# MASSACHUSETTS INSTITUTE OF TECHNOLOGY <br> Mechanical Engineering Department 

2.71/2.710 Final Exam

May 21, 2013
Duration: 3 hours ( 9 am-12 noon)

CLOSED BOOK

Total pages: 5

Name: $\qquad$

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WITH YOUR SOLUTION SHEET(S)

## Instructions:

Unless otherwise specified, feel free to treat the problems in one spatial dimension, and neglect apertures unless they are explicitly given in the problem statement. Multiplicative amplitude constants may be neglected when not required by the problem statement.

When a problem seeks a numerical result, you may leave it to a reduced fractional form without doing the numerical division, e.g. $4 / 7$ instead of 0.57143 .

When asked to "sketch" an optical system or a function, label your sketch clearly with as much quantitative detail as you can. If you are sketching a complex function, make sure to indicate the real and imaginary parts, or the magnitude and phase.

If a problem appears to be given insufficient data, make assumptions as necessary and state them clearly. When in doubt, make ample use of Occam's razor: "Reasons shall not be multiplied beyond necessity;" i.e., among all possible and adequate explanations the simplest one is the most likely to be correct.

1. ( 25 points) A horse is 2.25 m tall, and it stands with its face 15 m away from the plane of a thin lens camera whose focal length is 3 m .

(a) Determine the location of the image of the horse nose.
(b) What is the orientation of the image in the x and z direction? What is the magnification?
(c) How tall is the image of the horse?
(d) If the tail of the horse is 17.5 m from the lens, how long is the image of the horse (from nose to the tail)?
(e) If the lens has a diameter of 30 cm and we can neglect other apertures, what is the depth of focus of the image of the horse taken by the camera?
2. ( 20 points) The imaging system shown below was designed to be a telescope comprised of two identical positive thin lenses $\mathrm{L}_{1}, \mathrm{~L}_{2}$ of focal length $f$ and a pupil plane aperture S of full width $a$. However, due to an error during assembly, $\mathrm{L}_{2}$ was placed at distance $f+\Delta$, where $|\Delta|$ « $f$ from the pupil plane.
a) Using Geometrical Optics, locate the image plane for the defocused telescope.
b) Recalculate the image plane location using Fourier Optics and verify your result from question (a).

3. ( 25 points) The following figure illustrates a setup used for testing plano-convex lenses using a famous interference effect, known as Newton's rings. A plano-convex lens ( $\mathrm{n}=1.5$, thickness $\mathrm{t}=1 \mathrm{~cm}$, and radius of curvature $\mathrm{R}=20 \mathrm{~cm}$ ) is placed on an flat mirror, and illuminated at normal incidence with a plane monochromatic light at wavelength $\lambda=500 \mathrm{~nm}$. For the approximation, assume the path difference between the two reflected beams (one at the curved surface of the plano-convex lens and the other at the mirror surface) is much smaller than the radius of curvature R of the lens, so we are looking at the inner zones of Netwon's rings.

a) Assume both the plano-convex lens ( $\mathrm{n}=1.5$ ) and the beam splitter are large enough to accommodate the entire lateral width of the expanding paraxial wave, write an analytical expression for and sketch the interference pattern as function of the coordinate $x$ at the observation plane (CCD detector).
b) Calculate the radius of 10th bright ring on the CCD detector.
c) ( 2.710 only) How does the interference pattern change if the plano-convex lens is tilted by a small angle $\alpha$ ?
4. (30 points) We are given a 4F imaging system consisting of two identical lenses $\mathrm{L}_{1}, \mathrm{~L}_{2}$ with focal length $f=10 \mathrm{~cm}$. As an experiment, you attached a tilted grating as amplitude mask at the pupil plane of the imaging system. In all questions below, the illumination is assumed to be at wavelength $\lambda=0.5 \mu \mathrm{~m}$. Assume spatial period $\Lambda=10 \mu \mathrm{~m}$, stripe size $d=2 \mu \mathrm{~m}$, tilt $\theta=$ $30^{\circ}$ with respect to the aperture, and edge lengths $a=5 \mathrm{~mm}, b=3 \mathrm{~mm}$.

Hint: First calculate the Fourier transforms of the tilted grating and the aperture individually. Then use the convolution theorem.


Figure 2. 4F imaging system with a tilted grating as amplitude mask (dark area is completely opaque, and white area is transparent) placed at pupil plane( $\mathrm{X}, \mathrm{Y}$ ).
a) Let's consider a coherent plane wave illuminating the input plane. Please calculate the output field and sketch the intensity pattern.
b) Now a coherent point source is placed at the origin of the input plane ( $\mathrm{x}=0$ ), please calculate the point spread function (PSF) and sketch the Modulation Transfer Function (MTF).
c) ( 2.710 only) When the illumination is spatially incoherent, please calculate and plot the intensity point spread function (iPSF).

Good Luck and Enjoy your Summer Time!

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### 2.71 / 2.710 Optics

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