22.101 Applied Nuclear Physics (Fall 2006)

Lecture 23 (12/6/06)

Nuclear Reactions: Energetics and Compound Nucleus

References:

W. E. Meyerhof, *Elements of Nuclear Physics* (McGraw-Hill, New York, 1967), Chap 5.

A. Foderaro, *The Elements of Neutron Interaction Theory* (MIT Press, Cambridge, 1971), Chap. 6.

Among the many models of nuclear reactions there are two opposing basic models which we have encountered. These are (i) the compound nucleus model proposed by Bohr (1936) in which the incident particle interacts strongly with the entire target nucleus, and the decay of the resulting compound nucleus is *independent* of the mode of formation, and (ii) the independent particle model in which the incident particle interacts with the nucleus through an effective averaged potential. A well-known example of the former is the liquid drop model, and three examples of the latter are a model proposed by Bethe (1940), the nuclear shell model with spin-orbit coupling (cf. Chap 9), and a model with a complex potential, known as the optical model, proposed by Feshbach, Porter and Weisskopf (1949). Each model describes well some aspects of what we now know about nuclear structure and reactions, and not so well some of the other aspects. Since we have already examined the nuclear shell model is some detail, we will focus in the brief discussion here on the compound nucleus model, which in some sense may be considered to be in the same class as the liquid drop model. As we will see, this approach is well suited for describing reactions which show single resonance behavior, a sharp peak in the energy variation of the cross section. In contrast, the optical model, which we will not discuss in this course, is good for gross behavior of the cross section (in the sense of averaging over an energy interval).

Energetics

Before discussing the compound nucleus model we first summarize the energetics of nuclear reactions. We recall the Q-equation introduced in the study of neutron interactions (cf. Chap 15) for a general reaction depicted in Fig. 21.1,
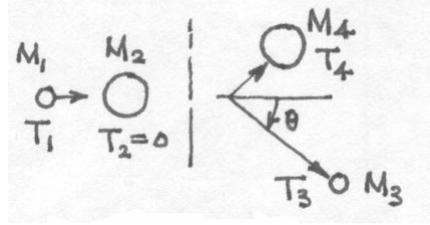
Fig. 21.1. A generic two-body nuclear reaction with target nucleus at rest.

$$Q = T_3\left(1 + \frac{M_3}{M_4}\right) - T_1\left(1 - \frac{M_1}{M_4}\right) - \frac{2}{M_4}\left(M_1 M_3 T_1 T_3\right)^{1/2}\cos\theta \qquad (21.1)$$

Since $Q = T_3 + T_4 - T_1$, the reaction can take place only if M_3 and M_4 emerge with positive kinetic energies (all kinetic energies are LCS unless specified otherwise),

$$T_3 + T_4 \geq 0, \quad \text{or} \quad Q + T_1 \geq 0 \qquad (21.2)$$

We will see that this condition, although quite reasonable from an intuitive standpoint, is *necessary but not sufficient* for the reaction to occur.

We have previously emphasized in the discussion of neutron interaction that a fraction of the kinetic energy brought in by the incident particle M_1 goes into the motion of the center-of-mass and is therefore not available for reaction. To see what is the energy available for reaction we can look into the kinetic energies of the reacting particles in CMCS. First, the kinetic energy of the center-of-mass, in the case where the target nucleus is at rest, is

$$T_o = \frac{1}{2}(M_1 + M_2)v_o^2 \qquad (21.3)$$

where the center-of-mass speed is $v_o = [M_1/(M_1 + M_2)]v_1$, v_1 being the speed of the incident particle. The kinetic energy available for reaction is the kinetic energy of the incident particle T_1 minus the kinetic energy of the center-of-mass, which we denote as T_i,

$$T_i = T_1 - T_o = \frac{M_2}{M_1 + M_2}T_1$$

$$= \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2v_o^2 \qquad (21.4)$$

The second line in (21.4) shows that T_i is also the sum of the kinetic energies of particles 1 and 2 in CMCS (we follow the same notation as before in using capital letters to denote velocity in CMCS). In addition to the kinetic energy available for reaction, there is also the rest-mass energy available for reaction, as represented by the Q-value. Thus the total energy available for reaction is the sum of T_i and Q. A *necessary and sufficient* condition for reaction is therefore

$$E_{avail} = Q + T_i \geq 0 \qquad (21.5)$$

We can rewrite (21.5) as

$$T_1 \geq -Q\frac{M_1 + M_2}{M_2} \qquad (21.6)$$

If $Q > 0$, (21.6) is always satisfied, which is expected since the reaction is exothermic. For $Q < 0$, (21.6) shows that the threshold energy, the minimum value of the incident particle kinetic energy for reaction, is greater than the rest-mass deficit. The reason for needing more energy than the rest-mass deficit, of course, is that energy is needed for the kinetic energy of the center-of-mass.

At threshold, $Q + T_i = 0$. So M_3 and M_4 both move in LCS with speed v_o (V_3 and $V_4 = 0$). At this condition the total kinetic energies of the reaction products is

$$(T_3 + T_4)_{thres} = \frac{1}{2}(M_3 + M_4)v_o^2 \tag{21.7}$$

Since we have $M_3V_3 = M_4V_4$ from momentum conservation, we can say in general

$$Q + T_i = \frac{1}{2}M_3V_3^2 + \frac{1}{2}\frac{(M_3V_3)^2}{M_4} \tag{21.8}$$

With Q and T_1 given, we can find V_3 from (21.8) but not the direction of $\underline{V}_3$. It turns out that for T_1 just above threshold of an endothermic reaction, an interesting situation exists where at a certain scattering angle in LCS one can have *two different* kinetic energies in LCS. A situation which violates the one-to-one correspondence between scattering angle and outgoing energy. How can this be? The answer is that the one-to-one correspondence that we have spoken of in the past applies *strictly only* to the relation between the kinetic energy T_3 and the scattering angle in CMCS (and not with the scattering angle in LCS). Fig. 21.2 shows how this special situation, which corresponds to the double-valued solution to the Q-equation, can arise.
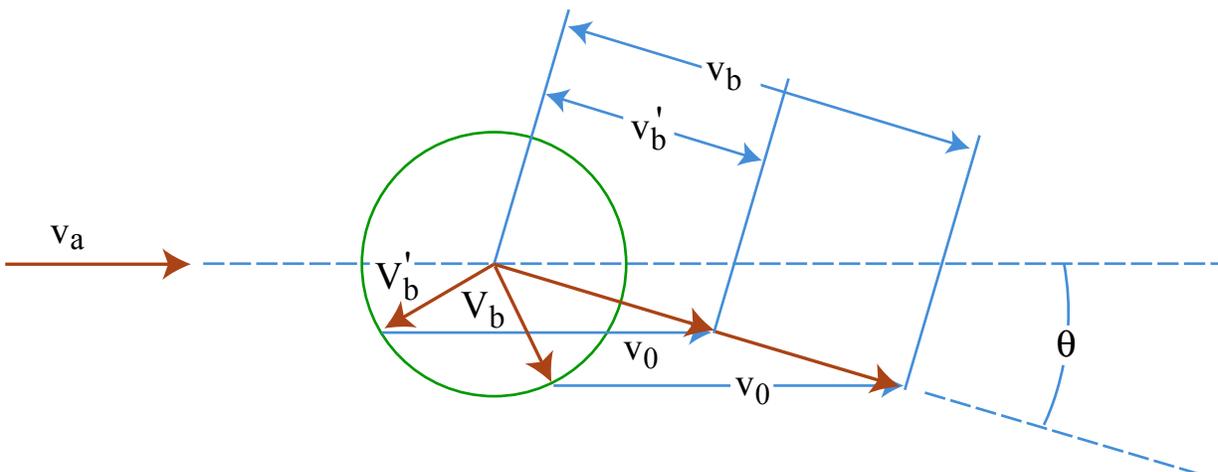
Figure by MIT OCW.

Fig. 21.2. A special condition where a particle can be emitted at the same angle but with two different kinetic energies, which can occur only in LCS.

Energy-Level Diagrams for Nuclear Reactions

We have seen in the previous chapter how the various energies involved in nuclear decay can be conveniently displayed in an energy-level diagram. The same argument applies to nuclear reactions. Fig. 21.3 shows the energies involved in an
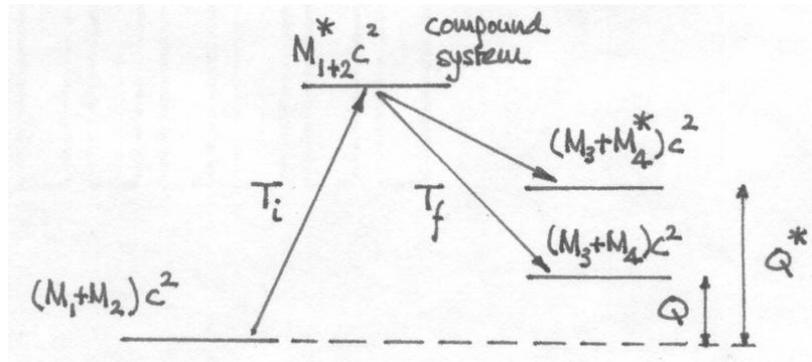
Fig. 21.3. Energy-level diagram for an endothermic reaction.

endothermic reaction. In this case the reaction can end up in two different states, depending on whether the product nucleus M_4 is in the ground state or in an excited state (*). T_f denotes the kinetic energy of the reaction products in CMCS, which one can write as

$$T_f = Q + T_i$$

$$= \frac{1}{2} M_3 V_3^2 + \frac{1}{2} M_4 V_4^2 \qquad (21.9)$$

Since both T_i and T_f can be considered kinetic energies in CMCS, one can say that the kinetic energies appearing in the energy-level diagram should be in CMCS.

Compound Nucleus Reactions

The concept of compound nucleus model for nuclear reactions is depicted in Fig. 21.4. The idea is that an incident particle reacts with the target nucleus in two ways, a

scattering that takes place at the surface of the nucleus which is, properly speaking, not a reaction, and a reaction that takes place after the incident particle has entered into the nucleus. The former is what we have been studying as elastic scattering, it is also known
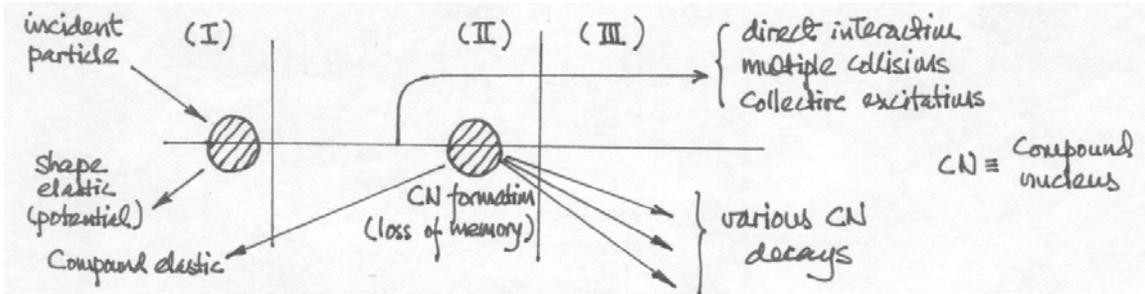
Fig. 21.4. Compound nucleus model of nuclear reaction – formation of compound nucleus (CN) and its subsequent decay are assumed to be decoupled.

as shape elastic or potential scattering. This process is always present in that it is allowed under any circumstances, we will leave it aside for a while in the following discussion. The interaction which takes place after the particle has penetrated into the target nucleus can be considered an absorption process, leading to the formation of a compound nucleus (this need not be the only process possible, the others can be direct interaction, multiple collisions, and collective excitations, etc.). This is the part that we will now consider briefly.

In neutron reactions the formation of compound nucleus (CN) is quite likely at incident energies of ~ 0.1 – 1 Mev. Physically this corresponds to a large reflection coefficient in the inside edge of the potential well. Once CN is formed it is assumed that it will decay in a manner that is independent of the mode in which it was formed (complete loss of memory). This is the basic assumption of the model because one can then treat the formation and decay as two separate processes. The approximation of two processes being independent of each other is expressed by writing the interaction as a two-stage reaction,

$$a + X \rightarrow C^* \rightarrow b + Y$$

the asterisk indicating that the CN is in an excited state. The first arrow denotes the formation stage and the second the decay stage. For this reaction the cross section $\sigma(a,b)$ may be written as

$$\sigma(a,b) = \sigma_C(T_i)P_b(E) \tag{21.10}$$

where $\sigma_C(T_i)$ is the cross section for the CN formation at kinetic energy T_i, which is the available kinetic energy for reaction as discussed above, and $P_b(E)$ is the probability that the CN at energy level E will decay by emission of particle b. It is implied that σ_C and P_b are to be evaluated separately since the formation and decay processes are decoupled. The energy-level diagram for this reaction is shown in Fig. 21.5 for an endothermic reaction $(Q < 0)$. Notice that E is the CN excitation and it is measured relative to
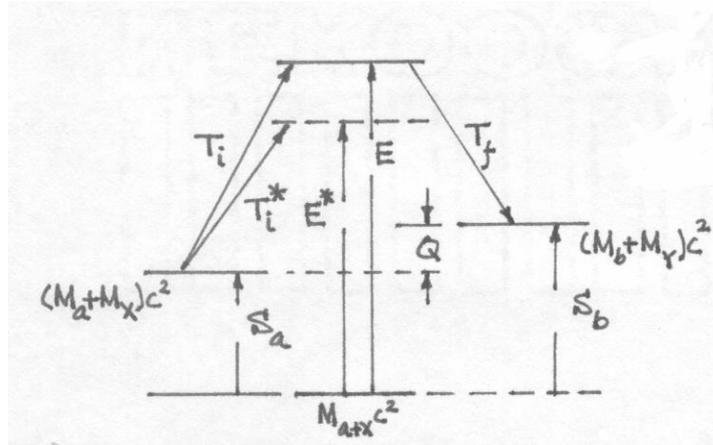
Fig. 21.5. Energy-level diagram for the reaction $a + X \rightarrow b + Y$ via CN formation and decay.

the rest-mass energy of the nucleus (a+X). If this nucleus should have an excited state (a virtual level) at E* which is close to E, then one can have a resonance condition. If the incoming particle a should have a kinetic energy such that the kinetic energy available for reaction has the value T_i*, then the CN excitation energy matches an excited level of the

nucleus (a+X), E = E*. Therefore the CN formation cross section $\sigma_C(T_i)$ will show a peak in its variation with T_i as an indication of a resonance reaction.

The condition for a reaction resonance is thus a relation between the incoming kinetic energy and the rest-mass energies of the reactants. Fig. 21.5 shows that this relation can be stated as Ti = Ti*, or

$$(M_a + M_X)c^2 + T_i^* = M_{a+X}c^2 + E* \qquad (21.11)$$

Each virtual level E* has a certain energy width, denoted as Γ, which corresponds to a finite lifetime of the state (level), $\tau = \hbar/\Gamma$. The smaller the width means the longer the lifetime of the level.

The cross section for CN formation has to be calculated quantum mechanically [see, for example, Burcham, *Nuclear Physics*, p. 532, or for a complete treatment Blatt and Weisskopf, *Theoretical Nuclear Physics*, pp. 398]. One finds

$$\sigma_C(T_i) = \pi \lambdabar^2 g_J \frac{\Gamma_a \Gamma}{\left(T_i - T_i^*\right)^2 + \Gamma^2/4} \qquad (21.12)$$

where $g_J = \dfrac{2J+1}{(2I_a+1)(2I_X+1)}$ and $\underline{J} = \underline{I}_a + \underline{I}_X + \underline{L}_a$. In this expression $\lambdabar$ is the reduced wavelength (wavelength/2π) of particle a in CMCS, $\underline{J}$ is the total angular momentum, the sum of the spins of particles a and X and the orbital angular momentum associated with particle a (recall particle X is stationary), Γ_a is the energy width (partial width) for the incoming channel a+X, and Γ (without any index) is the total decay width, the sum of all partial widths. The idea here is that CN formation can result from a number of channels, each with its own partial width. In our case the channel is reaction with particle a, and the partial width Γ_a is a measure of the strength of this channel. Given our relation (21.11) we can also regard the CN formation cross section to be a

function of the excitation energy E, in which case $\sigma_C(E)$ is given by (21.12) with $\left(E - E*\right)^2$ replacing the factor $\left(T_i - T_i^*\right)^2$ in the denominator.

To complete the cross section expression (21.10) we need to specify the probability for the decay of the compound nucleus. This is a matter that involves the excitation energy E and the decay channel where particle b is emitted. Treating this process like radioactive decay, we can say

$$P_b(E) = \Gamma_b(E)/\Gamma(E) \tag{21.13}$$

where $\Gamma(E) = \Gamma_a(E) + \Gamma_b(E)$ + width of any other decay channel allowed by the energetics and selective rules. Typically one includes a radiation partial width Γ_γ since gamma emission is usually an allowed process. Combining (21.12) and (21.13) we have the cross section for a resonance reaction. In neutron reaction theory this result is generally known as the Breit-Wigner formula for a single resonance. There are two cross sections of interest to us, one for neutron absorption and another for neutron elastic scattering. They are usually written as

$$\sigma(n,\gamma) = \pi \hat{\lambda}^2 g_J \frac{\Gamma_n \Gamma_\gamma}{\left(T_i - T_i^*\right)^2 + \Gamma^2/4} \tag{21.14}$$

$$\sigma(n,n) = 4\pi a^2 + \pi \hat{\lambda}^2 g_J \frac{\Gamma_n^2}{\left(T_i - T_i^*\right)^2 + \Gamma^2/4} + 4\pi \hat{\lambda} g_J a \Gamma_n \frac{\left(T_i - T_i^*\right)}{\left(T_i - T_i^*\right)^2 + \Gamma^2/4} \tag{21.15}$$

In $\sigma(n,n)$ the first term is the potential scattering contribution, what we had previously called the s-wave part of elastic scattering, with *a* being the scattering length. The second term in (21.15) is the compound elastic scattering contribution. It is the term responsible for the peak behavior of the cross section. The last term represents the interference between potential scattering and resonant scattering. Notice the interference is *destructive* at energy below the resonance and *constructive* above the resonance. Below

we will see that these are characteristic signatures of the presence of interference in a resonance reaction. In Fig. 21.6 we show schematically the energy behavior of the absorption cross section in the form of a resonance peak. Below the peak the cross section varies like $1/v$ as can be deduced from (21.14) by noting the energy dependence of the various factors, along with $\Gamma_n \sim \sqrt{T}$, and $\Gamma_\gamma \sim$ constant. Notice also the full width at half maximum is governed by the total decay width Γ. Fig. 21.7 shows a well-known absorption peak in Cd which is widely used as an absorber of low-energy neutrons. One can see the resonance behavior in both the total cross section, which is dominated by absorption, and the elastic scattering cross section in the inset.
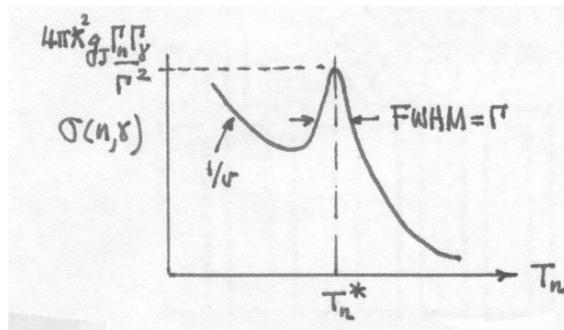
Fig. 21.6. Schematic of Breit-Wigner resonance behavior for neutron absorption.
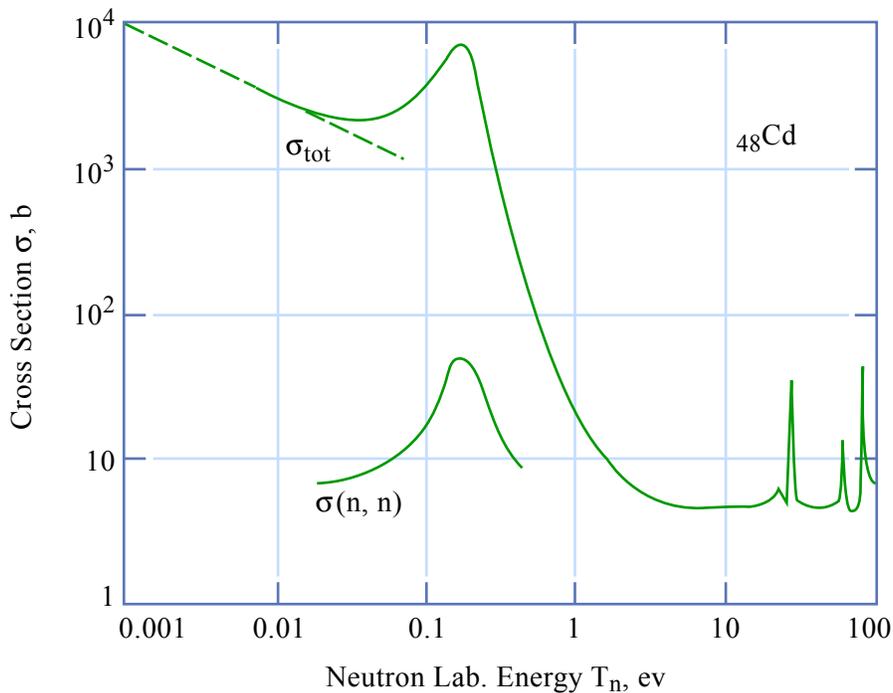
Figure by MIT OCW.

Fig. 21.7. Total and elastic neutron scattering cross sections of Cd showing a resonant absorption peak and a resonant scattering peak, respectively.

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We conclude our brief discussion of compound nucleus reactions by returning to the feature of constructive and destructive interference between potential scattering and resonance scattering in the elastic scattering cross section. Fig. 21.8 shows this behavior schematically, and Fig. 21.9 shows that such effects are indeed observed [J. E. Lynn, *The Theory of Neutron Resonance Reactions* (Clarendon Press, Oxford, 1968]. Admittedly this feature is not always seen in the data; the present example is carefully chosen and should not be taken as being a typical situation.
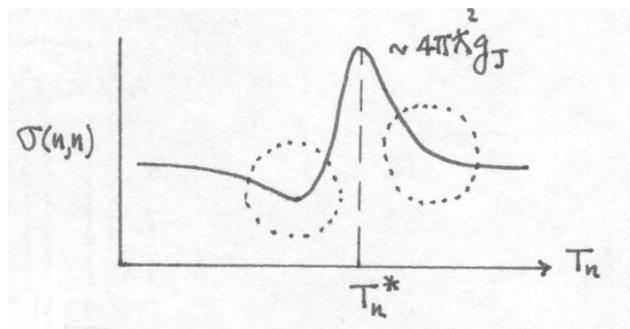
Fig. 21.8. Interference effects in elastic neutron scattering, below and above the resonance.
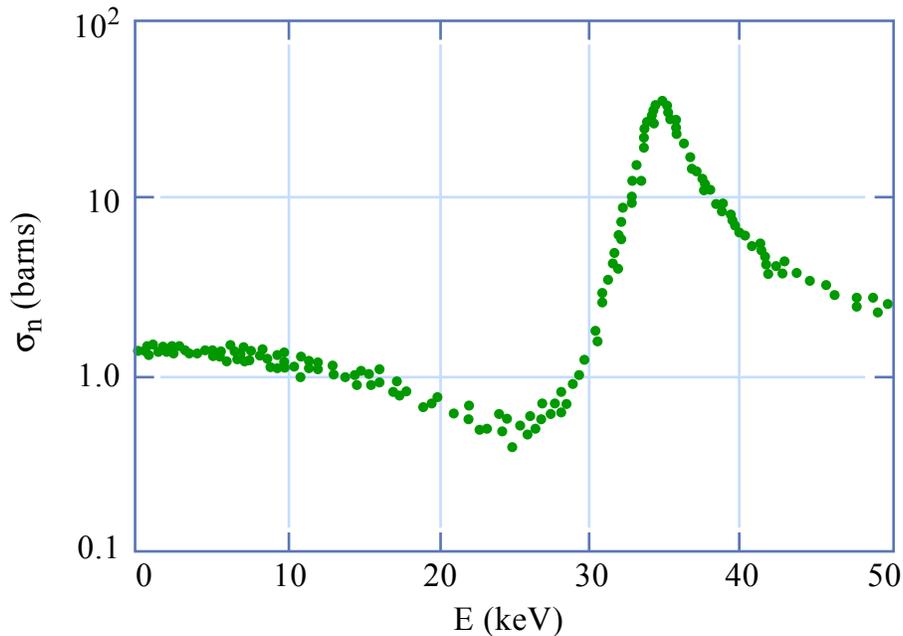
Fig. 21.9. Experimental scattering cross section of Al27 showing the interference effects between potential and resonance scattering, and an asymptotically constant value (potential scattering) sufficiently far away from the resonance. (from Lynn)