Planar X-ray Imaging

Measure the integeral of the linear attenuation coefficient over the beam path through the object.

$$P(x,y) = I_o e^{\int \mu dz}$$

$$where \;\; \mu = \mu(x,y,z)$$

μ has two main contributions

1. Photoelectric effect - this removes photons from the beam and has the properties we want for imaging
2. Compton Scattering - scatters photons which may end up in the detector. Can lead to noise.

Calculation of Attenuation Coefficient

To the extent that Compton scattered photons do not reach the detector, they also contribute to the signal, but contracts is low.

A reasonable (approximate) analytical expression for the attenuation coefficient is:

$$\mu = \rho N_g \left[f(E) + C_p \frac{Z^m}{E^n} \right]$$

$$N_g = N_A \frac{Z}{A}$$

$$Z = atomic\ \ number$$

$$A = atomic\ \ mass$$

$$N_A = Avogadro's\ \ number$$

$$\rho = density$$

$$f(E) = 0.597 \times 10^{-24} e^{-0.0028(E-30)}$$

$$= Compton\ \ scattering\ \ part\ \ for\ \ low\ \ E$$

$$C_p = 9.8 \times 10^{-24}$$

$$m = 3.8, n = 3.2$$

Calculation of Effective Z

This is quite approximate but does permit simple computation provided that the energy is high enough, the simple scattering is not an issue, and less than, or equal to, 200 keV.

For practical problems, still need to calculate an effective Z.

$$Z_{eff} = \left(\sum_i \alpha_i Z_i^m\right)^{1/m}$$

$$where \ \ \alpha_i \ \ is \ \ the \ \ electron \ \ fraction \ \ of \ \ the \ \ i^{th} \ \ element.$$

$$\alpha_i = \frac{N_{g_i}}{\sum_j N_{g_j}}$$

$$N_{g_i} = N_A w_i \left(\frac{Z_i}{A_i}\right)$$

w_i → *fraction by weight of the element*

Sample Effective Z Calculations

$Calculate\ \ Z_{eff}\ \ \ for:$

1. $water,\ H_2O$
2. $oil,\ CH_3(CH_2)_4CH_3 \rightarrow \rho = 0.66,\ mw = 86.2$
3. $calcium\ \ carbonate,\ CaCO_3 \rightarrow \rho = 2.930,\ mw = 100.09$

$Plot\ \ \mu_p\ \ and\ \ \mu_c\ \ for\ \ each$

	Atomic # (Z)	Atomic Weight (A)
H	1	1.008
C	6	12.011
O	8	15.994
Ca	20	40.08

Sample Effective Z Calculations (Water)

$$N_{g_o} = N_A\left(\frac{16}{18}\right)\left(\frac{8}{16}\right) \quad ; \quad N_{g_H} = N_A\left(\frac{2}{18}\right)\left(\frac{1}{1.008}\right)$$

$$= 0.444\,N_A \quad ; \quad = 0.111\,N_A$$

$$\alpha_o = \frac{0.444}{0.555} = 0.8 \quad ; \quad \alpha_H = 0.2$$

$$Z_{eff} = \left[0.8(8)^{3.8} + 0.2(1)^{3.8}\right]^{1/3.8}$$

$$= 7.54$$

A_{eff} *is just a normal weighted sum.*

Sample Effective Z Calculations (Hexane)

$$N_{g_o} = N_A\left(\frac{72}{86.2}\right)\left(\frac{6}{12}\right) \quad ; \quad N_{g_H} = N_A\left(\frac{14}{86.2}\right)(1)$$

$$= 0.42 N_A \quad ; \quad = 0.16 N_A$$

$$\alpha_o = \frac{0.42}{0.58} = 0.724 \quad ; \quad \alpha_H = \frac{0.16}{0.58} = 0.275$$

$$Z_{eff} = \left[0.724(6)^{3.8} + 0.275(1)^{3.8}\right]^{1/3.8}$$

$$= 5.51$$

$$A_{eff} = 12$$

Sample Effective Z Calculations (Calcium Carbonate)

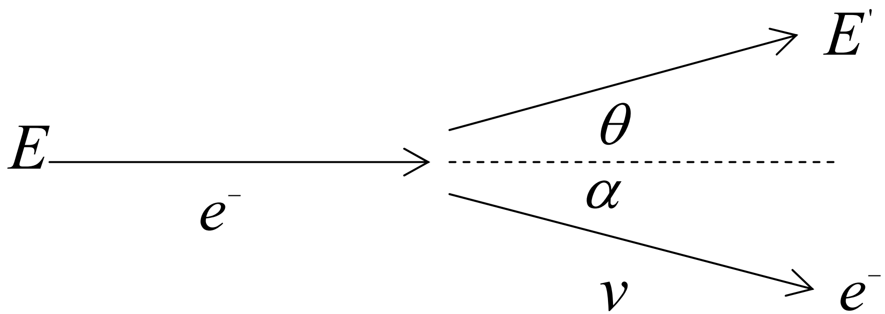
$$\left.\begin{aligned}
N_{g_{Ca}} &= N_A\left(\frac{20}{100}\right)\left(\frac{20}{40}\right) &&= 0.10\,N_A \\
N_{g_C} &= N_A\left(\frac{12}{100}\right)\left(\frac{6}{12}\right) &&= 0.06\,N_A \\
N_{g_O} &= N_A\left(\frac{48}{100}\right)\left(\frac{8}{16}\right) &&= 0.24\,N_A
\end{aligned}\right\}\sum = 0.40$$

$$\alpha_{Ca} = \frac{0.1}{0.4} = 0.25 \quad ; \quad \alpha_C = \frac{0.06}{0.4} = 0.15 \quad ; \quad \alpha_O = \frac{0.24}{0.4} = 0.6$$

$$\begin{aligned}
Z_{eff} &= \left[0.25(20)^{3.8} + 0.15(6)^{3.8} + 0.6(8)^{3.8}\right]^{1/3.8} \\
&= \left[21{,}971 + 135 + 1{,}621\right]^{1/3.8} \\
&= 14.2 \\
A_{eff} &= 21.4
\end{aligned}$$

Determining The Signal

We would like to know what the signal is at the detector, but this depends on the geometry since Compton scattering is important.



Conservation of energy:

$$E = E' + (m - m_o)c^2$$

where c^2 is the velocity of light

and m_o is the rest mass of electron

$$m = \frac{m_o}{\sqrt{1-\left(v/c\right)^2}} = \text{ mass of moving electrons}$$

Determining The Signal

Conservation of momentum:

$$\frac{E}{c} = \frac{E^{'}}{c}\cos(\theta) + mv\cos(\alpha)$$

$$0 = \frac{E^{'}}{c}\sin(\theta) - mv\sin(\alpha)$$

Solving these together yield:

$$\left(E - E^{'}\right) = \frac{EE^{'}}{m_{o}c^{2}}\left(1 - \cos(\theta)\right)$$

Angular Dependence of Compton Scattering (Low Energy)

At low energies the scatter angle distribution is approximately isotropic.

Plot ΔE vs angle for various energies

Note: Δτ = 0.0241(1 - cos(θ)) where Δτ is in Angstroms. To convert to keV, recall that 50keV is about 0.2Å.

$$E = h\nu = \frac{hc}{\lambda} \quad ; \quad \Delta E = \frac{1}{0.241(1-\cos(\theta))} \times \left(50keV \bullet 0.2\,\overset{o}{A}\right)$$

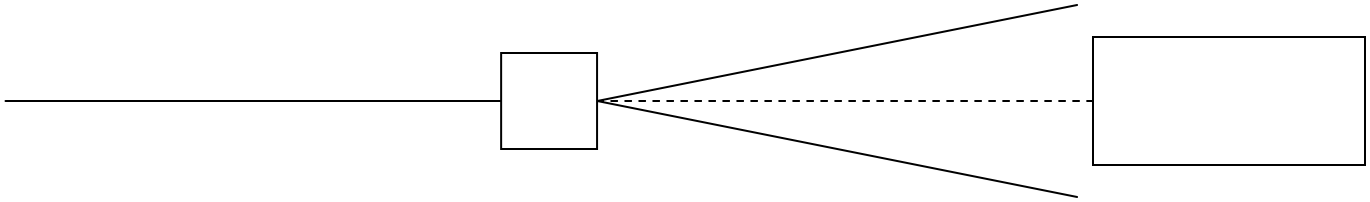
$$\lambda = \frac{1}{E_{in}} \bullet 50keV \bullet 0.2\,\overset{o}{A} \quad ; \quad \lambda_{out} = \frac{1}{E_{in}}\left(50keV \bullet 0.2\,\overset{o}{A}\right) - 0.0241(1-\cos(\theta))$$

$$E_{out} = \frac{1}{\lambda_{out}}\left(50keV \bullet 0.2\,\overset{o}{A}\right)$$

$$\Delta E = E_{in} - E_{out} = E_{in} - \frac{50keV \bullet 0.2\,\overset{o}{A}}{\frac{1}{E_{in}}\left(50keV \bullet 0.2\,\overset{o}{A}\right) - 0.0241(1-\cos(\theta))}$$

Angular Dependence of Compton Scattering (High Energy)

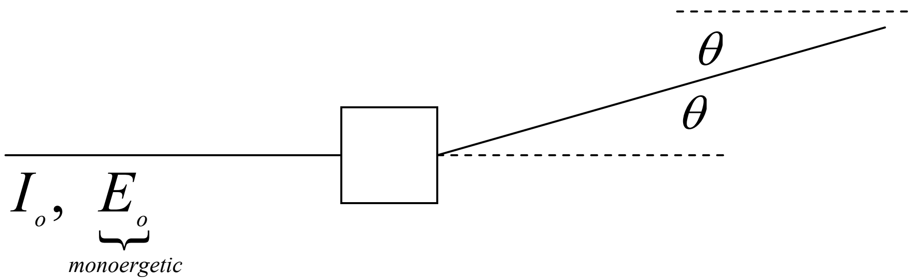
At high energies, the photon energy changes significantly with scatter angle. This results in most scatter being forward-directed and thus high-energy X-ray is extremely challenging since we can not distinguish scattered from transmitted radiation.



Photon electron (transmitted radiation) reaches the detector with the original beam geometry. Compton reaches as the solid angle subtended by the detector.

Compton-based Imaging

Can try Compton-based imaging with energy detection

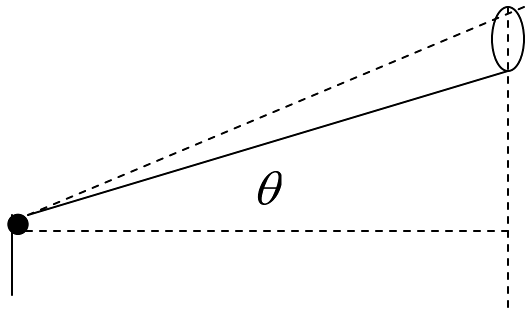


Detector at energy E' < E, thus scattering angle is θ.

This specifies a cone that the radiation can come form.

This can be reconstructed but it is difficult and only used in cosmology where the original source is at infinity.

Compton-based Imaging



θ

This specifies a cone that the radiation can come form.

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