

# 22.615 Spring 2007

## Homework 4

Distributed: April 19, 2007

Due: April 26, 2007

1. Problem 8.6
2. Problem 8.7
3. A cylindrical model of LDX consists of a cylindrical Z-pinch with a hard core, current carrying conductor located on the axis  $r = 0$ . The radius of the hard core is  $r = r_c$ . It carries a known current  $I = I_c$ . The outer boundary of the configuration is a perfectly conducting cylindrical wall of radius  $r = r_w$  where  $r_w \gg r_c$ . The pressure profile is given by

$$p(r) = K \frac{r^2 - r_c^2}{(r^2 + r_c^2)^3}$$

The geometry is illustrated below. For algebraic simplicity assume the ratio of specific heats is  $\gamma = 2$ . Calculate the MHD  $\beta$  limits for stability against the  $m = 0$  and  $m = 1$  modes. For  $\beta$  use the definition

$$\beta = \frac{2\mu_0 \bar{p}}{B_\theta^2(r_w)}$$
$$\bar{p} = \frac{2}{r_w^2 - r_c^2} \int_{r_c}^{r_w} p r dr$$

Wherever convenient simplify your results using the approximation  $r_w \gg r_c$ .