22.312 ENGINEERING OF NUCLEAR REACTORS

Wednesday, December 17th, 2008, 9:00am-12:00 pm

FINAL EXAM (SOLUTION)

PART A (20%) CLOSED BOOK

Flow boiling: boiling in a flow system. It occurs in both the PWR and BWR cores under normal operating conditions.

Subcooled boiling: boiling of a fluid whose bulk temperature is below the saturation temperature. It is the only type of boiling that takes place in the PWR core (hot channel) under normal operating conditions. It also occurs in the lower section of all BWR fuel assemblies.

Nucleate boiling: liquid-vapor phase transition via bubble nucleation, which typically occurs at microcavities present on the heated surface. It occurs in both PWR and BWR cores.

Film boiling: boiling in the presence of a continuous vapor film in contact with the heated surface. Does not occur in either PWR or BWR core under normal operating conditions.

Pool boiling: boiling of a fluid that is macroscopically stagnant. Does not occur in either PWR or BWR core under normal operating conditions.

Transition boiling: on the boiling curve it is the region between the DNB and the Leidenfrost points. Does not occur in either PWR or BWR core under normal operating conditions.

Departure from nucleate boiling (DNB): transition from the nucleate boiling regime to the film boiling regime. It results in a large sudden reduction of the heat transfer coefficient. This is the boiling crisis relevant to the PWR core situation. However, it does not occur in the PWR *under normal operating conditions*.

Dryout: disappearance of the liquid film in annular flow. It results in a large sudden reduction of the heat transfer coefficient. This is the boiling crisis relevant to the BWR core situation. However, it does not occur in the BWR *under normal operating conditions*.

Critical Flow: the maximum flow rate (or mass flux) that a (compressible) fluid can attain as it is discharged from a high-pressure component through a fixed-geometry nozzle. Does not occur in either PWR or BWR core under normal operating conditions.

ii)
For a given fluid, and fixed operating pressure and mass flow rate, nucleate boiling has the highest htc. The htc is generally higher at subcooled boiling conditions than saturated boiling conditions because the temperature difference driving the heat transfer is higher. Forced convection of a single-phase liquid comes next, and film boiling last, because in film boiling the surface is in contact with a slow moving vapor film, which has a very poor thermal conductivity. Therefore the correct htc ranking is:

Subcooled nucleate boiling: 1
Saturated nucleate boiling: 2
Single-phase (liquid) convection: 3
Film boiling: 4

PART B (80%)

Problem 1 (40%) – PWR fuel pin with a thin gap and no fill gas

i)
The pellet radial displacement is $u_f=0.005\ R_f \approx 20.5\ \mu m$, where $R_f=4.1$ mm is the as-manufactured pellet radius. Thus, the clad radial displacement is $u_c= u_f - t_{gap} \approx 10.5\ \mu m$, where $t_{gap} \approx 10\ \mu m$ is the as-manufactured gap thickness. Using the thin-shell theory, the principal stresses in the clad are:

$$\begin{aligned} \sigma_r &= -(P_i+P_o)/2 \qquad (1) \\ \sigma_\theta &= (P_i-P_o)R_c/t_c \\ \sigma_z &= -P_oR_c/(2t_c) \end{aligned}$$

where $t_c=0.4$ mm and $R_c=4.11$ mm are the as-manufactured thickness and inner radius of the clad, respectively, $P_o=15.5$ MPa, and P_i represents the force per unit area exerted by the pellet on the clad. (Note that there is no fill gas, so no internal pressure shows up in the equations for the stresses; also note that P_i is a radial force, so it does not show up in the expression for the axial stress, σ_z)

Hooke's law for the tangential strain is:

$$\varepsilon_\theta = 1/E\ [\sigma_\theta - \nu(\sigma_z+\sigma_r)] \qquad (2)$$

Substituting Eqs (1) into (2), and recognizing that $\varepsilon_\theta=u_c/R_c\approx 0.0026$, one gets one equation in the only unknown P_i:

$$\varepsilon_\theta = \frac{1}{E}\left\{(P_i - P_o)\frac{R_c}{t_c} - \nu\left[-P_o\frac{R_c}{2t_c} - \frac{P_i+P_o}{2}\right]\right\}$$

Which can be solved for P_i to give:

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$$P_i = \frac{E\varepsilon_\theta t_c + P_o[R_c - \nu(R_c + t_c)/2]}{R_c + \nu t_c/2} \approx 31.9 \text{ MPa}$$

ii)
Using Eqs. (1), one gets $\sigma_r \approx -23.7$ MPa, $\sigma_\theta \approx 168.2$ MPa and $\sigma_z \approx -79.6$ MPa.
The Tresca criterion is:

$$\max\{|\sigma_r - \sigma_\theta|, |\sigma_r - \sigma_z|, |\sigma_z - \sigma_\theta|\} \approx 247.8 \text{ MPa} > S_y = 200 \text{ MPa}$$

Therefore, the criterion is not satisfied and the stresses are **not** acceptable.

iii)
The idea is unattractive for the following reasons:
- As shown in part ‘ii’, the fuel-clad mechanical interaction is too severe, and will likely result in rupture of the clad
- It is questionable that the gap thermal resistance would be in fact reduced with this design, because the elimination of the fill gas means that, prior to clad-fuel contact, heat transfer across the gap can occur only by radiation, which is a poor heat transfer mechanism.
- Depending on the service lifetime of the fuel pin, a thin clad may not be acceptable from a corrosion viewpoint

The merits of the idea are:
- Fewer parasitic neutron absorptions in the clad, which is thinner
- Elimination of the fill gas, thus, possibly, a reduction in manufacturing costs

Problem 2 (40%) – Thermal-Hydraulic Analysis of a Boiling Channel

i)
The mass flow rate ($\dot{m}$=GA) and power ($\dot{Q} = q''\pi D_i L$) are related by the energy balance:

$$GAC_{p,f}(T_{out} - T_{in}) = q''\pi D_i L \qquad (3)$$

Where T_{in}=200°C, T_{out}=250°C, $A=\pi/4(D_o^2-D_i^2)=2.36\times10^{-4}$ m^2 is the flow area, and L=1 m.

Since the heat flux is uniform, the boiling crisis will occur at the channel outlet. As the outlet temperature is below the saturation temperature, the thermal-hydraulic conditions in the channel are subcooled, for which the relevant boiling crisis is departure from nucleate boiling (vs dryout). Therefore, the appropriate correlation for this problem is Tong-68.

$$q''_{DNB} = K_{Tong}\frac{G^{0.4}\mu_f^{0.6}h_{fg}}{D_e^{0.6}} \tag{4}$$

Where $D_e=D_o-D_i=1$ cm, $D_i=1$ cm, $D_o=2$ cm,

$K_{Tong} = [1.76-7.433x_{e,out}+12.222x_{e,out}^2]\cdot\left[1-\frac{52.3+80x_{e,out}-50x_{e,out}^2}{60.5+(10P)^{1.4}}\right] \approx 1.758$,

$x_{e,out} = \frac{C_{p,f}(T_{out}-T_{sat})}{h_{fg}} \approx -0.0427$ is the equilibrium quality at the exit, and P=5 MPa.

When the boiling crisis occurs, it is $q'' = q''_{DNB}$. Substituting Eq. (4) into Eq. (3) and solving for the mass flux G, we get:

$$G = \left[K_{Tong}\frac{\mu_f^{0.6}h_{fg}\pi D_i L}{C_{p,f}(T_{out}-T_{in})AD_e^{0.6}}\right]^{1/0.6} \approx 2048 \text{ kg/m}^2\text{s}$$

Therefore, the mass flow rate is $\dot{m}$=GA≈0.482 kg/s. Substituting G into Eq. (4), we get q''_{DNB}≈3.84 MW/m^2. Finally the channel power is $\dot{Q} = q''_{DNB}\pi D_i L \approx 120.6$ kW.

ii)

The energy balance for this situation is:

$$\dot{m}^*[x_{e,out}^* h_{fg} + C_{p,f}(T_{sat}-T_{in}^*)] = \dot{Q}^* \quad \Rightarrow \quad x_{e,out}^* = \frac{\frac{\dot{Q}^*}{\dot{m}^*} - C_{p,f}(T_{sat}-T_{in}^*)}{h_{fg}} \approx 0.0488$$

where T_{in}^*=260°C, $\dot{m}^*$=0.5 kg/s, and $\dot{Q}^*$=50 kW. Since the mixture is saturated, thermal equilibrium can be assumed, and thus the flow quality is equal to the equilibrium quality.

iii)

The void fraction is related to quality and slip ratio by the fundamental relation of two-phase flow, $\alpha = \frac{1}{1+\frac{\rho_g}{\rho_f}\cdot S\cdot\frac{1-x}{x}}$. As the flow rate, pressure, power and inlet temperature are fixed (as per the problem statement), the quality is also fixed. However, because of buoyancy it is S>1 in upward flow and S<1 in downward flow. Thus, the void fraction would be lower in upward flow than in downward flow.

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Fall 2015