13.002

Introduction to Numerical Methods for Engineers

Solution to Problem Set 4

$$\begin{bmatrix} e^{-\alpha} & 1 & 0 \\ -1 & e^{-\alpha} & -1 \\ 1 & e^{-\alpha} & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ e^{-\alpha} \\ e^{-\alpha} \end{bmatrix}$$

1. $\alpha = 0$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & -1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$L = \begin{bmatrix} -1 & 0 & 0 \\ -1 & 1 & -1 \\ 1 & 1 & -2 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & -2 \end{bmatrix}, \quad x = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

where $L\mathbf{c} = \mathbf{b}$ and $U\mathbf{x} = \mathbf{c}$ was used. For $\alpha \to \infty$

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

After swithching coulmns 1 and 2

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_2 \\ x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

and adding row3 to row2

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_2 \\ x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad x = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

2. See attach.
3. For α large the soluton becomes ill-conditioned as can be seen from the results attached.
4. Partial pivoting reduced that problem.
5. From the original set of the equations by interchanging colums 1 and 2

$$\begin{bmatrix} 1 & e^{-\alpha} & 0 \\ e^{-\alpha} & -1 & -1 \\ e^{-\alpha} & 1 & -2 \end{bmatrix} \begin{bmatrix} x_2 \\ x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ e^{-\alpha} \\ e^{-\alpha} \end{bmatrix}$$

This particular rearrangement yields a stable solution due to the connection to the physical background of the problem. See attach.

```
% Gaussian elimination
clear
% Examples
%[a]=[2 1 0 0; 1 2 1 0; 0 1 2 1; 0 0 1 2]
 b]=[2;1;4;8]
   ]=[1 -1 0; -1 2 -1; 0 -1 2]
% b]=[1;0;0;]
%[a]=[0 1 0; -1 0 -1; 1 0 -2]
%[b]=[1;0;0;]

n=3;
%alpha is p
p=5
[a]=[exp(-p) 1 0;-1 exp(-p) -1;1 exp(-p) -2]
[b]=[1;exp(-p);exp(-p)]

%partial pivoting 1&2 row
 for j=1:n
  dum=a(2,j);
  a(2,j)=a(1,j);
  a(1,j)=dum;
 end

  dum=b(2);
  b(2)=b(1);
  b(1)=dum;

 a
 b

 elimination
  r k=1:n
 for i=k+1:n
   m(i,k)=a(i,k)/a(k,k);
 for j=k:n
  a(i,j)=a(i,j)- m(i,k)*a(k,j);
 end
b(i)=b(i)- m(i,k)*b(k);
 end
end

a
b

% Back substitution
x(n)=b(n)/a(n,n);

for i=n-1:-1:1
  sum=b(i) ;
      for k=i+1:n
      sum=sum-a(i,k)*x(k);
      end

 x(i)=sum/a(i,i);
 end

  i)=sum/a(i,i)
```

```
>> hwk4_3

%alpha is p
p =

     0


a =

     1     1     0
    -1     1    -1
     1     1    -2


b =

     1
     1
     1


%elimination
a =

     1     1     0
     0     2    -1
     0     0    -2


b =

     1
     2
     0

% Back substitution
x =

     0     1     0

>> hwk4_3

p =

     5

a =

    0.0067    1.0000         0
         0  148.4199   -1.0000
         0         0   -2.9999


b =

    1.0000
  148.4199
         0
```

```
>> hwk4_4

%alpha is p
p =

     0


a =

     1     1     0
    -1     1    -1
     1     1    -2


b =

     1
     1
     1

% patial pivoting rows 1&2
a =

    -1     1    -1
     1     1     0
     1     1    -2


b =

     1
     1
     1


% eliminaton
a =

    -1     1    -1
     0     2    -1
     0     0    -2


b =

     1
     2
     0

% Back substitution
x =

     0     1     0

>> hwk4_4

p =

     5
```