References:

W. E. Meyerhof, *Elements of Nuclear Physics* (McGraw-Hill, New York, 1967), Chap 4.

A nucleus in an excited state is unstable because it can always undergo a transition (decay) to a lower-energy state of the *same nucleus*. Such a transition will be accompanied by the emission of gamma radiation. A nucleus in either an excited or ground state also can undergo a transition to a lower-energy state of *another nucleus*. This decay is accomplished by the emission of a particle such as an alpha, electron or positron, with or without subsequent gamma emission. A nucleus which undergoes a transition *spontaneously*, that is, without being supplied with additional energy as in bombardment, is said to be radioactive. It is found experimentally that naturally occurring radioactive nuclides emit one or more of the three types of radiations, α – particles, β – particles, and γ – rays. Measurements of the energy of the nuclear radiation provide the most direct information on the energy-level structure of nuclides. One of the most extensive compilations of radioisotope data and detailed nuclear level diagrams is the *Table of Isotopes*, edited by Lederer, Hollander and Perlman.

In this chapter we will supplement our previous discussions of beta decay and radioactive decay by briefly examining the study of decay constants, selection rules, and some aspects of α – , β – , and γ – decay energetics.

Alpha Decay

Most radioactive substances are α – emitters. Most nuclides with A > 150 are unstable against α – decay. α – decay is very unlikely for light nuclides. The decay constant decreases exponentially with decreasing Q-value, here called the decay energy, $\lambda_\alpha \sim \exp(-c/v)$, where c is a constant and v the speed of the α – particle,

$v \propto \sqrt{Q_\alpha}$. The momentum and energy conservation equations are quite straightforward in this case, as can be seen in Fig. 20.1.
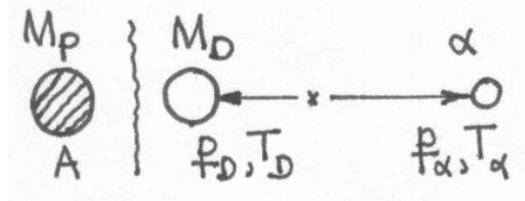

Fig. 20.1. Particle emission and nuclear recoil in α - decay .

$$\underline{p}_D + \underline{p}_\alpha = 0 \tag{20.1}$$

$$M_P c^2 = (M_D c^2 + T_D) + (M_\alpha c^2 + T_\alpha) \tag{20.2}$$

Both kinetic energies are small enough that non-relativistic energy-momentum relations may be used,

$$T_D = p_D^2 / 2M_D = p_\alpha^2 / 2M_D = (M_\alpha / M_D) T_\alpha \tag{20.3}$$

Treating the decay as a reaction the corresponding Q-value becomes

$$Q_\alpha = \left[M_P - (M_D + M_\alpha)\right]c^2$$

$$= T_D + T_\alpha$$

$$= \frac{M_D + M_\alpha}{M_D} T_\alpha \approx \frac{A}{A-4} T_\alpha \tag{20.4}$$

This shows that the kinetic energy of the α -particle is always less than Q_α . Since $Q_\alpha >$ 0 (T_α is necessarily positive), it follows that α -decay is an exothermic process.   The

various energies involved in the decay process can be displayed in an energy-level diagram shown in Fig. 20.2. One can see at a glance how the rest masses and
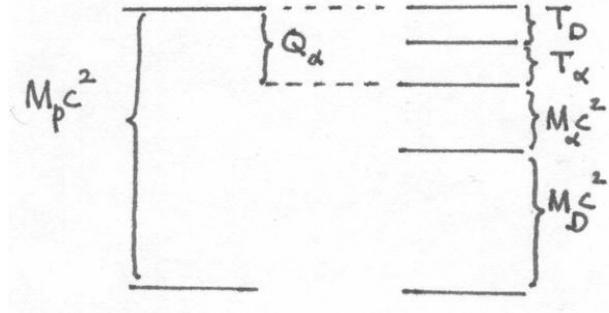

Fig. 20.2. Energy-level diagram for α -decay.

the kinetic energies combine to ensure energy conservation. We will see in the next lecture that energy-level diagrams are also useful in depicting collision-induced nuclear reactions. The separation energy S_α is the work necessary to separate an α -particle from the nucleus,

$$S_\alpha = \left[M(A-4,Z-2)+M_\alpha - M(A,Z)\right]c^2$$

$$= B(A,Z) - B(A-4,Z-2) - B(4,2) = -Q_\alpha \qquad (20.5)$$

One can use the semi-empirical mass formula to determine whether a nucleus is stable against α -decay. In this way one finds $Q_\alpha > 0$ for A > 150. Eq.(20.5) also shows that when the daughter nucleus is magic, B(A-4,Z-2) is large, and Q_α is large. Conversely, Q_α is small when the parent nucleus is magic.

Estimating α -decay Constant

An estimate of the decay constant can be made by treating the decay as a barrier penetration problem, an approach proposed by Gamow (1928) and also by Gurney and Condon (1928). The idea is to assume the α -particle already exists as a particle inside the daughter nucleus where it is confined by the Coulomb potential, as illustrated in Fig. 20.3. The decay constant is then the probability per unit time that it can tunnel through the potential,
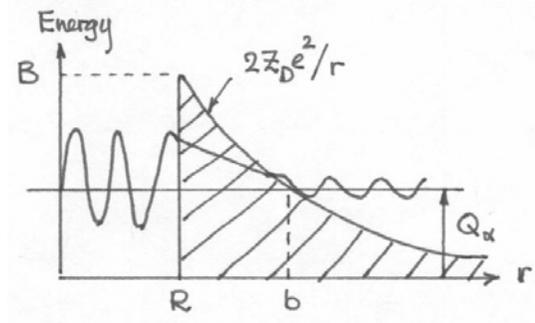
Fig. 20.3. Tunneling of an $\alpha-$particle through a nuclear Coulomb barrier.

$$\lambda_\alpha \sim \left(\frac{v}{R}\right)P \tag{20.6}$$

where v is the relative speed of the α and the daughter nucleus, R is the radius of the daughter nucleus, and P the transmission coefficient. Eq.(20.6) is a standard form for describing tunneling probability in the form of a rate. The prefactor v/R is the attempt frequency, the rate at which the particle tries to tunnel through the barrier, and P is the probability of tunneling for each try. Recall from our study of barrier penetration (cf. Chap 5, eq. (5.20)) that the transmission coefficient can be written in the form

$$P \sim e^{-\gamma} \tag{20.7}$$

$$\gamma = \frac{2}{\hbar}\int_{r_1}^{r_2} dr(2m[V(r)-E])^{1/2}$$

$$= \frac{2}{\hbar}\int_R^b dr\left[2\mu\left(\frac{2Z_De^2}{r}-Q_\alpha\right)\right]^{1/2} \tag{20.8}$$

with $\mu = M_\alpha M_D/(M_\alpha + M_D)$. The integral can be evaluated,

$$\gamma = \frac{8Z_D e^2}{\hbar v}\left[\cos^{-1}\sqrt{y} - \sqrt{y}(1-y)^{1/2}\right] \tag{20.9}$$

where y = R/b = Q_α/B, B = 2Z$_D$e^2/R, $Q_\alpha = \mu v^2/2 = 2Z_D e^2/b$. Typically B is a few tens or more Mev, while Q_α ~ a few Mev. One can therefore invoke the thick barrier approximation, in which case b >> R (or Q_α << B), and y << 1. Then

$$\cos^{-1}\sqrt{y} \sim \frac{\pi}{2} - \sqrt{y} - \frac{1}{6}y^{3/2} - \ldots \tag{20.10}$$

the square bracket in (20.9) becomes

$$[\ ] \sim \frac{\pi}{2} - 2\sqrt{y} + O(y^{3/2}) \tag{20.11}$$

and

$$\gamma \approx \frac{4\pi Z_D e^2}{\hbar v} - \frac{16Z_D e^2}{\hbar v}\left(\frac{R}{b}\right)^{1/2} \tag{20.12}$$

So the expression for the decay constant becomes

$$\lambda_\alpha \approx \frac{v}{R}\exp\left[-\frac{4\pi Z_D e^2}{\hbar v} + \frac{8}{\hbar}\left(Z_D e^2 \mu R\right)^{1/2}\right] \tag{20.13}$$

where μ is the reduced mass. Since Gamow was the first to study this problem, the exponent is sometimes known as the Gamow factor G.

To illustrate the application of (20.13) we consider estimating the decay constant of the 4.2 Mev α-particle emitted by U^{238}. Ignoring the small recoil effects, we can write

$$T_\alpha \sim \frac{1}{2}\mu v^2 \rightarrow v \sim 1.4 \text{ x } 10^9 \text{ cm/s}, \quad \mu \sim M_\alpha$$

$$R \sim 1.4 \, (234)^{1/3} \times 10^{-13} \sim 8.6 \times 10^{-13} \text{ cm}$$

$$-\frac{4\pi Z_D e^2}{\hbar v} = -173, \qquad \frac{8}{\hbar}\left(Z_D e^2 \mu R\right)^{1/2} = 83$$

Thus

$$P = e^{-90} \sim 10^{-39} \tag{20.14}$$

As a result our estimate is

$$\lambda_\alpha \sim 1.7 x 10^{-18} \text{ s}^{-1}, \quad \text{or } t_{1/2} \sim 1.3 \times 10^{10} \text{ yrs}$$

The experimental half-life is $\sim 0.45 \times 10^{10}$ yrs. Considering our estimate is very rough, the agreement is rather remarkable. In general one should not expect to predict λ_α to be better than the correct order of magnitude (say a factor of 5 to 10). Notice that in our example, B $\sim$ 30 Mev and $Q_\alpha = 4.2$ Mev. Also b = RB/Q_α= 61 x 10^{-13} cm. So the thick barrier approximation, B >> Q_α or b >> R, is indeed well justified.

The theoretical expression for the decay constant provides a basis for an empirical relation between the half-life and the decay energy. Since t$_{1/2}$ = 0.693/α , we have from (20.13)

$$\ell n(t_{1/2}) = \ell n\left(0.693R/v\right) + 4\pi Z_D e^2 / \hbar v - \frac{8}{\hbar}\left(Z_D e^2 \mu R\right)^{1/2} \tag{20.15}$$

We note R $\sim$A$^{1/3}$ $\sim$ $Z_D^{1/3}$, so the last term varies with Z$_D$ like $Z_D^{2/3}$. Also, in the second trerm v $\propto$ $\sqrt{Q_\alpha}$. Therefore (20.15) suggests the following relation,

$$\log(t_{1/2}) = a + \frac{b}{\sqrt{Q_\alpha}} \tag{20.16}$$

with a and b being parameters depending only on Z_D. A relation of this form is known as the Geiger-Nuttall rule.

We conclude our brief consideration of α-decay at this point. For further discussions the student should consult Meyerhof (Chap 4) and Evans (Chap 16).

Beta Decay

Beta decay is considered to be a weak interaction since the interaction potential is $\sim 10^{-6}$ that of nuclear interactions, which are generally regarded as strong. Electromagnetic and gravitational interactions are intermediate in this sense. β-decay is the most common type of radioactive decay, all nuclides not lying in the "valley of stability" are unstable against this transition. The positrons or electrons emitted in β-decay have a continuous energy distribution, as illustrated in Fig. 20.4 for the decay of Cu^{64},
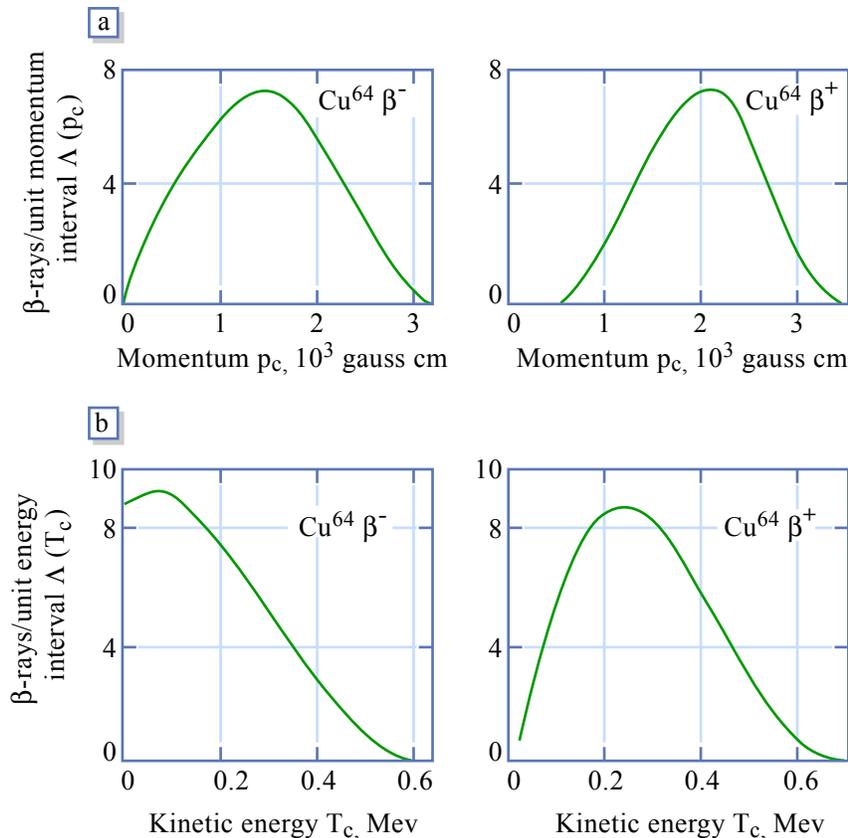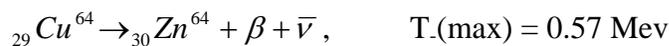

Figure by MIT OCW.

Fig. 20.4. Momentum (a) and energy (b) distributions of beta decay in Cu^{64}. (from Meyerhof)

$$_{29}Cu^{64} \rightarrow {}_{30}Zn^{64} + \beta + \bar{\nu}\,, \qquad T_-(\text{max}) = 0.57 \text{ Mev}$$

$$\rightarrow {}_{28}Ni^{64} + \beta^+ + \nu\,, \qquad T_+(\text{max}) = 0.66 \text{ Mev}$$

The values of $T_{\pm}(\max)$ are characteristic of the particular radionuclide; they can be considered as signatures.

If we assume that in β-decay we have only a parent nucleus, a daughter nucleus, and a β-particle, then we would find that the conservations of energy, linear and angular moemnta cannot be all satisfied. It was then proposed by Pauli (1933) that particles, called neutrino ν and antineutrino $\overline{\nu}$, also can be emitted in β-decay. The neutrino particle has the properties of zero charge, zero (or nearly zero) mass, and intrinsic angular momentum (spin) of $\hbar/2$. The detection of the neutrino is unusually difficult because it has a very long mean-free path. Its existence was confirmed by Reines and Cowan (1953) using the *inverse β-decay* reaction induced by a neutrino, $p + \overline{\nu} \rightarrow n + \beta^-$. The emission of a neutrino (or antineutrino) in the β-decay process makes it possible to satisfy the energy conservation condition with a continuous distribution of the kinetic energy of the emitted β-particle. Also, linear and angular momenta are now conserved.

The energetics of β-decay can be summarized as

$$\underline{p}_D + \underline{p}_\beta + \underline{p}_{\overline{\nu}} = 0 \tag{20.17}$$

$$M_P c^2 = M_D c^2 + T_\beta + T_{\overline{\nu}} \qquad \text{electron decay} \tag{20.18}$$

$$M_P c^2 = M_D c^2 + T_{\beta^+} + T_\nu + 2m_e c^2 \qquad \text{positron decay} \tag{20.19}$$

where the extra rest mass term in positron decay has been discussed previously in Chap 11 (cf. Eq. (11.9)). Recall also that electron capture (EC) is a competing process with positron decay, requiring only the condition $M_P(Z) > M_D(Z-1)$. Fig. 20.4 shows how the energetics can be expressed in the form of energy-level diagrams.
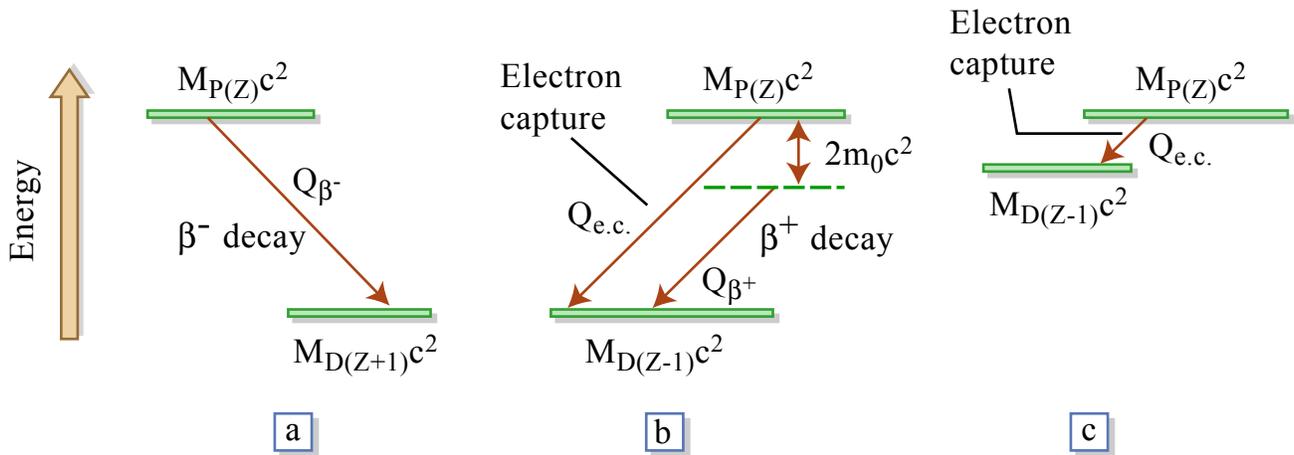
Figure by MIT OCW.

Fig. 20.5. Energetics of β – decay processes. (from Meyerhof)

Typical decay schemes for   -emitters are shown in Fig. 20.6. For each nuclear level
there is an assignment of spin and parity. This information is essential for determining
whether a transition is allowed according to certain selection rules, as we will discuss
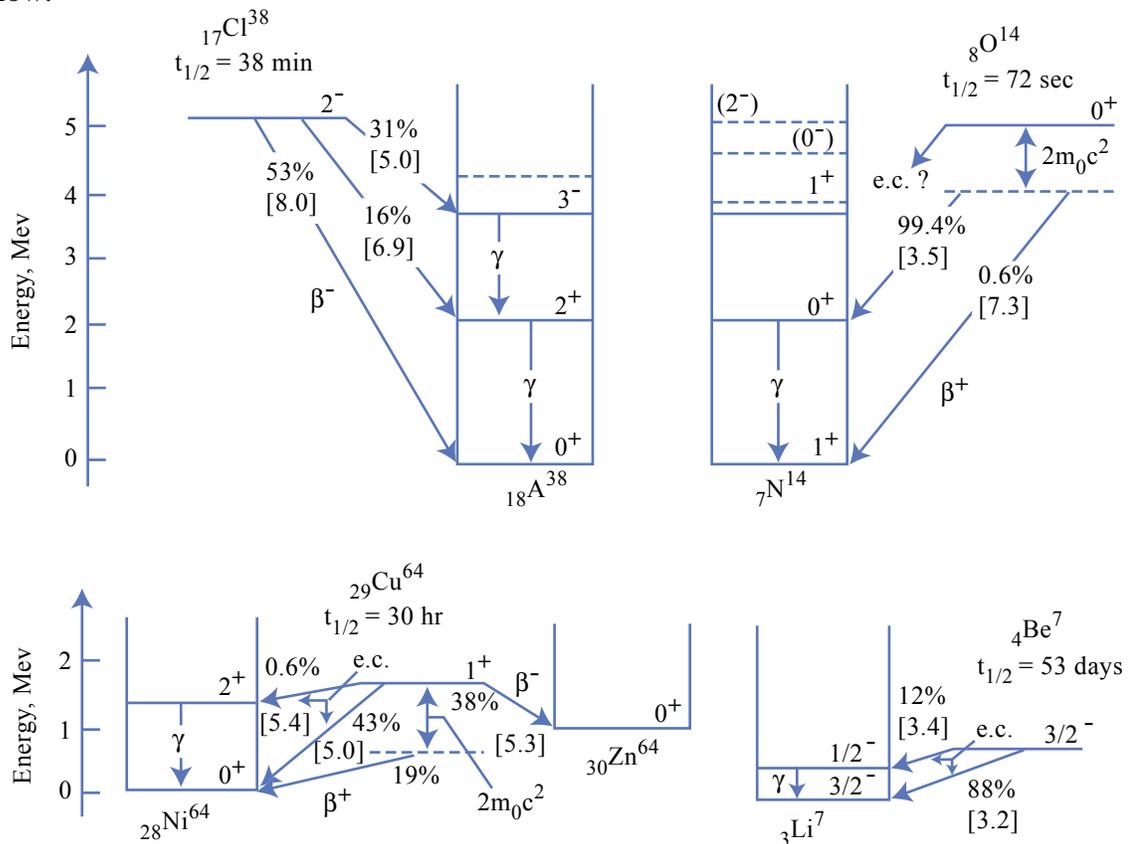below.

Figure by MIT OCW.

Fig. 20.6. Energy-level diagrams depicting nuclear transitions involving beta decay.
(from Meyerhof)

Experimental half-lives of β-decay have values spread over a very wide range, from 10^{-3} sec to 10^{16} yrs. Generally, $\lambda_\beta \sim Q_\beta^5$. The decay process cannot be explained classically. The theory of β-decay was developed by Fermi (1934) in analogy with the quantum theory of electromagnetic decay. For a discussion of the elements of this theory one can begin with Meyerhof and follow the references given therein. We will be content to mention just one aspect of the theory, that concerning the statistical factor describing the momentum and energy distributions of the emitted β particle. Fig. 20.7 shows the nuclear coulomb effects on the momentum distribution in β-decay in Ca (Z = 20). One can see an enhancement in the case of β^--decay and a suppression in the case of β^+-decay at low momenta. Coulomb effects on the energy distribution are even more pronounced.
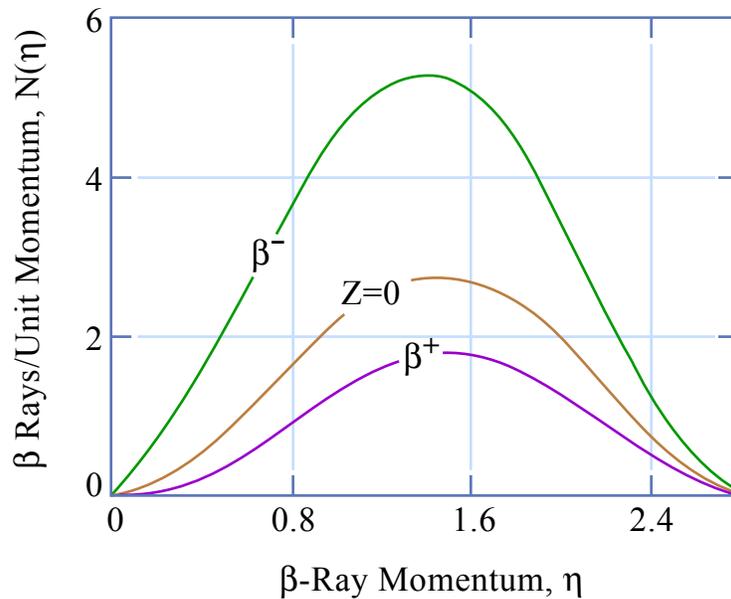

Fig. 20.7. Momentum distributions of β-decay in Ca.

Selection Rules for Beta Decay

Besides energy and linear momentum conservation, a nuclear transition must also satisfy angular momentum and parity conservation. This gives rise to selection rules which specify whether a particular transition between initial and final states, both with specified spin and parity, is allowed, and if allowed what mode of decay is most likely. We will work out the selection rules governing $\beta-$ and γ-decay. For the former conservation of angular momentum and parity are generally expressed as

$$\underline{I}_P = \underline{I}_D + \underline{L}_\beta + \underline{S}_\beta \tag{20.17}$$

$$\pi_P = \pi_D(-1)^{L_\beta} \tag{20.18}$$

where L_β is the orbital angular momentum and S_β the intrinsic spin of the electron-antineutrino system. The magnitude of angular momentum vector can take integral values, 0, 1, 2, …, whereas the latter can take on values of 0 and 1 which would correspond the antiparallel and parallel coupling of the electron and neutrino spins. These two orientations will be called *Fermi* and *Gamow-Teller* respectively in what follows.

In applying the conservation conditions, the goal is to find the lowest value of L_β that will satisfy (20.17) for which there is a corresponding value of S_β that is compatible with (20.18). This then identifies the most likely transition among all the allowed transitions. In other words, all the other allowed transitions with higher values of L_β, which makes them less likely to occur. This is because the decay constant is governed by the square of a transition matrix element, which in term can be written as a series of contributions, one for each L_β (recall the discussion of partial wave expansion in cross section calculation, Appendix B, where we also argue that the higher order partial waves are less likely the low order ones, ending up with only the s-wave),
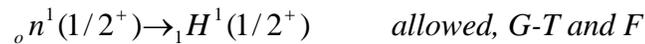
$$\lambda_\beta \propto |M|^2 = |M(L_\beta = 0)|^2 + |M(L_\beta = 1)|^2 + |M(L_\beta = 2)|^2 + ... \tag{20.19}$$

Transitions with $L_\beta = 0, 1, 2, …$ are called *allowed, first-borbidden, second-forbidden,…*etc. The magnitude of the matrix element squared decreases from one order to the next higher one by at least a factor of 10^2. For this reason we are interested only in the lowest order transition that is allowed.

To illustrate how the selection rules are determined, we consider the transition

$$_2He^6(0^+) \rightarrow {}_3Li^6(1^+)$$
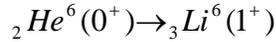
To determine the combination of L_β and S_β for the first transition that is allowed, we begin by noting that parity conservation requires L_β to be even. Then we see that $L_\beta = 0$ plus $S_\beta = 1$ would satisfy both (20.17) and (20.18). Thus the most likely transition is the transition designated as *allowed, G-T*. Following the same line of argument, one can arrive at the following assignments.

$$_8O^{14}(0^+) \rightarrow {}_7N^{14}(0^+) \qquad \textit{allowed, F}$$

$$_0n^1(1/2^+) \rightarrow {}_1H^1(1/2^+) \qquad \textit{allowed, G-T and F}$$

$$_{17}C\ell^{38}(2^-) \rightarrow {}_{18}A^{38}(2^+) \qquad \textit{first-borbidden, GT and F}$$

$$_4Be^{10}(3^+) \rightarrow {}_5B^{10}(0^+) \qquad \textit{second-forbidden, GT}$$
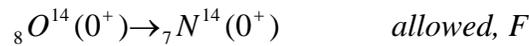
Parity Non-conservation

The presence of neutrino in β-decay leads to a certain type of non-conservation of parity. It is known that neutrinos have instrinsic spin antiparallel to their velocity, whereas the spin orientation of the antineutrino is parallel to their velocity (keeping in mind that v and $\bar{v}$ are *different* particles). Consider the mirror experiment where a neutrino is moving toward the mirror from the left, Fig. 20.8. Applying the inversion symmetry operation
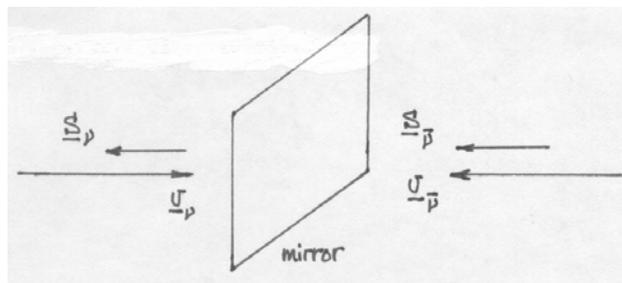
Fig. 20.8. Mirror reflection demonstrating parity non-sonserving property of neutrino. (from Meyerhof)

($x \rightarrow -x$), the velocity reverses direction, while the angular momentum (spin) does not. Thus, on the other side of the mirror we have an image of a particle moving from the right, but its spin is now parallel to the velocity so it has to be an antineutrino instead of a neutrino. This means that the property of ν and $\bar{\nu}$, namely definite spin direction relative to the velocity, is not compatible with parity conservation (symmetry under inversion).

For further discussions of beta decay we again refer the student to Meyerhof and the references given therein.

Gamma Decay

An excited nucleus can always decay to a lower energy state by γ-emission or a competing process called internal conversion. In the latter the excess nuclear energy is given directly to an atomic electron which is ejected with a certain kinetic energy. In general, complicated rearrangements of nucleons occur during γ-decay.

The energetics of γ-decay is rather straightforward. As shown in Fig. 20. 9 a γ is emitted while the nucleus recoils.
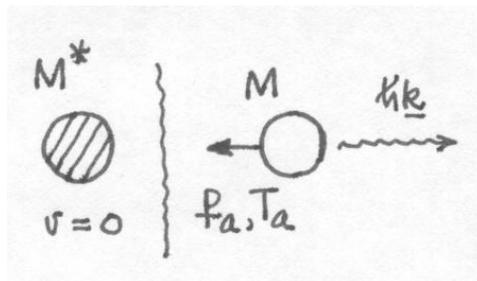

Fig. 20.9. Schematics of γ-decay

$$\hbar \underline{k} + \underline{p}_a = 0 \tag{20.20}$$

$$M * c^2 = Mc^2 + E_\gamma + T_a \tag{20.20}$$

The recoil energy is usually quite small,

$$T_a = p_a^2/2M = \hbar^2 k^2/2M = E_\gamma^2/2Mc^2 \qquad (20.21)$$

Typically, $E_\gamma \sim 2$ Mev, so if A $\sim$ 50, then T$_a$ $\sim$ 40 ev. This is generally negligible.

Decay Constants and Selection Rules

Nuclear excited states have half-lives for γ-emission ranging from 10^{-16} sec to > 100 years. A rough estimate of λ_γ can be made using semi-classical ideas. From Maxwell's equations one finds that an accelerated point charge e radiates electromagnetic radiation at a rate given by the Lamor formula (cf. Jackson, *Classical Electrodynamics*, Chap 17),

$$\frac{dE}{dt} = \frac{2}{3}\frac{e^2 a^2}{c^3} \qquad (20.22)$$

where a is the acceleration of the charge. Suppose the radiating charge has a motion like the simple oscillator,

$$x(t) = x_o \cos \omega t \qquad (20.23)$$

where we take $x_o^2 + y_o^2 + z_o^2 = R^2$, R being the radius of the nucleus. From (20.23) we have

$$a(t) = R\omega^2 \cos \omega t \qquad (20.24)$$

To get an average rate of energy radiation, we average (20.22) over a large number of oscillation cycles,

$$\left(\frac{dE}{dt}\right)_{avg} = \frac{2}{3}\frac{R^2\omega^4 e^2}{c^3}\left(\cos^2 \omega t\right)_{avg} \approx \frac{R^2\omega^4 e^2}{3c^3} \qquad (20.25)$$

Now we assume that each photon is emitted during a time interval τ (having the physical significance of a mean lifetime). Then,

$$\left(\frac{dE}{dt}\right)_{avg} = \frac{\hbar\omega}{\tau} \qquad (20.26)$$

Equating this with (20.25) gives

$$\lambda_\gamma \approx \frac{e^2 R^2 E_\gamma^3}{3\hbar^4 c^3} \qquad (20.27)$$

If we apply this result to a process in atomic physics, namely the de-excitation of an atom by electromagnetic emission, we would take R ~ 10^{-8} cm and E_γ ~ 1 ev, in which case (20.27) gives

$$\lambda_\gamma \sim 10^6 \text{ sec}^{-1}, \quad \text{or } t_{1/2} \sim 7 \text{ x } 10^{-7} \text{ sec}$$

On the other hand, if we apply (20.27) to nuclear decay, where typically R ~ 5 x 10^{-13} cm, and E_γ ~ 1 Mev, we would obtain

$$\lambda_\gamma \sim 10^{15} \text{ sec-1}, \quad \text{or } t_{1/2} \sim 3 \text{ x } 10^{-16} \text{ sec}$$

These result only indicate typical orders of magnitude. What Eq.(20.27) does not explain is the wide range of values of the half-lives that have been observed. For further discussions we again refer the student to references such as Meyerhof.

Turning to the question of selection rules for γ-decay, we can write down the conservation of angular momenta and parity in a form similar to (20.17) and (20.18),

$$\underline{I}_i = \underline{I}_f + \underline{L}_\gamma \qquad (20.28)$$

$$\pi_i = \pi_f \pi_\gamma \qquad (20.29)$$

Notice that in contrast to (20.27) the orbital and spin angular momenta are incorporated in $\underline{L}_\gamma$, playing the role of the *total* angular momentum. Since the photon has spin $\hbar$ [for a discussion of photon angular momentum, see A. S. Davydov, *Quantum Mechanics* (1965(, pp. 306 and 578], the possible values of L_γ are 1 (corresponding to the case of zero orbital angular momentum), 2, 3, …For the conservation of parity we know the parity of the photon depends on the value of L_γ. We now encounter two possibilities because in photon emission, which is the process of electromagnetic multipole radiation, one can have either electric or magnetic multipole radiation,

$$\pi_\gamma \;=\; (-1)^{L_\gamma} \qquad \text{electric multipole}$$

$$-(-1)^{L_\gamma} \qquad \text{magnetic multipole}$$

Thus we can set up the following table,

Radiation	Designation	Value of L_γ	π_γ
electric dipole	E1	1	-1
magnetic dipole	M1	1	+1
electric quadrupole	E2	2	+1
magnetic quadrupole	M2	2	-1
electric octupole	E3	3	-1
etc.			

Similar to the case of β-decay, the decay constant can be expressed as a sum of contributions from each multipole [cf. Blatt and Weisskopf, *Theoretical Nuclear Physics*, p. 627],

$$\lambda_\gamma = \lambda_\gamma(E1) + \lambda_\gamma(M1) + \lambda_\gamma(E2) + \dots \qquad (20.30)$$

provided each contribution is allowed by the selection rules. We are again interested only in the lowest order allowed transition, and if both E and M transitions are allowed, E will dominate. Take, for example, a transition between an initial state with spin and parity of 2^+ and a final state of 0^+. This transition requires the photon parity to be positive, which means that for an electric multipole radiation L_γ would have to be even, and for a magnetic radiation it has to be odd. In view of the initial and final spins, we see that angular momentum conservation (20.28) requires L_γ to be 2. Thus, the most likely mode of γ-decay for this transition is E2. A few other examples are:

$$1^+ \to 0^+ \qquad\qquad \text{M1}$$

$$\frac{1}{2}^- \to \frac{1}{2}^+ \qquad\qquad \text{E1}$$

$$\frac{9}{2}^+ \to \frac{1}{2}^- \qquad\qquad \text{M4}$$

$$0^+ \to 0^+ \qquad\qquad \text{no } \gamma\text{-decay allowed}$$

We conclude this discussion of nuclear decays by the remark that internal conversion (IC) is a competing process with γ-decay. The atomic electron ejected has a kinetic energy given by (ignoring nuclear recoil)

$$T_e = E_i - E_f - E_B \qquad\qquad (20.31)$$

where $E_i - E_f$ is the energy of de-excitation, and E_B is the binding energy of the atomic electron. If we denote by λ_e the decay constant for internal conversion, then the total decay constant for de-excitation is

$$\lambda = \lambda_\gamma + \lambda_e \qquad\qquad (20.32)$$