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Massachusetts Institute of Technology<br>Department of Mechanical Engineering<br>2.830/6.780J Control of Manufacturing Processes<br>Quiz 1<br>March 16, 2006<br>Open Book and Notes

Problem 1 (25\%) "Economical" SPC.
The cost and time required to make careful output measurements often determines how and how many samples will be taken.

An injection molding process is used to make small electrical connector components. The parts are made in a 4 -cavity mold; so 4 ostensibly identical parts are made in a single cycle of the machine. Since the parts are made rapidly, and they have to cool before they can be gauged, two competing methods have been proposed:

1. The weight of each part is measured and the four parts are used as a subgroup of $\mathrm{n}=4$ for charting purposes
2. The parts are removed from the sprue and dropped into a bin, from which parts are randomly sampled and used to create charts based on $n=4$
a) (5\%) How well will each of these methods be able to track changes in the part dimensions?
b) ( $10 \%$ ) How will each method affect assessment of the true process capability?
c) (5\%) What assumption(s) of SPC are violated by each method?

In a similar vein, a sheet rolling process is being monitored by making thickness measurements at three locations across the sheet: left, middle and right. These measurements are then used to form sample subgroups of $\mathrm{n}=3$ and charted.
d) (5\%) Comment on how such measurements will conform to or violate the SPC assumptions. What alternative might you suggest? Please explain carefully how the two methods would differ in performance.

## Problem 2 (40\%) Estimated Dimensions.

Consider the output of a process for making wheel shafts for automobiles. As it turns out, the critical dimensions for wheel-bearing fits occur at $\mathrm{W}_{1}$ and at $\mathrm{W}_{2}$. The problem is that it is very hard to measure the shaft diameter at $\mathrm{W}_{2}$. It is proposed that we measure the shaft diameter at each end, points $\mathrm{W}_{1}$ and $\mathrm{W}_{3}$, and assume a linear relationship between them.


|  | Shaft Diameter at <br> point $W_{1}$ | Shaft diameter at <br> point $W_{3}$ | Wheel Bearing |
| :---: | :---: | :---: | :---: |
| $\bar{x}$ | 5.05 | 5.025 | 5.04 |
| $\hat{\sigma}_{x}$ | 0.003 | 0.006 | 0.004 |

Table 1: Process data gathered for a sample size, $\mathbf{n}=15$
a) $(10 \%)$ Determine the mean and standard deviation of the shaft diameter at point $\mathrm{W}_{2}$
b) $(15 \%)$ Determine the percentage of Bearings that will fit on the shaft at point $\mathrm{W}_{2}$ with a clearance of at most 0.012 . Did we buy the right size bearings from our supplier?
c) $(5 \%)$ Find an estimate of the shaft manufacturing process capability, $\mathrm{C}_{\mathrm{pk}}$, at both $\mathrm{W}_{1}$ and $\mathrm{W}_{2}$ given that the specifications are $D=5.02 \pm 0.04$
d) (5\%) Given that $\mathrm{n}=15$, what is the $95 \%$ confidence interval on $\mathrm{C}_{\mathrm{pk}}$ at $\mathrm{W}_{1}$ ?
e) (5\%) Without doing a hypothesis test, comment on whether the two estimates of $\mathrm{C}_{\mathrm{pk}}$ could belong to the same distribution at the $95 \%$ confidence level. Use a sketch to illustrate your answer.

Problem \#3 (35\%) SPC Using Running Averages

The classic Shewhart Chart uses sample averages taken sequentially from the run data for the process. However, this means that if we have a sample size of $n$, the number of points plotted on the chart is $1 / n$ times the number of runs, thereby delaying or masking some of the true process changes. One solution to this problem is to use a running average of the data where a sample size of $n$ is still used, but the running average of $n$ points is plotted, and updated as each new run data point is measured.

Assume that a running average SPC chart is to be constructed with $\mathrm{n}=4$. If we define the run data as a sequence $\mathrm{x}_{i}$ and that running average as the variable $\mathrm{RA}_{\mathrm{j}}$ :
a) $(5 \%)$ Write an expression for calculating $\mathrm{RA}_{\mathrm{j}}$ recursively as each new $x_{I}$ is acquired.
b) $(10 \%)$ What is the expected distribution function $p(R A)$ ? Please explain.
c) (5\%) What are the control limits for the RA chart?
d) $(5 \%)$ If you were to plot both the Shewhart xbar data and RA for the same run data on the same plot, what would it look like? Please provide a sketch of at least 12 run data points to illustrate your answer.
e) $(10 \%)$ Now consider a sudden change in the mean value of the process: i.e.. $\mu_{\mathrm{x}}=\mu_{\mathrm{x}}+\Delta$.. How will the average run length (ARL) of the RA chart (and the subsequent time to detect the mean shift) compare to that of the conventional xbar chart?

