# Lab \#5 <br> Ocean Acoustic Environment 

2.S998 Unmanned Marine Vehicle Autonomy, Sensing and Communications

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## 1 The ocean acoustic environment

### 1.1 Ocean Acoustic Waveguide

The ocean is an acoustic waveguide limited above by the sea surface and below by the seafloor. The speed of sound in the waveguide plays the same role as the index of refraction does in optics. In the ocean, density is related to static pressure, salinity, and temperature. The sound speed in the ocean is an increasing function of temperature, salinity, and pressure, the latter being a function of depth. In general the temperature and salinity are close to constant at depth, but vary at the surface due to heating and cooling by the atmosphere and freshwater runoff from rivers. This depth-dependence is the dominant feature, and for most applications it may be assumed that the environment is range-independent, i.e. constant in the horizontal.

The Munk profile is an idealized ocean sound-speed profile which captures this depth-dependence and allows us to illustrate many features that are typical of deep-water propagation. In its general form, the profile is given by

$$
\begin{equation*}
c(z)=1500.0\left[1.0+\epsilon\left(\tilde{z}-1+e^{-\tilde{z}}\right)\right] . \tag{1}
\end{equation*}
$$

The quantity $\epsilon$ is taken to be

$$
\epsilon=0.00737
$$

while the scaled depth $\tilde{z}$ is given by

$$
\tilde{z}=\frac{2\left(z-z_{C}\right)}{z_{C}} .
$$

where $z_{C}$ is the depth of the minimum sound speed, the socalled SOFAR Channel depth, typically in the range 1000-1500 m depth.

The resulting profile for $z_{C}=1300 \mathrm{~m} / \mathrm{s}$ is plotted in Fig. 1 .

### 1.2 Ray tracing

As in optics, a ray of sound propagates in a depth-dependent medium according the Snell's law

$$
\begin{equation*}
\frac{\cos \theta(z)}{c(z)}=\text { Const. } \tag{2}
\end{equation*}
$$

where $\theta(z)$ is the grazing angle, i.e. the angle with horizontal for a ray at depth $z$.
For a sound speed profile that is linear in depth, i.e. when temperature and salinity are constant, then this leads to ray paths that are circle section as described below.

For the general sound speed variation, the ray tracing is performed by solving the coupled ordinary differential equations. In cylindrical coordinates $(r, z)$ these ray equations are of the form

$$
\begin{array}{ll}
\frac{d r}{d s}=c \xi(s), & \frac{d \xi}{d s}=-\frac{1}{c^{2}} \frac{d c}{d r} \\
\frac{d z}{d s}=c \zeta(s), & \frac{d \zeta}{d s}=-\frac{1}{c^{2}} \frac{d c}{d z} \tag{4}
\end{array}
$$

where $[r(s), z(s)]$ is the trajectory of the ray in the range-depth plane, with $s$ being the arclength, shown schematically in Fig. 2. Note that we assume here that the sound speed profile is rangeindependent, i.e. $d c / d r=0$. We have introduced the auxiliary variables $\xi(s)$ and $\zeta(s)$ in order to


Figure 1: The Munk sound-speed profile.
write the equations in first-order form. Recall that the tangent vector to a curve $[r(s), z(s)]$ is given by $[d r / d s, d z / d s]$. Thus from the above equations the tangent vector to the ray is $c[\xi(s), \zeta(s)]$.

This set of ordinary differential equations is solved numerically using numerical methods such as Euler's method and Runge-Kutta integration. However, to complete the specification of the rays we also need initial conditions. As indicated in Fig. 2, the initial conditions are that the ray starts at the source position $\left(r_{0}, z_{0}\right)$ with a specified take-off angle $\theta_{0}$. Thus we have

$$
\begin{array}{lll}
r=r_{0}, & \xi=\frac{\cos \theta_{0}}{c\left(z_{0}\right)} \\
z=z_{0}, & \zeta=\frac{\sin \theta_{0}}{c\left(z_{0}\right)} \tag{6}
\end{array}
$$

The source coordinate is of course a given quantity whereas the take-off angle $\theta_{0}$ for the moment is an unknown variable.

### 1.3 Part 3. Acoustic Pressure

The pressure field amplitude along each ray path is

$$
\begin{equation*}
p(s)=\frac{1}{4 \pi}\left|\frac{c(z(s)) \cos \theta_{0}}{c\left(z_{0}\right) J(s)}\right|^{1 / 2} \tag{7}
\end{equation*}
$$

where $J(s)$ is a measure of the relative cross-sectional area of the 'ray tube', varying as the ray propagates, and $\theta_{0}$ is the launch angle of the ray at the source. One can see from the geometry in


Figure 2: Schematic of 2-D ray geometry.

Fig. 3 that the cross-sectional area is just the hypotenuse:

$$
\begin{equation*}
J=r\left[\left(\frac{\partial z}{\partial \theta}\right)^{2}+\left(\frac{\partial r}{\partial \theta}\right)^{2}\right]^{1 / 2} . \tag{8}
\end{equation*}
$$

The extra factor of $r$ accounts for the fact that we have assumed cylindrical symmetry so that Fig. 3 is really just showing a slice through a ray tube that is rotated around the $z$ axis.

The cross-sectional area may be calculated by putting calipers on the ray trajectories, i.e. using finite-difference approximations. This yields the approximation:

$$
\begin{equation*}
J(s)=\frac{r_{i}(s)}{\sin \theta} \frac{r_{i+1}(s)-r_{i}(s)}{d \theta_{0}}, \tag{9}
\end{equation*}
$$

where $r_{i}(s)$ and $r_{i+1}(s)$ are bracketing rays forming a ray tube, and $\theta$ is the local grazing angle at arclength $s$.

As in geometric optics, the rays hitting the surface or seabed will reflect. Assume the reflection is governed by the rule that reflected angle is equal to the incident angle on the interface. For your pressure computation, assume a ray reflected from the seabed undergoes a loss of 6 dB .

### 1.4 Linear Sound Speed Profile

It can be shown [?], that in an ocean with a sound speed that is linear in depth,

$$
\begin{equation*}
c(z)=c(0)+g z, \tag{10}
\end{equation*}
$$



Figure 3: The ray-tube cross section.
the ray paths will be circular arcs. The circles are centered along the line where the sound speed vanishes,

$$
\begin{equation*}
z=-c(0) / g \tag{11}
\end{equation*}
$$

and have an arbitrary radius

$$
\begin{equation*}
R=\frac{1}{a g}, \tag{12}
\end{equation*}
$$

depending on the choice of $a$, as shown in Fig. 4.
The value of a is determined by the initial conditions for the ray. Thus, for a ray launched from depth $z_{0}$ at grazing angle $\theta_{0}$, Snell's law, Eq. 2, requires the sound speed at the deepest point $z_{\text {max }}$ of the array, the turning point, to be

$$
\begin{equation*}
c\left(z_{\max }\right)=c(0)+g z_{\max }=\frac{c\left(z_{0}\right)}{\cos \theta_{0}} \tag{13}
\end{equation*}
$$

yielding

$$
\begin{equation*}
z_{\max }=\frac{c\left(z_{0}\right)}{g \cos \theta_{0}}-\frac{c(0)}{g} . \tag{14}
\end{equation*}
$$

The last term is the depth of the center of the circular arcs, and the first term is therefore the radius, yielding

$$
\begin{equation*}
R=\frac{c\left(z_{0}\right)}{g \cos \theta_{0}} . \tag{15}
\end{equation*}
$$

Thus, the arbitrary parameter $a$ is simply the constant in Snell's law, Eq. 2,

$$
\begin{equation*}
a=g / R=\frac{\cos \theta_{0}}{c\left(z_{0}\right)} . \tag{16}
\end{equation*}
$$



Figure 4: Ray path in a medium with linear sound-speed variation.

Now one can easily compute the raypaths in range and depth. Thus, the arc length is

$$
\begin{equation*}
s=R\left(\theta_{0}-\theta\right), \tag{17}
\end{equation*}
$$

yielding, together with simple projection of the circular arc,

$$
\begin{align*}
r(s) & =R \sin \theta_{0}-R \sin \theta \\
& =R\left[\sin \theta_{0}+\sin \left(s / R-\theta_{0}\right)\right]  \tag{18}\\
z(s) & =R \cos \theta-c(0) / g \\
& =R \cos \left(s / R-\theta_{0}\right)-c(0) / g \tag{19}
\end{align*}
$$

## 2 Lab Assignment

### 2.1 Problem Statement

Power consumption is a critical issue for the deployment of underwater vehicles for long-time surveillanvce in the open ocean. Acoustic communication is the only feasible approach to maintaining connectivity with other underwater nodes and the operators, but each transmission require significant power. A common way of conserving power is to use directional sources and receivers. Thus, instead of transmitting sound in all directions from a point source, the use of a vertical source array allows for steering the transmitted energy into a directional, conical beam of finite width, which, if properly designed, can reduce the power consumption by a couple of orders of magnitude. The same approach is also used for active sonars for target detection and tracking.

The use of directional beams obviously requires that the autonomy system is capable of predicting how a particular directional beam of sound is propagating through the environment, and more importantly solve the inverse problem of determining the vertical transmission angle that will make the sound beam reach the collaborator or the target of interest. The creation of a MOOS process that performes this task is the goal of this Lab assignment.

### 2.2 MOOS Process pCommunicationAngle

The process you will write and demonstrate shall use the navigation information available for the collaborator to compute and publish to the MOOSDB a current estimate of the vertical transmission angle to be used for the communication transmission. Only direct paths will in general be reliable for communication, i.e. paths reflected from the surface or the seabed will not be useful. Therefore the process must contain a check for a direct path existing.

### 2.3 Configuration Parameters

The process shall be configured for an environment with a linear sound speed profile and arbitrary depth, through the following configuration parameters,
surface_sound_speed Sound speed in $\mathrm{m} / \mathrm{s}$ at the sea surface
sound_speed_gradient Sound speed gradient with depth, in (m/s)/m.
water_depth Water depth in m.
time_interval Time interval in seconds between estimates.

### 2.4 MOOSDB Subscriptions

The process shall subscribe to the following MOOS variables, all of which contain values of double:
NAV_X ownship East location in in local metric coordinates.
NAV_Y ownship North location in in local metric coordinates.
NAV_DEPTH ownship depth in meters.

COL_X collaborator East location in in local metric coordinates.
COL_Y collaborator North location in in local metric coordinates.
COL_DEPTH collaborator depth in meters.

### 2.5 MOOSDB Publications

The process shall publish the following variable to the MOOSDB:
ELEV_ANGLE Double representing the estimated elevation angle in degrees, positive up.
LAB05_ANSWER String containing your answer, and your email for identifation, in the format elev_angle=xxx.x,id=user@mit.edu.

Note: In case no direct path exists, the published value must be a NaN .

### 2.6 Testing and Demonstration

A simulated MOOS community named Spermwhale for an underwater vehicle operating in the deep ocean will be provided for testing and demonstration of your process:

Server henrik.mit.edu
Port 9005

You should give your process a unique name during execution, e.g. pCommunicationAngle_userid to avoid interferrence with other student's processes.

The environmental parameters are as follows:

```
surface_sound_speed 1480 m/s
sound_speed_gradient 0.016 (m/s)/m
water_depth 6000 m
```

You may check your solution by comparing the reference solution, published in the MOOS variables:

## ELEV_ANGLE_REF

LAB05_ANSWER_REF

You may use uXMS to check these variables together with the ones you are publishing, using the command
uXMS pCommunicationAngle.moos ELEV_ANGLE ELEV_ANGLE_REF
Note that you need to make your comparison in real time, since the spermwhale vehicle is moving.

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## 2.S998 Marine Autonomy, Sensing and Communications

Spring 2012

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