never have been written at all. Edmond Halley (of Halley's comet fame) took on the delicate task of cajoling Newton into making his ideas public, and even picked up the cost of publishing, though he was far from rich. The Royal Society, which had agreed to pay publication costs, had used up all its money on a book called "History of Fishes."

By most accounts the Principia is the most important scientific book ever written. Its crowning achievements are the laws of motion and of universal gravitation, which spell out how objects move and how they attract one another. Newton explained the motions of the Earth, the moon, and the planets; he explained the path of comets; he explained the tides; he deduced the precise shape of the Earth.

Newton's theory is called "universal" because the same laws embrace the com-

NEWTON, Page 40

Paul Degen illustration

Isaac Newton: dour genius

■ NEWTON
Continued from Page 39

monplace and the cosmos. They explain the path of a pen accidentally knocked off a desk, the arc of a baseball lofted to the outfield, the orbit of a planet moving round the sun, the circling of a pair of stars round each other, and the shape of a galaxy made up of countless billions of stars.

"The main idea Newton contributed," said Dudley Shapere, a philosopher of science at Wake Forest University, "is that there are laws of nature – the universe operates according to iron-bound, deterministic laws."

Moreover, Shapere pointed out, Newton showed that man's intellect can grasp those laws and can find order in what had seemed chaos. The intellectual confidence born of that triumph helped inspire the Age of Reason, and it moved Enlightenment thinkers to set about hunting for the laws governing society and other human affairs. "People tried to come up with mathematical theories on how various social forces interact," Shapere said. "People were waiting for the Newton of this, and the Newton of that."

The theory of gravity was revolutionary, more startling in its day than Einstein's theories in his. Three hundred years of intoning Newton's name have dimmed the surprise – every child knows that apples fall "because of gravity" – but contemporaries found the idea hard to grasp. Before Newton, the accepted view was Descartes's. In Shapere's summary, "the universe was filled with something like a fluid, in which the planets were carried around like corks in a bathtub."

Newton asked that thinkers abandon that easy-to-picture notion, and replace it with a mathematical abstraction. Newton's gravity is so mysterious – it reaches instantly across empty space, across unimaginable distances – that Descartes's followers condemned it as occult. How can bodies like the Earth and sun act on each other without touching and without anything in between? To highlight the mystery, one modern physicist has calculated that "a steel cable of a thickness equalling the diameter of the Earth would not be strong enough to hold the Earth in its orbit."

Instant celebrity

The Principia made Newton famous at once, but the theory of gravitation did not win over all its critics for decades. Even so, Newton's contemporaries and the generations immediately after hailed him as almost superhuman. "Nature and Nature's laws lay hid in Night; God said 'Let Newton be!' and all was light," wrote Alexander Pope. For physicists, who can follow mathematical arguments in the way that musicians can read a score, the awe that Newton inspired in his contemporaries remains alive.

University of Chicago astrophysicist Subramanyan Chandrasekhar, who won a Nobel Prize for work that predicted the existence of black holes, has recently gone through the Principia in detail. "During the past year, I've taken proposition after proposition, written out my own proof, and then compared it with Newton's. In every case, his proofs are incredibly concise, there is not a superfluous word. The style is imperial, just written down as if the insights come from Olympus.

Courtesy/Smithsonian Institution

Japanese woodcut in 1800s. Inscription reads: "Isaac Newton was a profound scholar who was not boastful."

"And," Chandrasekhar went on, "Newton did it all in 18 months, an incredibly short time. He wrote mathematics the way Mozart wrote symphonies – it just came naturally." Einstein's opinion was similar. "Nature to [Newton] was an open book, whose letters he could read without effort."

"If you take great scientists," Chandrasekhar said, "even though they made discoveries that one could not have made oneself, one can imagine making them – people say, 'I could have done that but I was just stupid,' or such things. Normal scientists can think of greater men, and it is not difficult to imagine doing what they did. But I don't think it's possible for any scientist to imagine what it would have been like to be Newton."

How to explain that genius? No one knows.

Newton was born Christmas Day, 1642, to a family that Westfall, the University of Indiana historian, called "wholly without distinction and wholly without learning." Newton's father could not sign his name.

As a schoolboy, he was bright but showed no sign of genius. At 17, he was called home from school to manage his mother's farm. He made a botch of it, his schoolmaster convinced his mother that Newton had talent it would be a shame to bury, and he went off to Cambridge University. He had never been more than ten miles from home.

Farmboy to scientist

Within six years, teaching himself from books, the farmboy had become the most important scientist alive. Only one important thinker, a Cambridge mathematician, even knew his name. "The young man not yet 24," Westfall wrote, "without benefit of formal instruction, had become the leading mathematician of Europe. And the only one who really mattered, Newton himself, understood his position clearly enough. He had studied the acknowledged masters. He knew the limits they could not surpass. He had outstripped them all, and by far."

The peak years, according to Newton's recollection in his old age, were 1665 and 1666. England was ravaged by plague, Cambridge was closed, and Newton had returned to his mother's farm. While there, he invented the mathematical field called calculus, he did much of the work on the theory of gravity that he developed years later in the Principia (an apple really did fall), and he made the revolutionary discovery that white light is composed of rays of different colors. To scholars, the years are the "anni mirabiles," the miracle years.

Westfall said, in an interview, that though he had devoted 20 years to his biography, Newton had come to seem more and more remote. "The more I learned, the more he seemed to swell and the more I realized how far he was from me," he said. "It's not an issue of familiarity breeding contempt, but quite the contrary – familiarity teaching what a gulf there is."

The secret, according to John Maynard Keynes, the economist and a Newton scholar, is that "Newton was capable of greater sustained mental effort than any man, before or since." Once he began on a problem, he worked relentlessly, forgetting to eat or sleep. "His cat," a Cambridge roommate tells us, "grew very fat on the food he left standing on his tray."

No distractions were permitted. Newton had no interest in recreation or exercise or company. An assistant said he had seen Newton laugh only once in five years, when someone asked him what good it did to study Euclid.

Only once, shortly before his death, did Newton permit a peek behind the austere mask. "I don't know what I may seem to the world," he recalled to a friend, "but, as to myself, I seem to have been only like a boy playing on the seashore and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

Edward Dolnick is a freelance writer living in Maryland.